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"Health and Development with Imperfect Capital Markets"

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Health and Development with Imperfect Capital Markets

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Abstract

Preliminary draft

This paper investigates the interaction between investment in education and in life-expanding investments, in a simple two-period model in which individuals are liquidity constrained in the first period. We show that under low levels of health and capital, investments in human capital and in health are complement: since the probability of survival is small, there is little incentive to invest in human capital; therefore the return on health investment is also low. This reinforcing effect does not hold for higher levels of health or capital, and the two investments become substitute.

This property has many consequences. First, subsidizing health care may have dramatically different effects on private investment in human capital, depending on the initial level of health and capital. Second, the assumption that mortality is endogenous induces an increase in inequality of income: since health investment is a normal good, the return on education is also lower for poor individuals. Third,

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in a non-overlapping generation model with non-altruistic agents, the health level of the population has strong consequences on growth. For a very low level of health, mortality is too high for the investment on education to be profitable. For a higher, but still low, level of health the economy grows only if the initial stock of capital is high enough; bad health and low capital create a poverty trap. Fourth, we compare redistributive income policies versus public health measures. Redistributing income reduces both static and dynamic inequality, but slows growth. In contrast, a paternalistic health policy that forces the poor to invest in health reduces dynamic inequality and may foster growth.

Age Structure of Deaths				
age	< 5	5-19	20-64	65+
Developed market economies	1 %	1 %	21 %	77 %
Least developed countries	4 %	15 %	29 %	16 %

Table 1. Year 1995, source : World Health Report. WHO, 1998.

1 Introduction

In developed countries, it is often considered that additional health expenditures have a larger effect on the quality of life, which would be a luxury good, than on the quantity of life, which would be a necessary good (Newhouse, 1982). Moreover, the continuing upward trend in life-expectancy at birth in developed countries has been mostly due to reductions in mortality at higher ages.

In sharp contrast to that situation, mortality at young age is still very high in many developing countries (See Table 1). Infectious diseases are responsible for a large amount of total deaths, up to 50 % in some countries (See Table 2). Many of these diseases are essentially preventable by vaccination or sanitation measures, and mandatory vaccination programs can be shown to be socially efficient in some, though not all, situations (Geoffard and Philipson, 1997).

The effect of health on development has been extensively looked at. One particular element that has been given a considerable attention is the link between workers health condition and productivity[1]. The basic assumption of that literature is that health, and in particular nutrition, can affect productivity; therefore, a sufficiently high wage may help workers become more productive and hence be efficient, thereby reverting the traditional assumption that productivity determines earnings, and not the opposite. However, from an empirical point of view, as Straus and Thomas (1998) put it, it seems that the case for the existence of health-based efficiency wages is fairly weak.

[1]See in particular Leibenstein (1957), Stiglitz (1976), Dasgupta and Ray (1986,1987). Straus and Thomas (1998) provide a survey of this literature, its empirical issues and results.

Causes of Deaths			
	Infectious and parasitic diseases	Perinatal and maternal causes	Diseases of the circulatory system
Developed	4 %	1 %	46 %
Developing	43 %	10 %	24 %

Table 2. Year 1997, source : World Health Report, WHO, 1998.

Besides the effect on instantaneous welfare and productivity, another important element of health is longevity. The fact that mortality is endogenous, in the sense that health consumption affects mortality rates, is well documented[2] Indeed, the link between longevity and development has most often focused on the effects of development on mortality. Sen (1998) advocates the use of mortality data as a reliable *indicator* of economic, or social success.

But the reciprocal, that mortality rates affect development has been less systematically investigated; in particular, the effect that the mortality rate is itself endogenous, in that it depends on private and public investments in health, has not been identified as a key element.

The other element of human capital that has been extensively looked at is education (Handbook of Development Economics, 1995), in particular the effects on education on growth and development, as well as empirical measures of the private rate of return on education. A key aspect is that, even with high rates of return, imperfections in credit markets may prevent poor individuals from making profitable (both privately and socially) investments in education.

The present paper links these two aspects together, namely the fact that both education and longevity are endogenous, in a simple two-period model. We claim that mortality rates may have an important effect on development because of a crossed-effect with investment in education. The return on education investment depends in an essential way on the mortality rate, that determines the probability the individual may benefit from the investment.

We claim that this reinforcing effect may have strong implications, both

[2]Literature on endogenous mortality: effects on social insurance programs: Philipson and Becker (1998); effect on pricing and R& D: Geoffard and Philipson (1998). Literature linked to endogenous discounting in choice theory: Becker and Mulligan (1997), Geoffard (1996).

positive and normative.

In section 2, we present a two period model in which both investments in health and human capital are choice variables, and are traded off against consumption. In addition to the assumption that mortality is endogenous, we also assume that credit markets are imperfect. We characterize the solution to the individual maximization problem and show how the initial level of health and capital determine the complementarity/substitution relationship between the two kinds of investment.

A first consequence of that property is that individuals with different initial capital levels will have different mortality rates, even with identical health conditions at birth. Section 3 links this result to the literature on endogenous discounting, and examines some first consequences. In particular, subsidies to health care may have dramatically different effects on private investment in human capital, depending on the initial level of health and capital.

In section 4, we introduce a non overlapping generation model with non-altruistic agents who live at most two periods. The assumption that mortality is endogenous is a factor of dynamic inequality of income: since health investment is a normal good, the return on education is lower for poor individuals, and therefore the expected growth rate of human capital is positively linked to the initial capital stock. In an economy with a representative agent and a positive depreciation rate of capital, the health level also has important consequences on growth. For a very low level of health, mortality is too high for investment on education to be profitable, and capital decreases to zero. For a higher, but still low, level of health the economy grows only if the initial stock of capital is high enough; bad health and low capital create a poverty trap.

Lastly, section 5 compares redistributive income policies versus public health measures. Redistributing income reduces both static and dynamic inequality, but slows growth. In contrast, a paternalistic health policy that forces the poor to invest in health reduces dynamic inequality and may foster growth.

