The Limits of Political Compromise: Debt Ceilings and Political Competition*

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Abstract

We study the desirability of limits on the public debt and of political competition in an economy where political parties alternate in office. Due to rent-seeking motives, incumbents have an incentive to set public expenditures above the socially optimal level. Parties cannot commit to future policies, but they can forge a political compromise where each party curbs excessive spending when in office if it expects future governments to do the same. In contrast to the received literature, we find that strict limits on government borrowing can *exacerbate* political-economy distortions by rendering a political compromise unsustainable. This tends to happen when political competition is limited. Conversely, a tight limit on the public debt fosters a compromise that yields the efficient outcome when political competition is vigorous, saving the economy from immiseration. Our analysis thus suggests a legislative tradeoff between restricting political competition and constraining the ability of governments to issue debt.

Keywords: debt limits; political turnover; efficient policies; fiscal rules

JEL classification: E61, E62, H30, H63.

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1 Introduction

Whenever the public debt starts to rise quickly, as it has in most developed economies since 2008, a debate on the merits of debt limits resurfaces. The debate has been heightened by successive American "debt-ceiling crises" triggered by Congress' reluctance to relax the federal debt ceiling. In this paper we show that the desirability of limits on the public debt hinges on the degree of political competition in the economy. If political competition is intense, a tight debt ceiling facilitates the implementation of efficient policies, but not otherwise. In particular, in a bipartisan society where political economy frictions are severe, the efficient policy is most likely to be sustainable if government access to the public debt is left *unrestricted*. Thus, in contrast to the general view that fiscal rules can only weaken political-economy distortions (albeit at the cost of reducing flexibility), we show that a debt ceiling aggravates them when political competition is limited.

At a more general level, we uncover a legislative tradeoff for the sustainability of socially desirable economic policies. If the laws regulating the formation of political parties are loose, constraints on government borrowing must be tight. But if the restrictions on formal political participation are stringent, then the government should be left free to borrow.

Our results also underscore a subtle impact of political competition on economic performance. Political competition matters for economic outcomes because it allows voters to discipline bad governments and to find alternatives to unskilled/selfish incumbents. We shut both the disciplining and the selection effects of elections out: we model political parties as rent-seeking but identical and unable to commit to future economic policies. Despite abstracting from those issues, we still find that the degree of political competition is critical to determine the political feasibility of socially beneficial policies.[1]

The key mechanism rests on the possibility of intertemporal cooperation among political parties (a "political compromise") aimed at neutralizing the policy inefficiencies that stem from political frictions. The parties have an incentive to cooperate because policies affect their payoffs when they are out of office, when they do not enjoy the perks and rents created by the policies but bear the consequences of the inefficiencies they introduce in the economy. A political compromise puts a brake on the current gains of the incumbent but can improve its future payoff. Whether it is sustainable depends on both the degree of political competition and the constraints on government borrowing.

[1] In the literature, the terms *political competition*, *political instability*, *political turnover*, *political fragmentation* and *power persistence* are often used interchangeably to denote phenomena related to situations when the identity of those who hold power changes over time. We stick with 'political competition,' but occasionally also use the near-synonyms mentioned above.

We embed the analysis in a simple, standard, neoclassic economic structure. In each period households decide how much to work and consume, while competitive firms decide how much to produce under a constant returns to scale technology that uses labor as input. The government provides a public good that is financed through taxes. The political structure is possibly the simplest that allows us to study our main question. There is an exogenous number of competing parties, which are unable to commit to policies. The political friction stems from incumbents and opposition parties having different preferences. Specifically, the period payoff of opposition parties is proportional to the representative household period utility, whereas incumbents enjoy some extra gain from government consumption. This results in incumbents having quasi-hyperbolic preferences, as defined by Laibson (1997), with the implication that the party in power has an incentive to spend more than is socially optimal.[2] Political turnover is determined by a random process in which the probability that a given party holds power in each period is inversely related to the degree of political competition, proxied by the number of active political parties.

A tighter debt ceiling lowers the incumbent's short-run gain from not cooperating, since it limits how much it can extract from future resources. The size of this reduction is independent of the degree of political competition. Under a political compromise aimed at implementing the efficient policy (which maximizes society's welfare), the rent benefit for future governments falls under a tight constraint on government borrowing, but rises when access to the debt is loose. Critically, this difference is more important when competition is weaker (i.e., when future rents matter more). It follows that when electoral rules are such that few parties participate in the political process, tight constraints on the public debt tend to undermine the feasibility of a political compromise. If instead numerous political parties actively compete, strong limits on government borrowing tend to *foster* a compromise. The upshot is that the desirability of tight fiscal rules is inversely related to the stringency of the rules allowing political participation.

We build the analysis of the general case by developing the polar cases of no debt and unconstrained debt. When debt is unavailable, we find that the efficient policy is unachievable if politicians are too profligate, since in that case the short-run temptation to spend is too large. Otherwise, a political compromise where all parties implement the efficient policy when in power can be sustained provided there is enough political competition.

[2]Such preferences imply that, in period t, the marginal rate of substitution between t and $t+1$ is lower than the marginal rate of substitution between $t+s$ and $t+s+1$, $s \geq 1$. Government preferences with this property are common in recent political economy models (e.g. Aguiar and Amador 2011, Halac and Yared 2014).

The intuition is simple. With strong competition, the probability that the incumbent will return to power and enjoy office rents in the future is low, while the probability that it will suffer the economic consequences of government rent-seeking when out of power is high. Hence it pays to forge a compromise that limits rents (and improves the economy's performance) when competition is fierce. This is not advantageous, however, if political competition is weak so that each party expects to hold office frequently.

Now, if the government were free to issue public debt, and thus to shape the action space of future administrations, the intuitive result just described is then largely overturned. We concentrate on the more interesting case where politicians' prodigality is high enough so that there is an equilibrium in which the first incumbent increases government expenditures so much that the public debt reaches its maximum sustainable level. This would drive the country into immiseration: a permanent state of low consumption and high debt. Under the shadow of this bad equilibrium, we find that the efficient policy can be sustained as an equilibrium outcome only when political competition is *not* too intense. The intuition is as follows. Without cooperation, the incumbent would enjoy extraordinarily high rents in office, but would leave the economy stuck in such a bad equilibrium that future governments would have little benefit from holding office. If instead a political compromise were forged, the incumbent would enjoy lower rents today but higher rents in the future, if it returned to power. A political compromise therefore not only secures a healthier state for the economy; it also *preserves* some rents for future governments. Those gains from future incumbency are more relevant to political parties when political competition is less intense, so that they are more likely to hold power in the future. Therefore, when the government has unrestricted access to debt, curbing politicians' profligacy requires *weak*, not strong political competition.

Put together, our results suggest the existence of a trilemma between intense political competition, unrestricted government borrowing, and a political compromise that yields efficient policies. With intense political competition and free government borrowing, a political compromise becomes unreachable. To ensure an efficient compromise under unlimited access to the public debt, political competition must be kept in check. In turn, such a compromise can be sustained with intense competition only when access to the public debt is sufficiently restricted.

To convey the mechanism as clearly as possible, we make the model deliberately simple. The drawback of that simplicity is that the model abstracts from several real-world features and frictions that would be needed for a thorough empirical assessment. Such an exercise

is beyond the scope of this paper. Nevertheless, despite its parsimony, the model yields an entirely novel, and potentially important, positive implication. Specifically, our analysis implies that whenever one wishes to study the economic impact of political competition, or of fiscal constraints, one must account for the *interaction* between them. Interestingly, the model's main prediction seems consistent with the available data. Using plausible proxies for debt/fiscal restrictions and conventional measures of political competition, a regression with country and year fixed effects for the period 1991-2012 indicates that a tight fiscal constraint has a positive effect on GDP per capita *only* when political competition is sufficiently intense. Although one should avoid over-interpreting those partial correlations, they match the model's main prediction rather remarkably.

The paper is organized as follows. After relating our contribution to the literature in the next section, we study the relationship between political competition and economic policy first in a model without public debt (section 3), and then allow for unrestricted public debt (section 4). Generalizing the insights from those polar cases, in section 5 we develop our main result on the tradeoff between constraints on government borrowing and on political competition. In section 6 we provide partial correlations among our main variables using country-level data. We conclude in section 7.

2 Related literature

The impact of political institutions on economic performance has been the focus of a large body of literature.[3] Yet to our knowledge the interplay between the intensity of political competition, debt constraints and economic outcomes has not yet been analyzed. One way to understand our contribution within the existing literature is to think of our main result as a bridge between two (so far) unrelated lines of political economy research.

On one hand, the main insight from our analysis in the environment without public debt has its roots in Alesina's (1988) early analysis of how cooperation between two political parties that are unable to commit to policies can improve economic outcomes. Political compromises between political parties are a central feature of democratic societies. As Alesina elegantly demonstrates, while a party that follows its individually optimal policies when in power obtains a short-run gain, if both parties behave that way, economic performance suffers. With cooperation across the political spectrum, a better outcome for both

[3]See for example the comprehensive reviews of Drazen (2000) and Persson and Tabellini (2000).

parties may be achievable.[4] Alesina's environment and focus are however quite different from ours. For example, in his setting political parties have different preferences and their payoffs do not depend on whether they hold office or not.

Closest to our setup without debt is the study of Acemoglu, Golosov and Tsyvinski (2011a). In their setting, political groups alternate in office according to an exogenous probabilistic process. The incumbent allocates consumption across groups, and has an incentive to increase its own welfare at the expense of others not in power. Acemoglu et al. then study how the degree of power persistence affects the possibility of cooperation among the political groups. Their main finding is that greater turnover helps to reduce political economy distortions and to sustain efficient outcomes. A similar result arises here when public debt is ruled out. Acemoglu et al. do not study, however, situations where the current policy affects the set of actions of future governments.