2 The model

We first specify an individual choice model in a context where both educational decisions and investments in health are endogenous. An important result is that health and education may be complement or substitute goods, depending on the (exogenous) income and health level of the agent at birth.

We then consider an economy populated by a continuum of agents who differ in terms of initial income. Implications of the endogenous mortality model range from the dynamics of inequality to growth and poverty traps, and the relative efficiency of income redistribution and public health programs in terms of inequality reduction and/or growth.

2.1 The individual

2.1.1 Health and endogenous mortality

At the beginning of period 1 (i.e., at birth), an agent is endowed with an exogenous human capital stock k, and an exogenously determined health level $h \in \mathbb{R}^+$.

In the first period the agent receives a wage w normalized to 1[3], so that his revenue in period 1 is equal to $wk = k$.

In period 1, the agent may invest a share x of his total income in human capital, and a share s in health. Consumption in period 1 is equal to:

$$c_1 = k(1 - x - s).$$

Health is a capital good (Grossman [?], Ehrlich and Chuma [?]). The agent may either live one or two periods, depending on his health capital at the beginning of the second period.

We assume that the technology of production of health, that is, transformation of spending $d = ks$ into health, has constant return α. If the agent has an initial health level h, his health capital after investment is equal to $h + \alpha ks$. At the beginning of the second period, the agent is submitted to a random additive negative health shock θ, distributed according to some

[3]We assume that per capita productivity is equal accross agents, and in particuliar does not depend on health. This can be a very strong assumption, especially for poor economies where nutrition quality may be related to productivity [?].

constant cumulative distribution G. After spending and shock the health level of the agent, h', is equal to :

$$h' = (h + \alpha d) - \theta.$$

According to the theory of health capital, the individual survives in period 2 if and only if h' is above some critical value h^s. This condition determines a critical value $\theta^s = (h + \alpha d) - h^s$ for the shock θ ; this value is increasing in h, d, and decreasing in critical health level h^s. The probability to survive, conditional on (h, s) is denoted by π :

$$\pi \equiv P(h' > h^s | h, s) = P(\theta < \theta^s(h, s)) = G(\theta^s(h, s)).$$

The health level after spending and before shock is $(h + \alpha ks)$. For a given exogenous value of h^s, the probability of survival π is a function of $h + \alpha ks$.

2.1.2 Education

When he is still living in period 2, the individual receives an income of ϕk, where ϕ is the educational achievement reached in period 1. The cost paid in period 1 to reach $\phi \geq 1$, measured in units of the consumption good, is denoted $x(\phi)$. The agent dies at the end of period 2, independently of his health capital. Therefore in period 2, there is no investment in health nor in education, and the income is entirely consumed: $c_2 = k\phi$. The utility of the agent is supposed to be linear in consumption and his lifetime expected utility is equal to: $U = c_1 + \pi c_2$, i.e. :

$$U(\phi, s) = k\,(1 - x(\phi) - s) + \pi\,(h + \alpha ks)\,k\phi.$$

The agent maximizes lifetime utility over (ϕ, s), under the following physical constraints: $s \geq 0$ (the agent may not "sell" his health); $\phi \geq 1$ (the agent may not "unlearn"). Moreover, we assume that there are no capital markets in this economy: the agent cannot borrow nor lend and c_1 must remain positive:

Assumption 1 (No credit mqrket) *Any admissible (ϕ, s) must satisfy:*

$$1 - x(\phi) - s \geq 0.$$

The following assumption specifies the cost function x of investing in education:

Assumption 2 (Quadratic cost of education) *We assume that the cost function x is quadratic, i.e.:*

$$x(\phi) = \frac{\beta}{2}(\phi - 1)^2.$$

Under these two assumptions, we denote $e(s)$ the maximum educational level attainable by an agent spending s in health:

$$e(s) \equiv x^{-1}(1-s) = 1 + \sqrt{\frac{2(1-s)}{\beta}}.$$

The borrowing constraint writes: $\phi \leq e(s)$.

Therefore, if we denote $B \equiv \{(\phi, s) | s \geq 0, \phi \geq 1. \phi \leq e(s)\}$ the agent's "reduced form" budget set, the maximization problem writes[4] :

$$\max_{(\phi,s)\in B} U(\phi, s) = k\left[(1 - x(\phi) - s) + \pi(h + \alpha k s)\phi\right]. \tag{1}$$

So far, we have interpreted π as the probability to survive in the second period. An alternative interpretation in terms of quality of life (or morbidity) instead of mortality may be given. If health and other consumption goods are complement (individuals enjoy more intensively consumption of non-health goods when they are in good health), we may assume that in period 2 the utility of consuming c_2 when being in health state h'' is equal to $v(h'')u(c)$ with u linear; then with $\pi(h, s)$ denoting the expected value $E[v(h'')|h, s]$, the *ex ante* utility is formally identical.

A third alternative would be a mix of the two, in which case π would represent the probability to live in good health in period 2.

In order to make sure that the problem (1) has indeed a solution, the following assumption on π will imply concavity of the objective function U:

Assumption 3 (Concavity) *For any h:*

$$\beta\pi''(h) + \pi'^2(h) < 0.$$

[4]Notice, however, that the objective need not be monotonic in s or ϕ; therefore the solution need not be on the boundary of the budget set.

According to this assumption, the probability of survival must be sufficiently concave in health (especially for small values of β). This assumption implies two technical properties:

1. For any h, $\pi''(h) < 0$;
2. For any $h, \phi \geq 1$, $\beta\pi''(h)\phi + \pi'^2 < 0$.

The first property indicates that health investments have decreasing returns. The second property implies that the objective is concave, since the hessian matrix of U is:

$$\begin{pmatrix} (\alpha k)^2\pi''\phi & (\alpha k)\pi' \\ (\alpha k)\pi' & -\beta \end{pmatrix};$$

and this matrix is semi-definite negative if and only if: $-\beta\pi''(h)\phi - \pi'^2(h) \geq 0$.

2.1.3 Characterization of the solution

Under assumptions 2 and 3, the solution to the maximization problem (1) depends on k and h, and may be characterized as follows.

First notice that under a quadratic cost function for investment in education, we have: $U_\phi|_{\phi=1} = \pi(h+\alpha ks) > x'(1) = 0$, and therefore $\phi = 1$ is never optimal; the agent always invests some strictly positive amount $x(\phi^*)$ in education, and consequently we also have $s^* < 1$. Second, under assumption 3, the problem is globally strictly concave, and therefore admits a unique solution; however, the solution may either be interior, or a corner solution with either $s^* = 0$, $\phi^* = e(s^*)$, or both. As proposition 1 below makes clear, this defines four regions in the (h, k) plane.

The following lemma, proved in the appendix, characterizes the optimal investment in health s^* whenever it is interior, either with $c_1^* > 0$ (case $s^* = s^1$) or $c_1^* = 0$ (case $s^* = s^2$):

Lemma 1 *Define the following functions of* (h, k)*:*

1. $s^1(h,k)$ *is the solution, when it exists, to:*

$$-1 + \alpha k\left(1 + \frac{1}{\beta}\pi(h + \alpha ks)\right)\pi'(h + \alpha ks) = 0;$$

2. $s^2(h,k)$ *is the solution, when it exists, to:*

$$-\frac{1}{\sqrt{2\beta(1-s)}}\pi(h+\alpha ks)+\alpha k\pi'(h+\alpha ks)e(s)=0.$$

Then for $i=1,2$, s^i *decreases with* h, *and* $ks^i(h,k)$ *increases with* k.

PROOF: See Appendix.

The limit conditions $s^1(h,k)=0$, $s^2(h,k)=0$, and $s^1(h,k)=s^2(h,k)$ partition the plane (h,k) into four different regions, depending on whether s^* and ϕ^* are interior or not.

Lemma 2 *Set* $k^0\equiv\beta/(\alpha\pi'(0)(\beta+\pi(0)))$, $h^c\equiv\pi^{-1}(\sqrt{2\beta})$, *and* $k^c\equiv\beta/(\alpha\pi'(h^c)(\beta+\pi(h^c)))$. *Define the following functions of* k:

1. *For* $k^0\le k\le k^c$, $h=h^1(k)$ *is the solution to* $s^1(h,k)=0$;
2. *For* $k^0\le k\le k^c$, $h=H(k)$ *is the solution, when it exists, to* $s^1(h,k)=s^2(h,k)$;
3. *For* $k^c\le k$, $h=h^2(k)$ *is the solution to* $s^2(h,k)=0$.

Then: h^1 *is increasing with* k, *and such that* $h^1(k^0)=0$ *and* $h^1(k^c)=h^c$; H *is such that* $H(k)\le h^1(k)$ *and* $H(k^c)=h^c$; h^2 *is increasing with* k, *and such that* $h^2(k^c)=h^c$.

PROOF: See Appendix.

The positive orthant $\{h\ge 0,\ k\ge 0\}$ is partitioned into 4 regions A, B, C, and D, defined formally by:

region A: $\{(h,k)\mid h\ge h^c,\text{ and } h\ge h^2(k)\}$
region B: $\{(h,k)\mid k\le k^c \text{ and } h\le H(k),\text{ or } k\ge k^c \text{ and } h\le h^2(k)\}$
region C: $\{(h,k)\mid h\le h^c, k\le k^c \text{ and } H(k)\le h\le h^1(k)\}$
region D: $\{(h,k)\mid h\le h^c, k\le k^c \text{ and } h\ge h^1(k)\}$

The different regions are represented in figure 1. The following proposition fully characterizes the solution (s^*,ϕ^*) of our maximization problem in the plane (h,k):

Proposition 1 *The optimal solution* (s^*, ϕ^*) *is given by:*

-in region A:	$s^* = 0$	$\phi^* = e(0)$	$c_1^* = 0$
-in region B:	$s^* = s^2(h,k)$	$\phi^* = e(s^2(h,k))$	$c_1^* = 0$
-in region C:	$s^* = s^1(h,k)$	$\phi^* = 1 + \pi[h + \alpha k s^1(h,k)]/\beta$	$c_1^* > 0$
-in region D:	$s^* = 0$	$\phi^* = 1 + \pi(h)/\beta$	$c_1^* > 0$.

The intuition of proposition 1 is best illustrated in figure 1. In region A, (s^*, ϕ^*) is at the upper corner of the budget set. Individuals start with high enough initial health but may not be very rich. Without investment in health, their probability of survival is already reasonably high. Therefore the expected return to education is also initially important. Given the fact that these individuals face a relatively strict liquidity constraint, it is optimal for them not to invest in health and allocate all their income to human capital investment.

Individuals in region B are on the other hand relatively rich but may start with a relatively low level of initial health. In order to increase their probability of survival, it is then optimal to allocate their income between the two types of investment (human capital and health), and still consume $c_1^* = 0$; (s^*, ϕ^*) is on the upper boundary of the budget set. However because of decreasing returns in the probability of survival, the return to health investment decreases faster than the return to human capital so that at the optimum, individuals in region B stop to invest in health at a level $s_2(h,k)$ and allocate all the rest of their income to education.

Clearly individuals in region A or B are liquidity constrained (ie. consume $c_1^* = 0$) because of our assumption of linear utility of consumption and constant returns to education in the initial level of human capital k. Under a standard concave utility function for current consumption or decreasing returns in k in human capital investment, individuals in region B would consume positively in the first period. However, though this is illustrated in the present model in a very stark way, we may expect that the effect illustrated in this region remains. That is, as we move into region B, the trade-off between investment in health and education gets strongly in favor of education.

Interestingly, the solution in regions C and D is quite different and reflect a complementarity between health and education. Those regions correspond to situations of poor individuals starting with low initial health. In these situations, the probability of survival is not high.