Yet that is the focus of a large body of research that goes back to the seminal contributions of Persson and Svensson (1989) and Alesina and Tabellini (1990).[5] A recurrent theme in that literature is the policymaking distortions created by political competition. In particular, by making politicians less patient, competition can induce them to overborrow. Although the mechanism is distinct, this is also a key force in our analysis when public debt is unrestricted: incumbents are more likely to internalize the cost of overindebtedness—which constrains future rent-seeking—when they expect to return to office in the future. If that probability is very low, the incumbent will not internalize those costs, spend as much as possible when in power, and leave the bill to whoever comes next.

Our key result links those two views by showing how the availability of debt shapes the desirability of political competition. In contrast with the main message from Acemoglu et

[4] Dixit, Grossman and Gul (2000) extend Alesina's (1988) logic to a situation where the political environment evolves stochastically. As a result, the nature of the political compromise between the two parties changes over time, depending on the electoral strength of the party in office. Acemoglu, Golosov and Tsyvinski (2011b) study instead an infinitely repeated game between a self-interested politician who holds power and consumers. They show that society may be able to discipline the politician and induce him to implement the optimal taxation policy in the long run despite his self-interest, provided that the politician discounts the future as consumers do.

[5] Battaglini (2014) departs from those canonical models by extending the analysis to a two-party infinite horizon problem and by explicitly modeling elections. Thus manipulation of public debt by one party affects not only the policy space available to future governments but also electoral probabilities. Callander and Hummel (2014) show that intertemporal policy linkages can arise even if there is no state variable, provided that information about the actual outcomes of a policy is incomplete. Once the party in power decides the initial level of the policy variable, society learns the mapping between policy and outcome at that initial level. Because there is a correlation between policies and outcomes at different levels, the incumbent sometimes engages in preemptive policy experimentation, manipulating the public information in the policy-outcome space available to its successor. Bierbrauer and Boyer (2013) study the relationship between political competition and welfare, but focus on the mode of political competition. Bonomo and Terra (2010) develop a model where electoral competition is influenced by lobbies and gives rise to electoral cycles. Fiva and Natvik (2013) point out that strategic manipulation of state variables due to political turnover is not exclusive to public debt, also extending to investment in physical capital.

al. (2011a), greater turnover does not always help; unlike what the strategic debt literature often suggests, it does not necessarily hurt either. Rather, we establish a tradeoff between intense political competition and unrestricted access to debt. The bottom line is that political competition has very different implications depending on the government's ability to borrow—or equivalently, the desirability of a debt ceiling hinges on the existing level of political competition. To our knowledge, this point has not been made before either in a formal model or informally.

This tradeoff does relate to a result by Azzimonti, Battaglini and Coate (2015), who study the impact of a balanced budget rule. They show that by constraining the tax smoothing role of the public debt, the rule induces legislators to lower the debt in the long run to prevent excessive tax volatility. Otherwise the debt would be inefficiently high due to political frictions in the legislative process, especially when agents are less patient; hence the debt reduction is more socially beneficial precisely in that case. In our analysis a tight ceiling on the debt is most desirable also when politicians become less patient, which happens when political competition is intense. Despite the similarity, the sources of the political friction, as well as the main mechanisms—restrictions to tax smoothing in the analysis of Azzimonti et al., difficulties in building a political compromise here—are entirely different in the two papers.

More fundamentally, we believe to be the first to point out that a tight debt ceiling can *exacerbate* political-economy distortions. The prevalent view is that fiscal rules exist to mitigate distorted incentives in policymaking, providing a commitment mechanism to governments. Their cost is the resulting loss of flexibility to react to shocks. In our non-stochastic model, there is no need for flexibility. Still, a debt limit can in some cases hurt the economy by inhibiting an efficiency-enhancing political compromise. This indicates that the consequences of debt rules can be subtler than it seems.[6]

Several other authors seek to explain how political economy frictions distort policy-making through debt. For example, in an environment with both political turnover and economic volatility, Caballero and Yared (2010) find that rent-seeking motivations lead to

[6]Bisin, Lizzeri and Yariv (2015) provide another rationalization for the adoption of debt ceilings and balanced budget rules. They study the interaction between voters who have time inconsistent preferences and two candidates who choose political platforms to maximize their electoral prospects. Due to the voter's time inconsistency, candidates have an incentive to propose policies that entail debt-financed transfers that will allow consumers to increase their consumption above the level that was optimal ex-ante. As a consequence, the economy will have an inefficiently high level of public debt. In contrast, Besley and Smart (2007) point out that a drawback of a debt ceiling is that it can disturb the political equilibrium and aggravate adverse selection problems. Halac and Yared (2015) take a different approach, studying the desirability of centralized versus decentralized fiscal rules in a multi-country economy, where the redistributive and the disciplining effects of interest rates play an important role.

excessive spending when there is high political uncertainty relative to economic uncertainty. Yet a rent-seeking incumbent will tend to underspend relative to the social planner during a boom when economic uncertainty is high relative to political uncertainty. The intuition is that an incumbent who has a high probability of keeping power will save during a boom to assure higher rents in the future, when the economy is likely to weaken. This result relates to our finding under unrestricted debt that weak political competition promotes good economic policies because political parties want to preserve future rents in case they return to power.[7] Song, Storesletten and Zilibotti (2012) study an environment where excessive levels of debt originate not from conflict between long-living political parties, but from an intergenerational conflict. Despite the very different setup, both here and in Song et al. lack of cooperation can lead to immiseration in the long run, when all governments can do is service the debt while providing the minimum level of the public good.[8]

The empirical literature studying the effects of political competition on economic policies, on the other hand, is more sparse. Using data for U.S. states since the nineteenth century, Besley, Persson and Sturm (2010) find that lack of political competition is strongly associated with "bad," anti-growth, policies. In their American environment, more political competition means simply the difference between elections contested by two parties and elections won by a clearly dominant party, so in our setting this would be equivalent to moving from a single-party ("dictatorship") to a bipartisan society. Closer in spirit to our analysis, Acemoglu, Reed and Robinson (2014) explore the effects of varying degrees of local political competition in Sierra Leone, which were arguably exogenously determined by the British colonial authorities in the late nineteenth century. Acemoglu et al. find that the degree of political competition in a locality, as measured by the number of potential local political rulers ("chiefs"), is positively correlated with several measures of economic development. That finding closely resembles our result in the no-debt economy, which is a good approximation for those regions, where rulers lack the ability to borrow extensively.[9]

[7] This effect also resembles a force stressed by Azzimonti (2011) when studying how polarization and political instability affects government expenditures, investment and long-run growth. She finds that a greater probability of returning to power puts a brake on the inefficiencies due to political uncertainty.

[8] Aizenman and Powell (1998) develop a model where conflict happens instead *within* the government, and the presence of competing parties in elections lowers the inefficiency of policies by disciplining incumbents.

[9] Arvate (2013) finds a related result when studying local governments in Brazil, which are unable to borrow freely in the market and display varying levels of political competition. There is also an empirical literature investigating the effects of political turnover on economic performance indirectly, by looking at how it affects government expenditures and the public debt. Results typically vary with methodology and sample. For example, in a panel of 19 OECD countries over the 1970-95 period, Perotti and Kontopoulos (2002) find that larger coalition sizes in power (a proxy for the instability of the government) are associated with more public expenditures and deficits, but Ricciuti (2004) finds no evidence that faster turnaround in office leads to more government consumption and higher public debt. Similarly, Pettersson-Lidbom

There is also a—largely unrelated—empirical literature investigating the macroeconomic impact of budget rules and fiscal rules. As Canova and Pappa (2006) point out, "the existing evidence on the issue is, at best, contradictory" (p. 1392). To some extent this may reflect lack of theoretical guidance—as Azzimonti et al. (2015, p.1) highlight, there is "remarkably little economic analysis" of the economic impact of budget rules, in contrast with the widespread policy debate on the issue. As a result, much of the empirical research focuses on the effectiveness of the rules (i.e., on whether the rules can be easily circumvented by accounting gimmicks), rather than on their economic consequences.[10]

Now, in none of the empirical analyses mentioned above are political competition and fiscal constraints considered together. A very notable exception—the only one we are aware of—is the recent study of the effects of fiscal restraints in Italian municipalities by Grembi, Nannicini and Troiano (2014). They exploit an arguably exogenous relaxation of fiscal rules, decided at the national level, which did not affect small cities with a population below a given threshold, comparing municipalities just below and just above the threshold. Interestingly, Grembi et al. observe that the effect of relaxing the fiscal constraint varies systematically with the number of political parties in the city council and with whether the mayor can run for reelection. In particular, they find that relaxing the fiscal constraint induces a deficit bias, but only in municipalities where political competition is sufficiently intense. Although their study is not designed to test a specific model, the results point to sizeable interaction effects between the consequences of fiscal restraints and the degree of political competition, precisely in the direction predicted by our model.

3 A society without public debt

In this section we assume that the government does not have access to public debt, and therefore needs to balance the budget in every period.

3.1 The economic environment

There is a continuum of identical households with Lebesgue measure one. Each of them is endowed with one unit of time. A single competitive firm produces a homogenous good under constant returns to scale. Technology is described by $0 \leq c + g \leq l$, where l is the

(2001) finds that among Swedish municipalities a higher probability of political turnover induces right-wing incumbents to accumulate debt, but leads left-wing ones to lower the debt.

[10] A related literature analyzes the effects of different debt levels on economic performance. See for example Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012).

amount of time allocated to production, c corresponds to household consumption, and g denotes a publicly provided good. At each date t, feasibility requires

$$c_t + g_t = l_t. \tag{1}$$

A spot market for goods and labor services operates in every period. The government finances its expenditures by taxing labor income at a proportional rate τ_t. Since in this section we assume there is no public debt, the government's budget constraint is simply

$$g_t = \tau_t l_t. \tag{2}$$

The twice differentiable function $u = u(c, l, g)$ describes the typical household period utility function. It is strictly increasing in c and g and strictly decreasing in l. For a fixed g, u satisfies standard monotonicity, concavity, and Inada conditions. Each household is endowed with one unit of time per period. Intertemporal preferences are described by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t),\ \beta \in (0, 1). \tag{3}$$

A household's date-t budget constraint is

$$c_t \leq (1 - \tau_t) l_t. \tag{4}$$

Given $\{g_t, \tau_t\}_{t=0}^{\infty}$, at date $t = 0$ a household chooses a sequence $\{c_t, l_t\}_{t=0}^{\infty}$ to maximize (3) subject to (4) and $l_t \leq 1$.