Consider first region D, in which (s^*, ϕ^*) is on the left boundary of the budget set. On one hand, individuals are so poor that, whatever the amount of investment in health they may be able to do, their chance of survival remains small. Therefore the expected return to education remains also relatively small and there are little incentives to invest in human capital. On the other hand, little investment in education and a poor initial level of human capital implies that, if he survives, an individual will enjoy a low level of consumption in period 2. This, in turn, implies a low return to health investment and the fact that it does not pay to invest in health to increase the probability of survival. Hence in this region, low investment in health and education are self reinforcing; there is no investment in health, low human capital investment and a high mortality rate.

Indeed, individuals in region D would even be willing to "sell their health," and decrease their probability to survive, if such a sale could help them increase their current consumption; but this possibility would reduce even further their investment in education, since in that region health and education are complement.

Region C reflects a similar pattern. However in this region, the initial income is high enough, or the initial health level is sufficiently low, so that the return on health investment becomes high enough to be profitable. Consequently the probability of survival is also higher and there is more investment in education. Again, health and education are complement.

3 Income, mortality, and investment horizon

Notice that in both regions C and D, individuals end up with a positive level of consumption in the first period and are *not* liquidity constrained (neither in their health nor in their human capital investment). This result may seem strange at first sight as one would intuitively expect poor individuals to be more likely to face liquidity constraints. This is due to the fact that in the present framework the mortality risk, that determines the investment horizon (the patience), is endogenous and, as the following proposition makes clear, decreases with income.

Proposition 2 *Health is a normal good: for a given health level h, the total amount invested in health $ks^*(h,k)$ increases with k; therefore, the probability of survival increases with income.*

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PROOF:

Take $h < h^c$. For k low, s^* is equal to zero (region D). In region C, $s^* = s^1$ and lemma 1 proved that $ks^1(h,k)$ was increasing with k; the same holds in region B where $s^* = s^2$; moreover, for $h = h^1(k)$ (boundary between D and C) $s^1(h,k) = 0$, and for $h = H(k)$ (boundary between C and B) $s^1(h,k) = s^2(h,k)$. Therefore $s^*(h,k)$ is continuous in k, and by continuity $ks^*(h,k)$ never decreases. The same argument holds when $h \geq h^c$.

Since the probability of survival is equal to $\pi(h + \alpha k s^*(h,k))$, it also increases with k.

□

This proposition establishes a first empirically testable implication of our assumption that mortality is endogenous. An exogenous mortality corresponds to the assumption that $\alpha = 0$, so that the mortality rate is determined by the health stock at birth, independantly of income. Under an exogenous mortality rate, the empirically observed (REF : Sen) correlation between income and life expectancy may only be due to a correlation between health condition at birth and income; under endogenous mortality rates, even individuals with *identical* initial health condition at birth would nevertheless live longer the higher their income.

Notice that if individuals differ not only in terms of income k but also in terms of health production efficiency α, then the same result holds. It has been argued, in particular, that education programs targeted to mothers could increase children health, and reduce child mortality (REF). These programs might be interpreted as a way to increase α. Notice however that an increase in α has no effect on the poorest individuals, who do not spend at all in health for reasons detailed above. Therefore such educational programs would prove useful only for sufficiently rich households. To make it short, training people to use some resource more efficiently is void if they do not have this resource.

Also notice that in the present framework, poor individuals have, at the optimum, a low probability of survival and therefore are *endogenously impatient*; this is the reason why such individuals are not liquidity constrained, and prefer to consume immediately the rest of their income.

Our model therefore provides the example of a situation in which poor enough individuals (ie. low k) are endogenously myopic, invest little in health and education while, on the contrary, rich individuals (ie. high enough k) are more forward looking and ready to invest a larger fraction of their income

in education and possibly in health. This result is closely related to recent work on endogenous time preference (Becker and Mulligan 1997). Becker and Mulligan assume that individuals can devote some share of their initial wealth to "project themselves in the future." By devoting more resources to this mental exercise, agents can "buy" a lower discount factor and become more patient. A consequence of that theory is that it is the rich who are patient (and therefore save more and become even richer), and not the patient who are rich. We do obtain a similar interpretation, but in our model this phenomenon is a consequence of the endogeneity of mortality, and not of any mental exercise that may be, to say the least, difficult to observe and test empirically.

It also illustrates well the idea developed by social scientists like Comte and others (?) that, poor individuals, because intrinsically of their material conditions, are less future oriented than rich people, justifying therefore some kind of social paternalism from a policy point of view. In section (?) we discuss the implications of this result for redistributive policies.

3.1 Health and Education as Complements or substitutes

A central and immediate implication of the previous discussion provides circumstances under which health and education investments as likely to be complements or substitutes. More precisely:

Proposition 3 *Investment in education and health are substitute in regions A and B, and are complement in regions C and D.*

As discussed above, education and health investment are more likely to be complements for low levels of initial health and income and to be substitutes for high enough initial health and income.

3.1.1 Cross price and subsidies effects

Proposition 3 has immediate consequences in terms of cross prices effects and subsidization policies. To see this, consider first an increase in β reflecting an increase in the price of education. It is a simple matter to see that (abstracting from the fact that the boundary between the different regions is also affected by changes in β):

Corollary 1 *In regions C and D,* $\frac{\partial \phi^*}{\partial \beta} < 0$ *and* $\frac{\partial s^*}{\partial \beta} < 0$, *while in regions A and B,* $\frac{\partial \phi^*}{\partial \beta} < 0$ *and* $\frac{\partial s^*}{\partial \beta} > 0$.

Similarly consider that the price of health investment changes from 1 to $1 + \tau$; then:

Corollary 2 *In regions C and D,* $\frac{\partial s^*}{\partial \tau} < 0$ *and* $\frac{\partial \phi^*}{\partial \tau} < 0$, *while in regions A and B,* $\frac{\partial s^*}{\partial \tau} < 0$ *and* $\frac{\partial \phi^*}{\partial \tau} > 0$.

From a policy point of view, corollaries 1 and 2 imply that, depending on the initial level of health and income, the effect of subsidization of one type of investment has quite different effects on the pattern of investment in the other variable. For poor initial health and low income individuals, subsidizing one type of investment (education or health) positively affects the other investment. For initially healthy and rich individuals, on the opposite, subsidizing one type of investment generates a decrease in investment of the other type.