A *competitive equilibrium* is a sequence $\{c_t, l_t\}_{t=0}^{\infty}$ that satisfies (1) and solves the typical household's problem for given $\{g_t, \tau_t\}_{t=0}^{\infty}$. A sequence $\{g_t\}_{t=0}^{\infty}$ is *attainable* if there exist sequences $\{\tau_t\}_{t=0}^{\infty}$ and $\{c_t, l_t\}_{t=0}^{\infty}$ such that $\{c_t, l_t\}_{t=0}^{\infty}$ is a competitive equilibrium for $\{g_t, \tau_t\}_{t=0}^{\infty}$.

We now characterize the set of attainable allocations and policies. The household's first-order necessary and sufficient conditions are (4) taken as equality and

$$-\frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)} = 1 - \tau_t, \tag{5}$$

which is equivalent to

$$\tau_t = 1 + \frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)}.$$

Combining this expression with (2), we have that any attainable outcome $\{c_t, l_t, g_t\}_{t=0}^{\infty}$ must satisfy

$$g_t = \left[1 + \frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)}\right] l_t. \tag{6}$$

We can then use techniques similar to those in Chari and Kehoe (1999) to show that a sequence $\{c_t, l_t, g_t\}_{t=0}^{\infty}$ satisfies (1) and (6) if and only if it is attainable.

At each date t, there are two fiscal variables (g_t and τ_t) that the government can select. In general, there may be multiple tax rates that fund the same level of government expenditures. Yet for the sake of simplicity we want to turn the choice of a date-t fiscal policy into a unidimensional problem. Thus, for each attainable value of g, we define $U(g)$ according to

$$U(g) \equiv \max_{(c,l)} u(c, l, g) \tag{7}$$

subject to (1) and (6). Hence, whenever we say that a sequence $\{g_t\}_{t=0}^{\infty}$ is a *policy*, we are assuming that τ_t is the solution of (7) for the corresponding g_t. It should be clear that U resembles an indirect utility function. Built into that function is a tradeoff between increasing the provision of g and reducing the tax burden.

Government expenditures are bounded from above by the economy's maximum feasible output. Hence, $g \leq 1$. Furthermore, the constraints $l \leq 1$ and (1) imply that if $g = 1$, then $c = 0$ and $l = 1$. Thus, the Inada conditions on u imply that $U(1) \leq U(g)$ for all g. Moreover, if $u(0, 1, g) = -\infty$, then $U(1) = -\infty$; similarly, if $u(c, l, 0) = -\infty$, then $U(0) = -\infty$.

As we have just shown, $U(g)$ is equal to either a real number or $-\infty$. Therefore, U is a map from $[0, 1]$ into $\bar{\mathbb{R}}$. Under standard Inada conditions on households' preferences, it may happen that $U(0) = -\infty$ or $U(1) = -\infty$. Such unboundedness of U would lead to a severe but uninteresting problem of equilibrium multiplicity in the games studied in the next sections of this paper. To prevent that, we assume that g is bounded from below by a small positive number γ and from above by a number Γ smaller than one.[11] These bounds can be easily rationalized. Since the economy's maximum output is one, to achieve $g = 1$ the government would need to tax all income while households choose to devote all their available time to work despite the 100% tax. An upper bound on g below one

[11] Although this will become clearer after we describe the games played by the political parties, it is easy to see why such a restriction rules out a large family of uninteresting equilibria. Consider the upper bound Γ. Since $U(1) = -\infty$, if $g = 1$ at some date, then the household lifetime utility will be equal to $-\infty$. Hence, as long as political parties care to any extent about household welfare, trigger strategies that specify reversion to the policy $g = 1$ could support any policy as an equilibrium outcome in an infinitely repeated game. Similar reasoning justifies the introduction of the lower bound γ. An alternative to the introduction of the bounds γ and Γ consists of assuming that both $U(0)$ and $U(1)$ are larger than $-\infty$.

is therefore a natural consequence of the limits on the government's ability to raise taxes. The lower bound γ can be understood as the value that the public expenditures would take if the state were downsized to the minimum dimension allowed by law, since even such a minimalist entity would entail some expenditures.

We assume that U is strictly concave, twice differentiable, and attains a maximum at a point $g^* \in (\gamma, \Gamma)$. We call g^* the *efficient policy*.[12] An inspection of problem (7) shows that the second derivative of U depends on the third derivatives of u. Thus, unless extra assumptions are placed on u, one cannot ensure that U is strictly concave. But it is easy to provide conventional examples in which U is indeed strictly concave. For one, let $u(c, l, g) = \alpha_1 \ln c + \alpha_2 \ln(1-l) + \alpha_3 \ln g$, where α_1, α_2, and α_3 are positive numbers. Then

$$U(g) = \alpha_1 \ln[\alpha_1 - (\alpha_1 + \alpha_2)g] + \alpha_3 \ln g + \alpha_2 \ln \left[\frac{\alpha_2}{(\alpha_1 + \alpha_2)^{\alpha_1}} \right] \tag{8}$$

and

$$U''(g) = -\left\{ \frac{\alpha_1(\alpha_1 + \alpha_2)}{[\alpha_1 - (\alpha_1 + \alpha_2)g]^2} + \frac{\alpha_3}{g} \right\} < 0. \tag{9}$$

All that said, what we really need to take from this subsection is the function U and its properties. In short, U measures the utility that the typical household achieves in a competitive equilibrium. The economics underlying its properties is simple: households enjoy an increase in g, but this comes at the cost of higher taxes. Thus, U captures the tradeoff between the provision of g and its funding, concisely describing households' preferences over consumption, leisure and the public good.

3.2 The political environment

A *political party* is a coalition of agents ("politicians") who want to achieve power to enjoy some extra utility/rents while in office. The set of all politicians has measure zero. There is an exogenous natural number $n \geq 2$ of competing and identical political parties. We denote the set $\{1, 2, ..., n\}$ of political parties by $\mathcal{I}$ and use the letter i to denote a generic party in $\mathcal{I}$. We refer to the party that holds power in period t by p_t. We denote by $\mathcal{O}_t$ the set of opposition parties, i.e., the difference $\mathcal{I} - \{p_t\}$.

[12] Lump-sum taxes are not available. Thus, g^* is efficient in a second-best sense; that is, in the terminology of the optimal fiscal and monetary policy literature, g^* is a Ramsey policy. Had we allowed for lump-sum taxes, g^* would be a Pareto efficient policy.

The period preferences of party i are described by

$$V_i(g_t) = U(g_t) + \mathbf{1}_{it}\lambda g_t, \tag{10}$$

where $\lambda > 0$ and $\mathbf{1}_{it}$ is an indicator function taking the value of one when party i is in office and zero otherwise. The incumbent party cares about both the welfare of households and government expenditures, from which it extracts rents; parameter λ describes the additional weight that the incumbent places on g relative to consumers. In contrast, the interests of opposition parties and households are perfectly aligned, since $\mathbf{1}_{it} = 0$ for all $i \in \mathcal{O}_t$ and, as a consequence, the payoff of each of those agents is equal to $U(g_t)$.

We adopt this assumption only for simplicity. The feature of representation (10) that really matters is that political parties perceive a higher relative benefit from public expenditures when in power than when out of power.

There are at least two possible ways of interpreting the term λg. The first is to understand it as ego rents that increase as the government consumption grows. The second is to interpret it as extra income (e.g., through corruption) that a politician can obtain from public spending. The opportunities to enjoy those additional earnings increase with the level of public expenditures.

It is useful to define a benchmark where political competition is absent, which is equivalent to having $n = 1$. In this case, the function

$$V(g) = U(g) + \lambda g \tag{11}$$

corresponds to the period payoff of the everlasting ruling party. We define the maximizer g^D of $V(g)$ as the *dictatorial policy*. Since g must lie in the set $[\gamma, \Gamma]$, g^D satisfies $U'(g^D) \geq -\lambda$; furthermore, this condition holds with equality whenever $g^D < \Gamma$. Clearly, $g^D > g^*$, so a dictator overspends relative to the social optimum. Moreover, g^D is strictly increasing in λ whenever $g^D < \Gamma$. Thus, λ reflects the political parties' degree of profligacy, in the sense that an incumbent who does not strategically interact with other political parties sets $g = g^D$ and the difference $g^D - g^*$ is increasing in λ.

Political parties cannot commit to specific policies. Furthermore, they share the same preferences before knowing which of them will hold office. As our focus is on the intertemporal coordination of policies between current and future governments, we assume that an election is simply a randomizing device that, at the beginning of each period, selects party i to govern during that period with probability $\pi_i(n) \geq 0$, with $\sum_{i \in \mathcal{I}} \pi_i(n) = 1$

and $d\pi_i(n)/dn < 0$. For analytical convenience, with little additional loss of generality we assume further that $\pi_i(n) = 1/n$ for all $i \in \mathcal{I}$, so that all parties are equally popular.

We define units so that each period of time corresponds to an administration term. Players' lifetime payoff are the usual discounted sum of period payoffs. That it, the lifetime payoff of a political party is given by $\sum_{t=0}^{\infty} \beta^t V_i(g_t)$.[13]

Our model is fully characterized by the array $(\beta, U, \gamma, \Gamma, \lambda, n)$. Its first four components are purely economic factors, while the last two are political ones. Hence, we say that $(\beta, U, \gamma, \Gamma)$ is an *economy* and (λ, n) is a *polity*. We use the term *society* to denote a combination of an economy and a polity—that is, the entire array $(\beta, U, \gamma, \Gamma, \lambda, n)$.

We finish this subsection with a brief discussion of some features of the model. We will see that parameter n plays a pivotal role in the analysis. We will recurrently refer to n as our measure of "political competition," and carry out comparative statics exercises accordingly. The key assumption of our setup is that more political competition makes holding power in the future less likely. Thus, although in the model n measures simultaneously the number of political parties and the reciprocal of the probability that the incumbent will hold power in the future, the latter is its key role, proxying the degree of "power persistence" in the polity. It follows that the assumption that $\pi_i(n) = 1/n$ can be relaxed. For example, one could generalize the analysis to heterogeneous π_i, so features such as incumbency advantage could be considered. Although this would entail the cost of introducing a taxonomy, it would not yield fundamentally different insights, provided that $d\pi_i(n)/dn < 0$.

One may also wish to endogenize the probability of election, for example by letting voters decide based on both economic and non-economic issues in a probabilistic voting setting. At least for the symmetric case, however, little would change in the analysis. For example, if we maintain the assumption that parties cannot commit to policies, then without coordination among the parties the electoral probability would remain $\frac{1}{n}$ for each of them in every period, with each party implementing $g = g^D$ when in power.[14] If parties could commit to policies, then under conventional assumptions the chosen policy would lie between g^* and g^D for all parties in the absence of policy coordination, implying again a probability of election $\frac{1}{n}$ for each party. Naturally, such an electoral model could be extended and enriched in several directions. A large and important literature deals with such issues. We choose instead to keep the analysis as simple as possible in that dimension,

[13] We assume that politicians, like the typical household, live forever. As is well known from the repeated games literature, we could replace this hypothesis by an uncertain lifetime of politicians. Of course, that would also entail allowing the set of political players to change over time.

[14] We show in the next subsection that having $g_t = g^D$ for all t is indeed an equilibrium of the game we consider.

so that we can focus on the dimensions where we push the literature forward.

It is worth noting that our key assumptions are very similar, for example, to those of Aguiar and Amador (2011) in their analysis of investment and growth patterns when governments can expropriate foreign capital. Like here, their political friction stems from a situation where incumbents enjoy a higher payoff from government consumption than non-incumbents; governments do not have access to a commitment technology; and political turnover is exogenous (although they allow for—exogenous—incumbency advantage).[15]

3.3 The policy game

To study how political competition impacts policymaking, we consider a game in which the players are the political parties. The incumbent party selects current policies. Future policies are chosen by future governments.

Let $h^t = (g_0, g_1, ..., g_t)$ be a history of policies. At each date s, the incumbent p_s selects a date-s policy g_s as a function of history h^{s-1}. We denote that choice by $\sigma_{p,s}(h^{s-1})$. The incumbent also chooses plans $\{\sigma_{p,t}\}_{t=s+1}^{\infty}$ for future policies in case it later returns to office. An opposition party o selects only plans $\{\sigma_{o,t}\}_{t=s+1}^{\infty}$ for future policies. Given an array $[\{\sigma_{i,t}\}_{t=0}^{\infty}]_{i\in\mathcal{I}}$ of policy plans and a history h^{t-1}, the date-t policy follows the rule

$$g_t = \sum_{i\in\mathcal{I}} \mathbf{1}_{it}\sigma_{i,t}(h^{t-1}).$$

That is, the actual policy g_t is the choice of g for period t of the incumbent in period t.

At each date s, the lifetime payoff $\mathcal{V}_{i,s}$ of party i is given by

$$\mathcal{V}_{i,s} = \sum_{t=s}^{\infty} \beta^{t-s} V_i(g_t).$$

The incumbent's problem is the following. Given h^{s-1} and the other parties' plans, $[\{\sigma_{o,t}\}_{t=s+1}^{\infty}]_{o\in\mathcal{O}_s}$, it chooses a policy plan $\{\sigma_{p,t}\}_{t=s}^{\infty}$ to maximize the expected value of $\mathcal{V}_{p,s}$. Opposition parties solve an analogous problem.

Given the ex-ante symmetry of political parties, it is natural to concentrate on symmetric outcomes. A *symmetric political equilibrium* is a policy plan $\{\sigma_t\}_{t=0}^{\infty}$ with the property that if all opposition parties follow the policy plan $\{\sigma_t\}_{t=0}^{\infty}$, then the solution of the incumbent's problem at every period s for all histories h^{s-1} is $\{\sigma_t\}_{t=s}^{\infty}$. A sequence $\{g_t\}_{t=0}^{\infty}$ is a

[15] A similar observation applies to Azzimonti (2011), whose setup also features government and society having different objectives, with the former being unable to commit to policies.

symmetric political outcome if there exists a symmetric political equilibrium $\{\sigma_t\}_{t=0}^{\infty}$ such that $\sigma_t(g_0, ..., g_{t-1}) = g_t$ for all t.[16]

It is easy to see that g^D is a stationary symmetric political outcome. Define the *dictatorial plan* $\{\sigma_t^D\}_{t=0}^{\infty}$ so that, after any history h^{t-1}, every political party sets $g_t = g^D$ if it holds power. Suppose that, at some date t, party p_t believes that all parties in $\mathcal{O}_t$ will follow the plan $\{\sigma_t^D\}_{t=0}^{\infty}$. Clearly, the best course of action for party p_t is to implement the plan $\{\sigma_t^D\}_{t=0}^{\infty}$ as well. Therefore, $\{\sigma_t^D\}_{t=0}^{\infty}$ is a symmetric political equilibrium and the corresponding outcome is $g_t = g^D$ for every t.

Having identified an equilibrium for the policy game, it is natural to use trigger strategies to characterize other symmetric political outcomes. In particular, we consider the following *revert-to-dictatorship* policy plan. It specifies that if all previous governments implemented a certain policy $\{g_t\}_{t=0}^{\infty}$, then the current incumbent does the same; otherwise, the incumbent implements $g = g^D$ today and whenever it returns to office.

Denote by $\Omega_s(\{g_t\}_{t=s}^{\infty})$ the expected value of $\mathcal{V}_{p,s}$ when all parties follow the policy $\{g_t\}_{t=0}^{\infty}$. Thus,

$$\Omega_s(\{g_t\}_{t=s}^{\infty}) = U(g_s) + \lambda g_s + \sum_{t=s+1}^{\infty} \beta^{t-s} \left[U(g_t) + \frac{\lambda}{n} g_t \right]. \tag{12}$$

With some abuse of notation, let $\Omega(g)$ represent the payoff of party i when $g_t = g$ for all t:

$$\Omega(g) = \frac{1}{1-\beta} \left[U(g) + \left(1 - \beta + \frac{\beta}{n} \right) \lambda g \right]. \tag{13}$$

Then, if a policy $\{g_t\}_{t=0}^{\infty}$ satisfies

$$\Omega_s(\{g_t\}_{t=s}^{\infty}) \geq \Omega(g^D) \tag{14}$$

for every date s, then $\{g_t\}_{t=0}^{\infty}$ is a symmetric political outcome. The left-hand side of (14) is the payoff of the date-s incumbent if $\{g_t\}_{t=0}^{\infty}$ is implemented from date s onward, while the right-hand side corresponds to the payoff of that player if the dictatorial policy is implemented from date s onward.

To see that (14) is a sufficient condition for $\{g_t\}_{t=0}^{\infty}$ to constitute a symmetric political outcome, suppose that all parties in $\mathcal{O}_s$ follow the revert-to-dictatorship plan associated with $\{g_t\}_{t=s}^{\infty}$. Consider the decision of party p_s at some date s. If the prevailing history

[16] The symmetric political equilibrium is similar to the sustainable equilibrium introduced by Chari and Kehoe (1990). As those authors point out, such an equilibrium entails subgame perfection.

is $\{g_t\}_{t=0}^{s-1}$, then condition (14) ensures that implementing g_t is optimal for party p_s. If the prevailing history differs from $\{g_t\}_{t=0}^{s-1}$, then all parties in $\mathcal{O}_s$ implement the dictatorial policy g^D whenever they come to office. Consequently, the best action for party p_s is to implement g^D as well. Hence, the revert-to-dictatorship plan is a best-response strategy for party p_s.[17]

3.4 The political feasibility of the efficient policy

Politicians can do better than just follow the dictatorial policy if they coordinate policies, i.e., if they forge a *political compromise*. We now assess the conditions under which a political compromise can sustain the efficient policy.[18]

If $g_t = g^*$ for every t, then (14) can be written as $\Omega(g^*) \geq \Omega(g^D)$. This inequality is equivalent to

$$\frac{\beta}{1-\beta}\left[U(g^*) - U(g^D) + \frac{\lambda}{n}(g^* - g^D)\right] \geq V(g^D) - V(g^*). \tag{15}$$

Therefore, the efficient policy is a symmetric political outcome if (15) holds. Its left-hand side represents the present value of the future gains from cooperation for the incumbent, whereas the right-hand side denotes its short-run gain from implementing the dictatorial policy instead of the efficient one.

From the definitions of g^* and g^D, we have that $V(g^D) - V(g^*) > 0$, $U(g^*) - U(g^D) > 0$, and $(\lambda/n)(g^* - g^D) < 0$. Therefore, the right-hand side of (15) is strictly positive but its left-hand side, which is strictly increasing in n, may be negative for small values of n. Intuitively, the gains from cooperation for the incumbent come from preventing excessive public spending when it is not enjoying rents from those expenditures. If the incumbent expects to hold office often, the circumstances under which it would benefit from cooperation become relatively rare and its gain from cooperation may turn negative. This makes clear that the degree of political competition plays a crucial role when it comes to the sustainability of the efficient policy.

[17] Observe that (14) is a sufficient condition for a policy to be an equilibrium outcome. We cannot rule out that, by designing different punishments, it may be possible to implement policies that do not satisfy (14). Since solving this specific question will add little to the comprehension of the problems we deal with here, we do not address this matter in this paper.

[18] Even if the efficient policy were sustainable, the political parties may want to coordinate on another policy. A plausible alternative is that they seek to implement the best policy from their own perspective. In the online appendix (available at http://www.alexbcunha.com/research/papers/paper15oa.pdf), we develop the analysis for the case where the incumbent proposes the (time-invariant) policy over which the parties coordinate. Such an analysis yields some additional interesting insights, although the qualitative results are similar to those obtained when we focus on the efficient outcome.