4 The economy

Our model of investment in health and human capital also generates implications for growth and development. Again the complementarity between health and education at low levels of income will play an important role.

In this respect, the model can be simply extended in the following way. Consider a non overlapping generation structure of individuals living at most two periods (1 and 2). The population is made of I different "families." Each family i has a population L_t^i normalized to 1[5]. At the beginning of their life, agents of generation t and family i are endowed with a stock of human capital k_t^i and an identical level of initial health h. The individual then solves the optimization problem (1) with $k = k_t^i$. This provides an expected stock of human capital for the second period equal to: $E(k_{2,t}^i) = g^*(h, k_t^i)k_t^i$, where $g^*(h,k)$, the gross expected growth rate of human capital, is equal to:

$$g^*(h,k) = \pi[h + \alpha k s^*(h,k)]\phi^*(h,k).$$

[5]We implicitly assume, for simplicity, that fertility rates adapt to differences in mortality rates, so that each family reproduces itself exactly from one generation to the other.

We also assume that. because of social spillovers and externalities, the next generation's initial stock of human capital depends on the effective amount of human capital generated by the previous generation still alive:

$$k^i_{t+1} = E(k^i_{2,t}) - \delta k^i_t = g^*(h, k^i_t)k^i_t -$$

where δ is the depreciation rate of human capital from one generation to the next.

We assume that the individuals of a given generation are not altruistic, i.e., they only maximize their own lifetime utility[6].

4.1 Inequality dynamics

We first study the evolution of the distribution of income between period t and period $t+1$.

The fact that, if mortality is endogenous, health and education are complement for poor individuals (other things held constant), has important consequences in terms of inequality dynamics. The following proposition shows that the the inequality of income distribution increases between one generation to the next:

Proposition 4 *The gross rate of growth of expected human capital $g^*(h,k)$ is:*

- *constant in regions A and D*
- *increasing with k in the regions B and C.*

PROOF:

In regions A and D, $s^* = 0$ and therefore π is independent of k, as well as ϕ^* which is equal to $e(0)$ (region A) or $1 + \pi(h)/\beta$ (region D).

In region B we have $s^* = s^1$, and $\phi^* = (1 + \pi(h + \alpha k s^1)/\beta)$. Since ks^1 increases with k, so do π and ϕ^*.

In region C, consumption is equal to 0 independently of k. Therefore the value of the problem is equal to $\pi\phi^*$; since the objective function increases with k, the envelope theorem implies that the value of the problem also does, which proves the result.

[6]Assuming that bequests enter the utility function of each individuals would probably change the dynamics of growth and inequality, but this point is left for future research.

□

The fact that the expected human capital grows at a higher rate when income increases is, again, a consequence of endogeneity of mortality. When π does not depend on s (the limit case $\alpha = 0$), then all agents reach a similar educational level ϕ determined by the first order condition $x'(\phi) = \pi(h)$ if $h < h^c$, and by their borrowing constraint $x(\phi) = 1$ if $h \geq h^c$; therefore the expected growth rate of human capital is equal to $\pi(h)\min\{e(0), 1+\pi(h)/\beta\}$, and is independent of k.

Proposition 4 generates important implications in terms of income inequality dynamics, evaluated in expected terms (i.e., ex ante). We have for any family i:

$$\frac{E(k_{t+1}^i)}{k_t^i} = g^*(h, k_t^i) - \delta.$$

Thus, proposition 4 implies:

Corollary 3 *Consider two families i and j initially endowed with human capital $k_1^i < k_1^j$ and identical initial health h. Then for any t:*

- if (h, k_1^i) and (h, k_1^j) belong both to region D or region A:

$$\frac{E(k_{t+1}^i)}{k_t^i} = \frac{E(k_{t+1}^j)}{k_t^j}$$

- Otherwise:

$$\frac{E(k_{t+1}^i)}{k_t^i} < \frac{E(k_{t+1}^j)}{k_t^j}.$$

Apart from the case in which there is no investment in health (both individuals in regions A or D), this result illustrates a tendency to an increase in inequalities in human capital over time. The stock of human capital in ex ante terms tends to regress away from the mean. Obviously, in a model in which there are also decreasing returns in k in the technology of accumulation in human capital, there would be an opposite force tending to make the stock of human capital to regress to the mean. Our result however shows that there is also an effect which generates social polarization in human capital between, for instance, individuals starting with a level of human capital in regions D or A and individuals with an initial level of human capital in regions C or B.

Note also that this result comes crucially from the fact that the probability of survival is endogenous. As noticed above, in the case of exogenous mortality rate, the gross rate of growth of expected human capital $g^*(h,k)$ is equal to $\pi(h)\min\{e(0), 1+\frac{\pi(h)}{\beta}\}$, and does not depend on k. Hence two families i and j initially endowed with different human capital $k_1^i < k_1^j$ but identival initial health stock h would expect the same expected growth rate of human capital.

4.2 Growth and poverty traps.

We assume here that there is a single family in the economy, and study growth in such a representative agent context. Again a simple application of proposition 4 gives:

Corollary 4 *Denote*

$$\theta(h) \equiv g^*(h,0) = \pi(h)\min\{e(0), 1+\frac{\pi(h)}{\beta}\}.$$

Then since $g^*(h,k)$ is increasing with k and bounded above by $\max_h\{\pi(h)\}e(0)$, we may define:

$$g^*(h) \equiv \lim_{k\to+\infty} g^*(h,k),$$

and we have that $g^*(h) > \theta(h)$. The long run growth dynamics is given by:

- *If $\delta < \theta(h)$, the economy grows indefinitely with a growth rate converging to a finite value $g^*(h) - \delta$;*
- *If $\theta(h) < \delta < g^*(h)$, there exists a threshold level $k_T > 0$ such that: 1) for $k_{10} < k_T$ the economy converges to $k_\infty = 0$, 2) for $k_{10} > k_T$ the economy grows indefinitely with a growth rate converging to a finite value $g^*(h) - \delta$;*
- *If $g^*(h) < \delta$, the economy converges to $k_\infty = 0$.*

The most interesting case is obviously the case of an intermediate depreciation rate δ, such that $\theta(h) < \delta < g^*(h)$. In that situation, depending on the initial condition k_{10}, there is the possibility of a poverty trap. When

the economy is poor initially (and initial health h not too large), health and education are complements. Because of bad initial health conditions, the expected profitability of education in the economy is low, therefore there is little investment in human capital. As well, because of low education and low initial incomes, expected incomes in the future are also low and incentives to increase the probability of survival are conversely weak. Poor enough economies then get caught into a poverty trap with low health , high mortality and little human capital accumulation. On the contrary, when the economy is rich enough, then it starts to invest in health, increasing the prospects to survive and triggering an ever increasing process of human capital accumulation.

The third case corresponds to a case in which the maximal expected gross rate of human capital is lower than its depreciation rate; since $g^*(h) < \max_h\{\pi(h)\}$, this situation occurs in particular when the survival probability is always relatively small; this may give countries with endemic health problems such as high prevalence of infectious diseases a strong and reinforcing growth disadvantage.

Corollary 5. *Redistributive policies and public health*

A final implication of the complementarity between human capital and health at low levels of incomes and their substituability at higher levels concerns the impact of transfers, redistributive policies and public health measures.

Consider first transfers and direct redistributive policies. Suppose that one wants to increase to some extent the initial income level of a poor individual with characteristics (h,k) in region D who does not invest in health and has a low rate of human capital accumulation. The important insight of proposition 3 is the fact that at this income level k, health and education are so strongly complementary that the individual is not liquidity constrained. Hence any marginal increase in his income level k will not increase his investment in health nor education, but will be allocated to a consumption increase in period 1. This actually will certainly increase the well being of the individual but will not affect the growth rate of the economy and possibly can be detrimental to the future generations if there are social intergenerational spillovers of transmission of human capital and transfers are financed by individuals with higher income.

At the same time, a marginal decrease in income of an individual in any of the other regions will reduce, by proposition 4 his or her expected stock of human capital in the next period $E(k_2) = g^*(h, k_1)k_1$. The implication of this is the fact that a marginal direct redistribution of income from a individual b in region C, B, A to a poorer individual a in region D will reduce the human capital investment of b and will not increase the human capital of a. Hence we get the conclusion that:

Corollary 6 *A marginal redistribution of income from an individual of characteristics (h, k) in regions C, B, or A to a poorer individual of characteristics (h', k') in region D is detrimental to aggregate expected growth.*

Consider now a public health measure such that the government provides publicly a minimum level of health $h_{\min}$ in society. Then, for an individual characterized by (h, k) in region D with $h < h_{\min}$, as long as the new charcteristics $(h_{\min}, k)$ remain in D, that individual will not privately invest in health. However, because of a higher probability of survival, his investment in human capital will be increased from $\phi(h) = 1 + \frac{\pi(h)}{\beta}$ to the higher level $\phi(h_{\min}) = 1 + \frac{\pi(h_{\min})}{\beta}$. For an individual with a level of initial health h larger than $h_{\min}$, and abstracting from taxes used to finance the public health scheme, nothing is changed in the program of the agent. Hence his or her investments in health and education remain the same. Thus:

Corollary 7 *As a whole, a public health program ensuring a minimun health level improves the accumulation of human capital in the economy.*

Obviously, in reality,one may expect taxes to be distortionary and to affect also negatively the choice in health and education for rich individuals. Still, the important aspect which remains is the fact, in order to improve the average level of education in this economy, it is better to provide first health to the poor than to make a direct income redistribution.

6 Conclusion

To be completed.

A Proofs

A.1 Maximization over ϕ

The partial derivatives of U write (after dividing by k):

$$\begin{cases} U_\phi &= -\beta(\phi - 1) + \pi(h + \alpha k s) \\ U_s &= -1 + \alpha k \phi \pi'(h + \alpha k s) \end{cases}$$

Under a quadratic cost function (in fact as soon as $x'(1) < \pi(h)$), we have: $U_\phi|_{\phi=1} = \pi(h + \alpha k s) > 0$, and therefore $\phi^* > 0$.

By concavity of U in ϕ, the borrowing constraint $\phi \leq e(s)$ implies that $U_\phi(\phi, s) \geq U_\phi(e(s), s)$ for any (ϕ, s). Since $U_\phi(e(s), s) = -\sqrt{2\beta(1-s)} + \pi(h + \alpha k s)$, it is increasing with s and, as noticed, positive for $s = 1$.

If $\pi(h) > \sqrt{2\beta}$ (low cost of education, good health at birth), then we have $U_\phi(e(0), 0) > 0$: for any s, $U_\phi(e(s), s) \geq 0$, and $U_\phi(\phi, s) \geq 0$. In that case the borrowing constraint is binding: $\phi^* = e(s)$, and U as a function of s only (reduced form) becomes:

$$\tilde{U}(s) = \pi(h + \alpha k s)\left(1 + \sqrt{\frac{2(1-s)}{\beta}}\right).$$

If $\pi(h) \leq \sqrt{2\beta}$, then $U_\phi(e(0), 0) \leq 0$: there exists $\bar{s} \in [0, 1)$ such that $U_\phi(e(\bar{s}), \bar{s}) = 0$. Precisely, $\bar{s}$ is defined by:

$$\pi(h + \alpha k \bar{s}) = \sqrt{2\beta(1 - \bar{s})}.$$

For $s \geq \bar{s}$, we have $U_\phi(\phi, s) \geq U_\phi(e(s), s) \geq 0$; ϕ^* is again equal to $e(s)$, and the reduced form of U is $\tilde{U}$.