It helps to break down the analysis of (15) into two cases. We study each of them in turn. Suppose first that

$$\frac{\beta}{1-\beta}[U(g^*) - U(g^D)] \leq V(g^D) - V(g^*). \tag{16}$$

Since $(\lambda/n)(g^* - g^D) < 0$, inequality (15) would not hold regardless of the value of n. This happens when a high λ makes the short-run gain from implementing g^D too large relative to the future gains under coordination. In this case, the efficient policy is unachievable through the revert-to-dictatorship strategy.

Proposition 1 *For every economy $(\beta, U, \gamma, \Gamma)$, there exists a number λ_0 such that, if a polity (λ, n) satisfies $\lambda \geq \lambda_0$, then inequality (16) holds. As a result, the efficient policy cannot be implemented by the revert-to-dictatorship strategy for any level of n.*

Proof. See online appendix.

Consider now the case in which (16) does not hold:

$$\frac{\beta}{1-\beta}[U(g^*) - U(g^D)] > V(g^D) - V(g^*). \tag{17}$$

It is then possible to place conditions on n that ensure that (15) holds and, as a consequence, the efficient policy constitutes a symmetric political outcome. Define

$$N^0(\beta, \lambda) \equiv \frac{\lambda(g^* - g^D)}{\frac{1-\beta}{\beta}[V(g^D) - V(g^*)] - [U(g^*) - U(g^D)]}. \tag{18}$$

$N^0(\beta, \lambda)$ corresponds to the value of n that makes (15) hold with equality. Observe that $N^0(\beta, \lambda) > 0$ under (17).

Proposition 2 *If a society $(\beta, U, \gamma, \Gamma, \lambda, n)$ satisfies (17) and $n \geq N^0(\beta, \lambda)$, then the efficient policy g^* constitutes a symmetric political outcome.*

Proof. The left-hand side of (15) is strictly increasing in n, while its right-hand side does not depend on n. Furthermore, (15) holds with equality for $n = N^0(\beta, \lambda)$. Thus, if $n \geq N^0(\beta, \lambda)$, (15) is satisfied. As a consequence, g^* is a symmetric political outcome. ■

According to Proposition 2, $N^0(\beta, \lambda)$ defines the minimum number of parties that can sustain g^* as an equilibrium with the revert-to-dictatorship plan. Thus, if the efficient

policy is sustainable in a polity (λ, n), it is also sustainable in a polity (λ, n'), where $n' > n$. In that sense, political competition fosters good economic policy.

Our society can suffer from outcomes that differ from the efficient policy g^* because $\lambda > 0$ distorts the objectives of politicians away from those of society at large. Proposition 2 shows that a high n can offset the adverse effects of a positive λ. However, as Proposition 1 makes clear, such a conclusion holds only if politicians are not too profligate (i.e., λ is not too large). This result becomes particularly relevant when one observes that the differences $g^D - g^*$ and $U(g^*) - U(g^D)$ are weakly increasing functions of λ. Hence, exactly when the political distortions can be more severe, competition among the political agents fails to discipline them.

Summing up, in the context studied in this section, where the actions of the political party in office have no bearing on the options available to future governments, there is a clear sense in which more political competition can foster the implementation of better policies and improve economic performance. As we will see, this is no longer true when current policies can affect the set of actions available to future governments.

4 A society with unrestricted public debt

We now show how the public debt impacts the strategic interaction between politicians. In particular, we will see that it is no longer true that intense political competition fosters the implementation of better economic policies.

As in the previous section, we work with a reduced-form function $U(\cdot)$ that maps economic policies into household welfare in a competitive equilibrium. In section 3, the period payoff function of a typical household is similar to an indirect utility function that captures the structure of the underlying economy. That function is shaped by the tradeoff between the provision of the public good and its funding. The introduction of public debt affects that tradeoff. In particular, the vector (b_t, g_t, b_{t+1}), where b_t denotes the beginning-of-period t value of the public debt, takes on the role that up to now g_t played alone.

In this section we represent the payoff of a typical household by a function $U(b_t, g_t, b_{t+1})$. That function is shaped by the tradeoff between providing the public good, raising distortionary tax revenues[19], and managing the public debt. Of course, that tradeoff depends on

[19]The results of section 3 do not depend on whether or not the government has access to lump-sum taxes. However, we know since Barro (1974) that if lump-sum taxes are available, the government can relax any constraint imposed by the public debt by simply raising tax revenues that exactly match the value of its outstanding bonds. Therefore, lump-sum taxes are ruled out in the underlying economy.

the interest rate, which is a built-in component of U. In the next subsection we provide an example of a simple dynamic general equilibrium model for which the payoff representation we postulate here either (i) exactly describes or (ii) provides a steady-state approximation of the typical household's utility. In the latter case, the approximation perfectly matches the household's lifetime utility for every equilibrium we study.

4.1 The economic environment

4.1.1 Basic economic structure and competitive equilibrium

We modify the economy of the previous section by allowing the government to issue claims to one unit of the consumption good, redeemable in the next period. Therefore, b_t is the amount of those claims outstanding at the beginning of period t. This variable is measured in the same units as g_t and its initial value b_0 is exogenous and equal to zero.[20] The claims are traded at a price q_t. For notational convenience, we will often denote b_t and b_{t+1} by, respectively, b and b'.

The government period budget constraint is

$$g_t + b_t = \tau_t l_t + q_t b_{t+1}, \tag{19}$$

while households' budget constraint becomes

$$c_t + q_t b_{t+1} \leq (1 - \tau_t) l_t + b_t. \tag{20}$$

To avoid Ponzi schemes, the public debt must satisfy the constraint $|b_{t+1}| \leq M < \infty$, where M is large enough so that this constraint never binds.

Given $\{g_t, \tau_t, q_t\}_{t=0}^{\infty}$, at date $t = 0$ a household chooses a sequence $\{c_t, l_t, b_{t+1}\}_{t=0}^{\infty}$ to maximize (3) subject to (20) and $l_t \leq 1$. The necessary and sufficient conditions for this are (20) taken as equality, (5),

$$\beta \frac{u_c(c_{t+1}, l_{t+1}, g_{t+1})}{u_c(c_t, l_t, g_t)} = q_t, \tag{21}$$

and

$$\lim_{t \to \infty} \beta^t u_c(c_{t+1}, l_{t+1}, g_{t+1}) b_{t+1} = 0. \tag{22}$$

[20] The assumption $b_0 = 0$ is mostly a simplifying hypothesis. We will further discus it when analyzing the efficient policy.

A *competitive equilibrium* is composed of sequences $\{c_t, l_t\}_{t=0}^{\infty}$, $\{b_{t+1}\}_{t=0}^{\infty}$ and $\{q_{t+1}\}_{t=0}^{\infty}$ that satisfy (1) and the optimal behavior of the households for given $\{g_t, \tau_t\}_{t=0}^{\infty}$. A sequence $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ is *attainable* if there exist sequences $\{\tau_t\}_{t=0}^{\infty}$, $\{c_t, l_t\}_{t=0}^{\infty}$ and $\{q_{t+1}\}_{t=0}^{\infty}$ such that $\{c_t, l_t\}_{t=0}^{\infty}$, $\{b_{t+1}\}_{t=0}^{\infty}$ and $\{q_{t+1}\}_{t=0}^{\infty}$ constitute a competitive equilibrium for $\{g_t, \tau_t\}_{t=0}^{\infty}$.

Let $H(c, l, g) \equiv u_c(c, l, g)c + u_l(c, l, g)l$. Using the reasoning of Chari and Kehoe (1999), we have that the set of attainable sequences is fully characterized by (1) and

$$\sum_{t=0}^{\infty} \beta^t H(c_t, l_t, g_t) = 0. \tag{23}$$

Additionally, the public debt sequence must satisfy

$$\sum_{t=s}^{\infty} \beta^{t-s} H(c_t, l_t, g_t) = u_c(c_s, l_s, g_s)b_s. \tag{24}$$

4.1.2 The efficient and the dictatorial policies

The efficient allocation $\{c_t^*, l_t^*, g_t^*\}_{t=0}^{\infty}$ solves the problem of maximizing households' lifetime utility (3) subject to (1) and (23). The solution is characterized by those constraints plus the first-order conditions

$$\begin{cases} u_c(c_t, l_t, g_t) - \theta_t + \Theta H_c(c_t, l_t, g_t) = 0 \\ u_l(c_t, l_t, g_t) + \theta_t + \Theta H_l(c_t, l_t, g_t) = 0 \\ u_g(c_t, l_t, g_t) - \theta_t + \Theta H_g(c_t, l_t, g_t) = 0, \end{cases} \tag{25}$$

where θ_t and Θ are, respectively, Lagrange multipliers for (1) and (23), while H_c, H_l and H_g are partial derivatives.

Equations (25), together with (1), establish that $\{c_t^*, l_t^*, g_t^*\}_{t=0}^{\infty}$ is a static sequence. Thus, $\sum_{t=s}^{\infty} \beta^{t-s} H(c_t^*, l_t^*, g_t^*) = \sum_{t=0}^{\infty} \beta^t H(c_t^*, l_t^*, g_t^*) = 0$ for all s. Hence, (24) implies that $b_s^* = 0$ for every s.[21]

Finally, observe that if $b_0 \neq 0$, it would be necessary to add $u_c(c_0, l_0, g_0)b_0$ to the right-hand side of (23). As a consequence, the date-0 first-order conditions would be slightly different; the public debt would be constant and the efficient allocation static only for $t \geq 1$. In summary, if $b_0 \neq 0$, the efficient allocations and the debt levels would change from $t = 0$ to $t = 1$, and then reach a steady state.

[21] Naturally, since our environment is not stochastic, there is no role for the tax smoothing property of the public debt.