For $s < \bar{s}$, we have $U_\phi(e(s), s) < 0$: ϕ^* is interior and the first order condition $U_\phi = 0$ gives: $\phi^*(s) = 1 + \frac{1}{\beta}\pi(h + \alpha k s)$. In that case the reduced form becomes:

$$\tilde{V}(s) = (1 - s) + \pi(h + \alpha k s)\left(1 + \frac{1}{2\beta}\pi(h + \alpha k s)\right).$$

A.2 Maximization over s

The partial derivatives of $\tilde{U}$ and $\tilde{V}$ w.r.t. s are:

$$\begin{aligned}\tilde{V}_s(s) &= -1+\alpha k\left(1+\frac{1}{\beta}\pi(h+\alpha k s)\right)\pi'(h+\alpha k s)\\ \tilde{U}_s(s) &= -\frac{1}{\sqrt{2\beta(1-s)}}\pi(h+\alpha k s)+\alpha k\pi'(h+\alpha k s)e(s).\end{aligned}$$

Denote by $s^1(h,k)$ and by $s^2(h,k)$ the solution to $\tilde{V}_s(s)=0$ and to $\tilde{V}_s(s)=0$, respectively, as a function of (h,k). The solution will be interior (and therefore characterized by the first-order condition) if the boundary conditions are satisfied.

We always have $\lim_{s\to 1}\tilde{U}_s=-\infty$. Therefore $s=1$ is never optimal: when the borrowing constraint is saturated, a marginal investment in education is always more profitable than a marginal investment in health. Therefore, the optimal investment in health may either be 0 or interior, depending on the sign of $U_s(\phi^*(0),0)$. We have:

$$\begin{aligned}\tilde{V}_s(0) &= -1+\alpha k\left(1+\frac{1}{\beta}\pi(h)\right)\pi'(h)\\ \tilde{U}_s(0) &= -\frac{1}{\sqrt{2\beta}}\pi(h)+\alpha k\pi'(h)e(0),\end{aligned}$$

both of which may be positive or negative.

By concavity, $\tilde{V}_s(0)>\tilde{V}_s(\bar{s})$. In the first regime ($\pi(h)<\sqrt{2\beta}$), three cases may occur:

	$\tilde{V}_s(0)<0$	$\tilde{V}_s(\bar{s})<0\le\tilde{V}_s(0)$	$0\le\tilde{V}_s(\bar{s})$
s^*	0	$0\le s^1(h,k)<\bar{s}$	$\bar{s}\le s^2(h,k)$
ϕ^*	$1+\pi(h)/\beta$	$1+\pi(h+\alpha k s^*)/\beta$	$e(s^*)$
c_0^*	>0	>0	0

In the second regime ($\pi(h)\ge\sqrt{2\beta}$), two cases may occur:

	$\tilde{U}_s(0)<0$	$\tilde{U}_s(\bar{s})<0$
s^*	0	$0\le s^2(h,k)$
ϕ^*	$e(0)$	$e(s^*)$
c_0^*	0	0

A.3 Study of $s^1(h,k)$

s^1 is the solution to:

$$F^1(h,k;s) \equiv (\beta + \pi(h+\alpha k s))\pi'(h+\alpha k s) - \frac{\beta}{\alpha k} = 0.$$

We have:

$$\left\{ \begin{array}{lcl} F^1_s &=& \alpha k(\pi'^2 + (\beta+\pi)\pi'') \leq 0 \\ F^1_h &=& (\pi'^2 + (\beta+\pi)\pi'') < 0 \\ F^1_k &=& \alpha s(\pi'^2 + (\beta+\pi)\pi'') + \beta/(\alpha k^2). \end{array} \right.$$

Therefore, s^1 is decreasing with h (intial health stock and investment in health are substitute), but may be increasing or decreasing with k:

$$\left\{ \begin{array}{lcll} s^1_h &=& -F^1_h/F^1_s &= -1/(\alpha k) \leq 0 \\ s^1_k &=& -F^1_k/F^1_s &= -s/k - \beta/[\alpha^2 k^3(\pi'^2 + (\beta+\pi)\pi'')] \\ & & &= -\dfrac{s^1\alpha^2k^2(\pi'^2 + (\beta+\pi)\pi'') + \beta}{\alpha^2k^3(\pi'^2 + (\beta+\pi)\pi'')}. \end{array} \right.$$

Notice that in that case, investment in health (ks^1) is a normal good, since for any (h,k):

$$\partial(ks^1(h,k))/\partial k = ks^1_k + s^1 = -\beta/[\alpha^2k^2(\pi'^2 + (\beta+\pi)\pi'')] > 0.$$

A.4 Study of $s^1(h,k) = 0$

We have $s^1(h,k) = 0$ iff $F^1(h,k;0) = 0$, and we know that $F^1_h < 0$. Therefore, for any k there exists a (locally) unique $h^1(k)$ such that $s^1(h^1(k),k) = 0$. Moreover, since

$$F^1_k|_{s=0} = \frac{\beta}{\alpha k^2} > 0,$$

we have that $h^1(k)$ is increasing with k.

A.5 Study of $s^2(h,k)$

Since $\tilde{U}(s) = \pi(h+\alpha k s)e(s)$, $\tilde{U}_s(s) = 0$ if and only if:

$$F^2(h,k;s) \equiv \alpha k \frac{\pi'(h+\alpha k s)}{\pi(h+\alpha k s)} + \frac{e'(s)}{e(s)} = 0.$$

We have:

$$\begin{cases} F_s^2 &= [(\alpha k)^2(\pi''\pi - \pi'^2) + \pi^2(e''e - e'^2)/e^2]/\pi^2 \leq 0 \\ F_h^2 &= \alpha k(\pi''\pi - \pi'^2)/\pi^2 < 0 \\ F_k^2 &= [\alpha\pi'\pi + \alpha^2 k s^2(\pi''\pi - \pi'^2)]/\pi^2 \end{cases}$$

The sign of F_k is unclear, and therefore s^2 may increase or decrease with k. But s^2 non ambiguously decreases with h: again, initial health stock and investment are substitute:

$$\begin{cases} s_h^2 &= -F_h^2/F_s^2 &= -\dfrac{\alpha k(\pi''\pi - \pi'^2)}{(\alpha k)^2(\pi''\pi - \pi'^2) + \pi^2(e''e - e'^2)/e^2} < 0 \\ \\ s_k^2 &= -F_k^2/F_s^2 &= -\dfrac{\alpha\pi'\pi + \alpha^2 k s^2(\pi''\pi - \pi'^2)}{(\alpha k)^2(\pi''\pi - \pi'^2) + \pi^2(e''e - e'^2)/e^2}. \end{cases}$$

Notice that for any (h, k):

$$\begin{aligned} \partial(ks^2(h,k))/\partial k &= ks_k^2 + s^2 \\ &= \frac{-\alpha k\pi'\pi + s^2\pi^2(e''e - e'^2)/e^2}{(\alpha k)^2(\pi''\pi - \pi'^2) + \pi^2(e''e - e'^2)/e^2} > 0. \end{aligned}$$

Therefore, in that case also investment in health is a normal good.