As in section 3, assume that the period payoff of a dictator is given by $u(c_t, l_t, g_t) + \lambda g_t$. The characterization of the dictatorial policy $\{g_t^D, b_{t+1}^D\}_{t=0}^{\infty}$ requires finding a sequence $\{c_t^D, l_t^D, g_t^D\}_{t=0}^{\infty}$ that maximizes $\sum_{t=0}^{\infty} \beta^t [u(c_t, l_t, g_t) + \lambda g_t]$ subject to (1) and (23). The solution is given by the same equations as the efficient policy except that

$$u_g(c_t, l_t, g_t) + \lambda - \theta_t + \Theta H_g(c_t, l_t, g_t) = 0$$

replaces the third equation in (25). Thus, $b_{t+1}^D = 0$ for all t and the sequence $\{g_t^D\}_{t=0}^{\infty}$ is static. Hence, the efficient and dictatorial debt levels are identical. However, as in the economy without debt, government expenditures are inefficiently high under a dictatorship.

4.1.3 Constructing the function $U(b, g, b')$

Take an array $[b_0, \{g_t, b_{t+1}\}_{t=0}^{\infty}]$. Define $\mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^{\infty})$ according to

$$\mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^{\infty}) \equiv \max_{\{c_t, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \tag{26}$$

subject to (1), (22), and

$$c_t + \beta \frac{u_c(c_{t+1}, l_{t+1}, g_{t+1})}{u_c(c_t, l_t, g_t)} b_{t+1} = -\frac{u_l(c_t, l_t, g_t)}{u_c(c_t, l_t, g_t)} l_t + b_t.$$

This last expression is an implementability constraint for the typical household budget constraint. It was obtained by combining (20) holding with equality, (5) and (21). By construction, $\mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^{\infty})$ is the highest lifetime utility that the household can attain if the government implements the policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$.

If there is a function $\mathcal{W}(b, g, b')$ satisfying $\sum_{t=0}^{\infty} \beta^t \mathcal{W}(b_t, g_t, b_{t+1}) = \mathcal{U}(b_0, \{g_t, b_{t+1}\}_{t=0}^{\infty})$, then we simply set

$$U(b, g, b') = \mathcal{W}(b, g, b'). \tag{27}$$

If $\mathcal{U}$ cannot be decomposed as above, we construct U as follows. Let $\{g, b'\}$ denote a policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ in which $(g_t, b_{t+1}) = (g, b')$ for every t. Now take a generic vector (b, g, b'). If each of the arrays $[b, \{g, b'\}]$ and $[b', \{g, b'\}]$ is attainable, then define $U(b, g, b')$ so that

$$U(b, g, b') \equiv \mathcal{U}(b, \{g, b'\}) - \beta \mathcal{U}(b', \{g, b'\}). \tag{28}$$

Observe that $\mathcal{U}(b, \{g, b'\})$ is the household lifetime payoff when the economy starts with

debt b and reaches the steady-state (g, b') after a single period, while $\mathcal{U}(b', \{g, b'\})$ is the lifetime payoff in such a steady state. Hence, $U(b, g, b')$ captures the household utility gain (or loss) associated with that one-period transition. If $[b, \{g, b'\}]$ or $[b', \{g, b'\}]$ is not attainable, then we need to modify our definition. In that case, we set U according to

$$U(b, g, b') = \mathcal{U}(b, [(\tilde{g}(b, b'), b'), \{\hat{g}(b'), b'\}]) - \beta \mathcal{U}(b', \{\hat{g}(b'), b'\}), \tag{29}$$

where $\hat{g}(b')$ is the maximum attainable value for g in a steady state with debt b', $\tilde{g}(b, b')$ is the maximum attainable value for g_0 when the initial debt is b and the economy will be in state $(\hat{g}(b'), b')$ for every $t \geq 1$, and $[(\tilde{g}(b, b'), b'), \{\hat{g}(b'), b'\}]$ denotes the policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ in which $(g_0, b_1) = (\tilde{g}(b, b'), b')$ and $(g_t, b_{t+1}) = (\hat{g}(b'), b')$ for every $t \geq 1$. Observe that we replaced the values of g specified in the arrays $[b, \{g, b'\}]$ and $[b', \{g, b'\}]$ with the highest attainable values for that variable. Implicit in our definition is the assumption that b and b' are attainable values for the public debt.

It should be clear that if it is possible to define U as in (27), then U provides an exact measure of a household's lifetime utility. If U is defined as in (28) and (29), these two equalities ensure that U perfectly measures the typical household payoff in any steady state or any outcome in which there is a one period transition to a steady state. Since every equilibrium considered in this paper is either static or displays a one period transition to a steady state, then U is an exact metric of household welfare in those equilibria. We formalize these arguments in the online appendix.

4.1.4 Further economic features

We denote the partial derivatives of U by U_b, U_g and $U_{b'}$. Similar notation is used for the second-order derivatives. We assume that U possesses standard concavity features, so that $U_{gg}(b, g, b') < 0$. Furthermore, we postulate that the partial derivatives satisfy

$$U_{b'}(b, g, b') \geq 0,\ U_{bg}(b, g, b') < 0,\ U_{gb'}(b, g, b') > 0, \tag{30}$$

and $U_{bg}(b, g, b) + U_{gb'}(b, g, b) < 0$. Intuitively, if b and g are held constant, an increase in b' reduces the amount of distortionary taxes required to balance the government period budget constraint. This justifies the first inequality. Analogously, if g and b' are held constant, an increase in b leads to an increase in the tax burden, lowering the marginal utility of g. Similarly, U_g should be a strictly increasing function of b'. Furthermore, if the

public debt is held constant over time at a level b, then an increase in that level requires, for a fixed g, an increase in the tax burden to service the debt, lowering the marginal utility of g.

The government's ability to raise tax revenue places bounds on its consumption and interest expenditures. Let r denote the steady-state interest rate; as usual, r satisfies the equality $\beta = (1+r)^{-1}$. Let $\bar{B} > 0$ denote the maximum value the public debt can reach at any given date. It has the property that the sum $\gamma + r\bar{B}$ is equal to the maximum amount of tax revenue the government can raise in a single period. Observe that if $b_{t+1} = \bar{B}$ at some date t, then $(g_s, b_{s+1}) = (\gamma, \bar{B})$ for every $s \geq t+1$. That is, if the debt ever reaches its maximum attainable value, the economy becomes locked in the state $(\gamma, \bar{B})$ permanently.

The debt may also take negative values. In that case, the typical household becomes a debtor. The household's ability to repay its debt is bounded by the lifetime income that it could obtain by working all available time at every date t. Thus, there must be a real number $\bar{b} \geq 0$ such that $b_t \geq -\bar{b}$ for all t.

The sequence of period budget constraints (19) is the venue through which the date-t government impacts the set of admissible actions of the future administrators. However, since we want to represent the economic structure in a simple reduced form, we need an alternative way to model the relevant features of (19). As we show next, we achieve that with the help of two very generic functions, $f^b(b)$ and $f^g(b, b')$.

Let $f^b(b)$ be a strictly increasing and continuously differentiable function. The date-t government's choice of b_{t+1} must satisfy

$$b_{t+1} \in [f^b(b_t), \bar{B}]. \tag{31}$$

Since f^b is strictly increasing, a rise in b_t shrinks the set $[f^b(b_t), \bar{B}]$. Thus, by increasing the debt it leaves to its successor, the incumbent at date $t-1$ restricts the choice of b_{t+1} of the next administration. Furthermore, $f^b(\bar{B}) = \bar{B}$, because the economy becomes locked in state $(\gamma, \bar{B})$ if the public debt ever reaches the value $\bar{B}$.

We model the constraints that b_t and b_{t+1} place on g_t with the continuously differentiable function $f^g(b, b')$. This function is strictly decreasing in b and strictly increasing in b'. The choice of g_t must satisfy

$$g_t \in [\gamma, f^g(b_t, b_{t+1})]. \tag{32}$$

The role of the upper bound Γ in the previous section is now played by $f^g(b_t, b_{t+1})$. The set $[\gamma, f^g(b_t, b_{t+1})]$ shrinks when b_t increases. Since γ is the only admissible value for g_t

when $b_t = \bar{B}$, we must have $f^g(\bar{B}, \bar{B}) = \gamma$. Suppose now that b_t is equal to some generic value b for every t. The higher b is, the higher the interest the government must pay, so the tighter its budget constraint is. Hence, the partial derivatives of f^g must satisfy $f^g_b(b,b) + f^g_{b'}(b,b) < 0$.

The party in office at date t can increase b_{t+1} to enlarge the set from which g_t is selected. This would restrict the choices of the next administration by tightening constraints (31) and (32). Hence, the date-t incumbent can increase the end-of-period debt b_{t+1} to achieve two goals simultaneously: relax the constraints it faces when selecting g_t; and tighten the constraints the government at $t+1$ will face when selecting (g_{t+1}, b_{t+2}). In the limiting case in which $b_{t+1} = \bar{B}$, the date-t incumbent locks the society permanently in state $(\gamma, \bar{B})$.

Now let $g^*(b, b')$ denote the value of g that maximizes $U(b, g, b')$ under the constraint $\gamma \leq g \leq f^g(b, b')$. We assume that if $b < \bar{B}$, then $g^*(b, b') < f^g(b, b')$.[22] Furthermore, if the government keeps its debt constant at some generic level b, the amount of distortionary revenue needed to balance its budget will be a strictly increasing function of b. As a consequence,

$$b < \hat{b} \Rightarrow U(b, g^*(b,b), b) > U(\hat{b}, g^*(\hat{b}, \hat{b}), \hat{b}). \tag{33}$$

In this reduced form representation, the efficient policy is the attainable sequence $\{g^*_t, b^*_{t+1}\}_{t=0}^{\infty}$ that maximizes $\sum_{t=0}^{\infty} \beta^t U(b_t, g_t, b_{t+1})$. From our analysis of the optimal and dictatorial policies in 4.1.2, we know that $b^*_{t+1} = 0$. As a consequence, $g^*_t = g^*(0,0)$ for every t.

4.2 The political environment and the policy game

The political environment is essentially identical to the one of section 3.2; we only substitute $U(b, g, b')$ for $U(g)$. Therefore, in the present context an *economy* is an array $(\beta, U, \gamma, f^g, f^b, \bar{B})$, a *polity* is a vector (λ, n), and a *society* is the combination of an economy and a polity.

We modify the game of the previous section as little as possible. The players are the same. A *history* of policies is now an array $h^t = ((g_0, b_1), (g_1, b_2), ..., (g_t, b_{t+1}))$. After observing h^{t-1}, the date-t incumbent selects a policy (g_t, b_{t+1}). The probability that any given party will be elected is equal to $1/n$. A *symmetric political equilibrium* is defined exactly as before.[23] Finally, if $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ is a symmetric political outcome, then the

[22] This assumption implies that, given b and b', the optimal g is smaller than its attainable upper bound. Hence, a profligate government has room to overspend without increasing the public debt unless $B = \bar{B}$.

[23] At this point, one may feel compelled to consider a game in which date-t actions can depend only on

payoff of the date-s incumbent along the equilibrium path is

$$\Omega_s(\{g_t, b_{t+1}\}_{t=s}^{\infty}) = U(b_s, g_s, b_{s+1}) + \lambda g_s + \sum_{t=s+1}^{\infty} \beta^{t-s} \left[U(b_t, g_t, b_{t+1}) + \frac{\lambda}{n} g_t \right]. \tag{34}$$

4.3 The spendthrift equilibrium

We now turn to the characterization of an equilibrium outcome that we will use to support other equilibria by means of trigger strategies. Observe that the task here is not as simple as in the previous section. For example, even if the date-t incumbent believes that all other parties will implement the dictatorial policy regardless of the history h^{t-1}, it may find it optimal to issue debt to fund a level of g_t above the dictatorial level.

To characterize the equilibria set of our political game, it is convenient to define

$$G(b, b', \lambda) \equiv \arg\max_{g} [U(b, g, b') + \lambda g] \tag{35}$$

subject to[24]

$$g \leq f^g(b, b'). \tag{36}$$

Function $G(.)$ defines the level of g that maximizes the incumbent's period payoff, given (b, b'). The first-order condition associated with this problem is

$$U_g(b, G(b, b', \lambda), b') \geq -\lambda. \tag{37}$$

This condition holds with equality whenever (36) does not bind.

We show in Lemma 2 of the online appendix that $G(.)$ is strictly decreasing in b, strictly increasing in b', and weakly increasing in λ. The intuition behind these properties is simple. If g and b' are held constant, an increase in b requires the government to increase its distortionary revenues. Since the definition of G entails finding an optimal balance between government consumption and distortionary taxation, G decreases as b rises. Similar reasoning implies that G increases in b'. Furthermore, a simple inspection of (35) suggests that G should be increasing in λ.[25]

b_t and study its corresponding Markov perfect equilibrium. However, it turns out that the efficient policy will not be an equilibrium outcome in such a game. If the date-t incumbent believes that all other parties will implement the policy $(g_s, b_{s+1}) = (g^*(0,0), 0)$ whenever $b_s = 0$, then the best action for that player is to set (g_s, b_{s+1}) equal to $(g^D, 0)$. Given our interest in the implementability of efficient policies, we must consider a game that does not have a Markov structure.

[24] Another constraint is $g \geq \gamma$, but it never binds.

[25] The only hurdle in the process of formalizing that reasoning is that constraint (36) binds for some

Suppose that the date-t incumbent believes that all other parties will leave a debt $\bar{B}$ regardless of the debt they inherited. If under this assumption the best strategy for the date-t incumbent is to set $b_{t+1} = \bar{B}$, then we have an equilibrium in which the first incumbent enjoys a relatively high payoff and future governments have no option but to set $g_t = \gamma$ and $b_{t+1} = \bar{B}$. In particular, for the policy plan

$$\tilde{\sigma}_t(h^{t-1}) = (G(b_t, \bar{B}, \lambda), \bar{B}) \tag{38}$$

to be a symmetric political equilibrium for every h^{t-1}, λ must be sufficiently large. In this equilibrium, the corresponding outcome is $\{\tilde{g}_t, \tilde{b}_{t+1}\}_{t=0}^{\infty}$, where $\tilde{g}_{t+1} = \gamma$ and $\tilde{b}_{t+1} = \bar{B}$ for every t, while $\tilde{g}_0 = G(0, \bar{B}, \lambda)$. That is, the date-0 incumbent sets a value for g_0 high enough to drive the economy to a steady state characterized by γ and $\bar{B}$. We refer to this equilibrium and its outcome as the *spendthrift equilibrium*.[26]

Since λ measures politicians' degree of profligacy, at first glance it may seem obvious that the spendthrift policy would be an equilibrium outcome if λ were large enough. However, this need not be true. For instance, recall that $b_{t+1}^D = 0$ for every t. Hence, regardless of λ, a dictator would not expand the public debt. The reason is that a high λ represents a penchant for rents today but also in the future, and setting $b_1 = \bar{B}$ would decrease future rents to their minimum level.

Hence, for the spendthrift policy to be an equilibrium outcome, two conditions must be met:

($C1$) politicians are sufficiently profligate;

($C2$) the rate at which an incumbent can substitute g_t for g_{t+1} is not too small.

Recall that q_t denotes the price of b_{t+1}, in units of g_t. By issuing one unit of b_{t+1} the government can increase g_t by q_t units. To balance its date $t+1$ budget, the government can reduce g_{t+1} by exactly one unit. Hence, an incumbent can use the public debt to substitute g_t for g_{t+1} at a rate equal to q_t.

The partial derivatives of the date-t incumbent's payoff with respect to g_t and g_{t+1} are equal to, respectively, $U_g(b_t, g_t, b_{t+1}) + \lambda$ and $\beta[U_g(b_{t+1}, g_{t+1}, b_{t+2}) + \lambda/n]$. Therefore,

$$-\frac{dg_t}{dg_{t+1}} = \beta \frac{U_g(b_{t+1}, g_{t+1}, b_{t+2}) + \lambda/n}{U_g(b_t, g_t, b_{t+1}) + \lambda},$$

(b, b', λ). As a result, the partial derivatives G_b, $G_{b'}$, and G_λ may be undefined at those points.

[26] The spendthrift equilibrium shares some characteristics with the financial autarky equilibrium of Aguiar and Amador (2011), where a deviation by the government from its promised payments locks the country forever into financial autarky, and as a result the deviating government chooses to set the tax rate at its maximum possible level.

where $-dg_t/dg_{t+1}$ is a standard intertemporal marginal rate of substitution. Thus, the date-t incumbent has an incentive to increase g_t and to reduce g_{t+1} by issuing debt whenever:

$$q_t > \beta \frac{U_g(b_{t+1}, g_{t+1}, b_{t+2}) + \lambda/n}{U_g(b_t, g_t, b_{t+1}) + \lambda}. \tag{39}$$

Inequality (39) reveals how the combination of political competition with conditions $(C1)$ and $(C2)$ brings forth the spendthrift equilibrium. Make $\lambda \to \infty$. Since U_g is bounded, the right-hand side of (39) converges to β/n. Hence, for λ sufficiently large, (39) holds whenever

$$q_t > \beta/n. \tag{40}$$

It is well known from basic macroeconomics that if an economy is in a deterministic steady state, $q_t = \beta$. Thus, if λ is large and q_t is not considerably smaller than its steady-state value, the date-t incumbent will have an incentive to issue debt and increase g_t.

We formally establish in the online appendix that if conditions $(C1)$ and $(C2)$ are satisfied, then the spendthrift policy is an equilibrium outcome. For condition $(C1)$, we require that

$$\lambda > \tilde{\lambda}, \tag{41}$$

where $\tilde{\lambda}$ is a real number whose existence is established in that appendix.[27] In turn, condition $(C2)$ entails placing a lower bound on q_t. Since that variable is not an explicit component of our political game, we must disentangle it from the whole structure of the game.

To do so, take a policy $\{g_t, b_{t+1}\}_{t=s}^{\infty}$ with the property that $g_t = G(b_t, b_{t+1}, \lambda)$. For simplicity, assume that the partial derivatives G_b and $G_{b'}$ are defined at every point (b, b', λ). Let t be any date and δ be a small positive number. If b_{t+1} increases by δ, g_t will grow by approximately $\delta G_{b'}(b_t, b_{t+1}, \lambda)$, while g_{t+1} will fall by approximately $-\delta G_b(b_{t+1}, b_{t+2}, \lambda)$. Hence, a policymaker can substitute g_t for g_{t+1} at the rate

$$-\frac{\delta G_{b'}(b_t, b_{t+1}, \lambda)}{\delta G_b(b_{t+1}, b_{t+2}, \lambda)} = -\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)}.$$

However, the rate at which a policymaker can substitute g_t for g_{t+1} is also equal to q_t.

[27] When $\lambda \leq \tilde{\lambda}$, one can show that if $U(.)$ satisfies some regularity conditions, then there exists an equilibrium outcome in which the public debt converges monotonically to $\bar{B}$. Given that outcome, it is possible to characterize a subset of all equilibrium outcomes. By contrast, under (41), one can characterize the entire set, in addition to allowing for a faster transition to the state $(\gamma, \bar{B})$.

Therefore, we can express q_t as

$$q_t = -\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)}.$$

Substituting back into (40), that condition becomes

$$-\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)} > \frac{\beta}{n}.$$

For this condition to hold for all n, we need that

$$-\frac{G_{b'}(b_t, b_{t+1}, \lambda)}{G_b(b_{t+1}, b_{t+2}, \lambda)} > \frac{\beta}{2}. \tag{42}$$

In the online appendix we provide a more general version of (42) that takes into consideration, among other technical issues, that G_b and $G_{b'}$ may be undefined at some points (b, b', λ).

All that being said, henceforth we assume that conditions $(C1)$ and $(C2)$ are satisfied, so that the spendthrift policy is an equilibrium outcome.