A.6 Study of $s^2(h, k) = 0$

This condition matters only in the case $\pi(h) \geq \sqrt{2\beta}$. We have

We have $s^2(h, k) = 0$ iff $F^2(h, k; 0) = 0$, and we know that $F_h^2 < 0$. Therefore, for any k there exists a (locally) unique $h^2(k)$ such that $s^2(h^2(k), k) = 0$. Moreover, since

$$F_k^2\big|_{s=0} = \alpha\pi'/\pi > 0,$$

we have that $h^2(k)$ is increasing with k.

A.7 Study of $\bar{s}(h, k)$

Since $\bar{s}$ is defined as the solution to:

$$\bar{F}(h, k; s) \equiv \pi^2(h + \alpha k s) - 2\beta(1 - s) = 0.$$

We have:

$$\begin{cases} \bar{F}_s &= 2\alpha k\pi'\pi + 2\beta > 0 \\ \bar{F}_h &= 2\pi'\pi \geq 0 \\ \bar{F}_k &= 2\alpha\bar{s}\pi'\pi \geq 0 \end{cases}$$

Therefore $\bar{s}$ decreases with h or k:

$$\begin{cases} \bar{s}_h &= -\bar{F}_h/\bar{F}_s &= -\dfrac{\pi'\pi}{\alpha k\pi'\pi + \beta} \leq 0 \\ \bar{s}_k &= -\bar{F}_k/\bar{F}_s &= -\dfrac{\alpha\bar{s}\pi'\pi}{\alpha k\pi'\pi + \beta} \leq 0 \end{cases}$$

Notice that $\bar{s}(h,k) = 0$ iff $\pi^2(h) = 2\beta$: this condition is independent of k and equivalent to the separating condition between the two regimes.

A.8 Study of $s^1(h,k) = s^2(h,k) = \bar{s}(h,k)$

$$F(h,k) \equiv s^1(h,k) - \bar{s}(h,k).$$

Then:

$$F_h = -\frac{\beta}{\alpha k(\alpha k\pi\pi' + \beta)} < 0.$$

Therefore there exists (locally) a unique $h = H(k)$ such that $s^1(H(k),k) = s^2(H(k),k)$. But is this H increasing or decreasing? This depends on F_k, which may be positive or negative without additional assumptions, since we have:

$$\begin{aligned} F_k &= -\frac{s^1\alpha^2k^2(\pi'^2 + (\beta+\pi)\pi'') + \beta}{\alpha^2k^3(\pi'^2 + (\beta+\pi)\pi'')} + \frac{\alpha\bar{s}\pi'\pi}{\alpha k\pi'\pi + \beta} \\ &= -\beta\left(\frac{s^1\alpha^2k^2(\pi'^2 + (\beta+\pi)\pi'') + (\alpha k\pi'\pi + \beta)}{[\alpha^2k^3(\pi'^2 + (\beta+\pi)\pi'')][\alpha k\pi'\pi + \beta]}\right). \end{aligned}$$

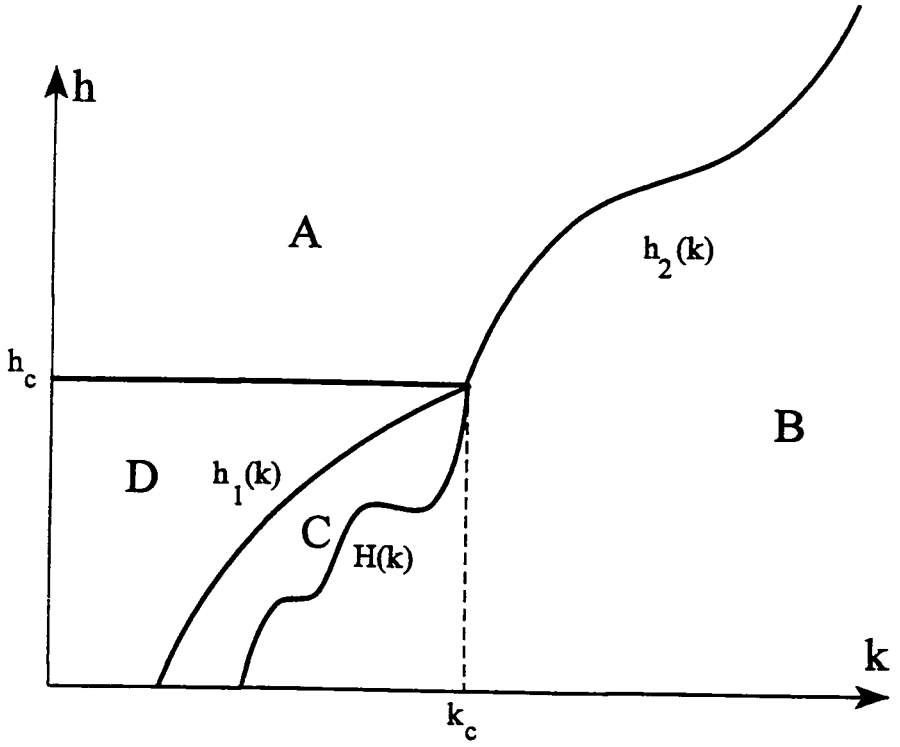

Figure 1

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