4.3.1 Example

Let

$$U(b, g, b') = -\frac{1}{2}\left(g - a_1 e^{-a_2 b}\right)^2 + a_3(b' - b)g + W(b, b') \tag{43}$$

and

$$f^g(b, b') = \gamma + (2a_4 - a_5)\bar{B} - 2a_4 b + a_5 b', \tag{44}$$

where a_1, a_2, a_3, a_4, and a_5 are positive constants and W is any differentiable function increasing in b'. Those functions, which exhibit reasonable and easily understandable features, lead to a G satisfying (42).

Consider first the function U. Observe that its partial derivatives satisfy (30). To grasp the meaning of parameters a_1, a_2, and a_3, take debt values b and b' smaller than $\bar{B}$. Then, $g^*(b, b') = a_3(b' - b) + a_1 e^{-a_2 b}$. Since $g^*(0, 0) = a_1$, we can interpret a_1 as a parameter that defines the efficient level of g. Parameter a_2 defines the impact of equal variations in b' and b over g^*, while a_3 measures the impact of the public debt growth $(b' - b)$ on g^*.

The function f^g is a plain affine relation. In line with our description of the economic features of the model, $f^g_b < 0$, $f^g_{b'} > 0$, and $f^g(\bar{B}, \bar{B}) = \gamma$. Its parameters a_4 and a_5 define the respective impacts of changes in b and b' on the upper bound of g_t.

The function G induced by (43) and (44) satisfies inequality (42) for several vectors $(a_1, a_2, a_3, a_4, a_5)$. Take a point (b, b', λ) such that $G(b, b', \lambda) = f^g(b, b')$. It follows that (42) holds whenever $a_5 > \beta a_4$. Now take a point (b, b', λ) such that $G(b, b', \lambda) < f^g(b, b')$. Then (36) does not bind and $G(b, b', \lambda) = a_3(b' - b) + a_1 e^{-a_2 b} + \lambda$. For simplicity, let the lower bound on b satisfy $\bar{b} = 0$; therefore, (42) holds for every value of b' and b if $(2 - \beta)a_3 > \beta a_1 a_2$.

4.4 The political feasibility of the efficient policy

We use trigger strategies that specify reversion to the spendthrift policy plan $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$ to characterize a set of equilibrium outcomes. Define the *revert-to-spendthrift plan* associated with a policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ as a plan such that, if the prevailing history is exactly $\{g_t, b_{t+1}\}_{t=0}^{s-1}$, a player sticks to the policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$; otherwise, the player implements the policy specified in (38). If the policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ satisfies

$$\begin{aligned} \Omega_s(\{g_t, b_{t+1}\}_{t=s}^{\infty}) \;\geq\; & U(b_s, G(b_s, \bar{B}, \lambda), \bar{B}) + \\ & \lambda G(b_s, \bar{B}, \lambda) + \sum_{t=s+1}^{\infty} \beta^{t-s} \left[U(\bar{B}, \gamma, \bar{B}) + \frac{\lambda}{n}\gamma \right] \end{aligned} \tag{45}$$

for every s, then $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ is a symmetric policy outcome. This is so because inequality (45) ensures that the corresponding revert-to-spendthrift plan is an equilibrium strategy. Observe that condition (45) is not only sufficient, but also necessary for a policy $\{g_t, b_{t+1}\}_{t=0}^{\infty}$ to be an equilibrium outcome. Indeed, if (45) were not satisfied at some date s, the incumbent could implement $(G(b_s, \bar{B}, \lambda), \bar{B})$ and achieve the payoff specified in the right-hand side. Since (45) is a necessary and sufficient condition that any symmetric policy outcome must satisfy, it provides a complete characterization of the set of all symmetric political outcomes. This allows us to obtain stronger results than those of section 3.

With some abuse of notation, let $\Omega(g, b)$ denote the payoff of the incumbent party if all parties implement the static policy (g, b). Hence,

$$\Omega(g, b) = \frac{1}{1 - \beta} \left[U(b, g, b) + \left(1 - \beta + \frac{\beta}{n} \right) \lambda g \right]. \tag{46}$$

It follows from (45) that the efficient policy $(g^*(0, 0), 0)$ is a symmetric political outcome if and only if $\Omega(g^*(0, 0), 0) \geq \Omega_0(\{\tilde{g}_t, \tilde{b}_{t+1}\}_{t=0}^{\infty})$. Let $\Delta U \equiv U(0, g^*(0, 0), 0) - U(\bar{B}, \gamma, \bar{B})$ and

$\Delta V \equiv V(0, G(0, \bar{B}, \lambda), \bar{B}) - V(0, g^*(0,0), 0)$. The last inequality is equivalent to

$$\frac{\beta}{1-\beta}\left[\Delta U + \frac{\lambda}{n}(g^*(0,0) - \gamma)\right] \geq \Delta V. \tag{47}$$

The right-hand side is the short-run gain for an incumbent from selecting the spendthrift policy instead of the efficient one. The left-hand side corresponds to its future payoff gain from the implementation of the efficient instead of the spendthrift policy. Since $g^*(\bar{B}, \bar{B}) = \gamma$, an appeal to (33) establishes that $U(0, g^*(0,0), 0) > U(\bar{B}, \gamma, \bar{B})$. Moreover, $g^*(0,0) > \gamma$. Therefore, the left-hand side is strictly positive and strictly decreasing in n. The right-hand side is also positive, since

$$V(0, G(0, \bar{B}, \lambda), \bar{B}) > V(0, g^*(0,0), \bar{B}) \geq V(0, g^*(0,0), 0).$$

Consider the inequalities

$$\frac{\beta}{1-\beta}\Delta U \geq \Delta V \tag{48}$$

and

$$\frac{\beta}{1-\beta}\Delta U < \Delta V. \tag{49}$$

Like in section 3, the analysis depends on which of the two inequalities holds. However, the comparison between g^* and the level of g achieved in the absence of coordination—γ in the current setting and g^D in section 3, where $\gamma < g^*(0,0) < g^D$—has critical implications for the consequences of political competition.

Proposition 3 *If a society* $(\beta, U, \gamma, f^g, f^b, \bar{B}, \lambda, n)$ *satisfies (48), then the efficient policy* $(g^*(0,0), 0)$ *constitutes a symmetric political outcome.*

Proof. Combine inequalities (48) and $(\lambda/n)(g^*(0,0) - \gamma) > 0$ to conclude that (47) holds. As a consequence, $(g^*(0,0), 0)$ is an equilibrium outcome. ■

If politicians are sufficiently profligate and the payoffs satisfy (48), the efficient policy is an equilibrium outcome regardless of the degree of political competition. There was no such a result in section 3. It arises here because the efficient policy yields a level of rents greater than the level under the spendthrift equilibrium after date zero.

The analysis is richer when inequality (49) holds. Define $N^b(\beta, \lambda)$ as

$$N^b(\beta, \lambda) \equiv \frac{\lambda(g^*(0,0) - \gamma)}{\frac{1-\beta}{\beta}\Delta V - \Delta U}. \tag{50}$$

Observe that under (49), $N^b(\beta, \lambda) > 0$.

Proposition 4 *If a society* $(\beta, U, \gamma, f^g, f^b, \bar{B}, \lambda, n)$ *satisfies (49), then the efficient policy* $(g^*(0,0),0)$ *constitutes a symmetric political outcome if and only if* $n \leq N^b(\beta, \lambda)$.

Proof. We start with the "if" part. Condition (47) holds with equality when $n = N^b(\beta, \lambda)$. Since the left-hand side of (47) is strictly decreasing in n, it holds whenever $n \leq N^b(\beta, \lambda)$. Thus, the efficient policy is a symmetric political outcome. For the "only if" part, assume that $(g^*(0,0),0)$ is an equilibrium outcome. Therefore, (47) must hold. As a consequence, $n \leq N^b(\beta, \lambda)$. ■

Some implications of Proposition 4 are quite different from those of its counterpart under no public debt, Proposition 2. First, while the latter lays down a sufficient condition, Proposition 4 establishes one that is necessary and sufficient. Second, and more importantly, the function $N^b(\beta, \lambda)$ establishes the *maximum* number of parties that makes it possible to sustain the efficient policy through trigger strategies. Thus, when the government is free to borrow, the implementation of the efficient policy requires an upper bound—instead of a lower bound—on the number of competing parties.

The combination of Propositions 3 and 4 implies that, provided politicians are sufficiently profligate, the efficient policy can be an equilibrium outcome regardless of whether (48) or (49) holds. If the former prevails, the efficient policy is an equilibrium outcome for every value of n. But if the latter holds, then political competition cannot be too intense. Therefore, it is important to understand the conditions that determine which of those inequalities will prevail.

Lemma 1 *For every economy* $(\beta, U, \gamma, f^g, f^b, \bar{B})$, *there exists a number* λ_0^b *such that, if a polity* (λ, n) *satisfies* $\lambda > \lambda_0^b$, *then inequality (49) holds.*

Proof. See online appendix.

If $N^b(\beta, \lambda) < 2$ and $\lambda > \lambda_0^b$, Proposition 4 and Lemma 1 imply that the efficient policy cannot be an equilibrium outcome. However, note that $N^b(\beta, \lambda) > 2$ if β is sufficiently close to one. Henceforth we concentrate on this more interesting case. It follows that if politicians are very profligate but consumers are sufficiently patient, then some, but not too much, political competition is necessary and sufficient to ensure that the efficient policy is a symmetric political outcome.

Corollary 1 *For every economy* $(\beta, U, \gamma, f^g, f^b, \bar{B})$*, if a polity* (λ, n) *satisfies* $\lambda > \lambda_0^b$*, the efficient policy* $(g^*(0,0),0)$ *constitutes a symmetric political outcome if and only if* $n \leq N^b(\beta, \lambda)$*.*

We conclude this section with a synthesis of its results. We study the strategic interactions of competing political parties in a dynamic political game where the public debt links the incumbent's choices to the action space of future governments. If politicians are sufficiently profligate ($\lambda > \tilde{\lambda}$), there is an equilibrium in which the date-0 incumbent sets current public expenditures very high, pushing the public debt up to the point of immiserizing the economy forever, in the sense of leaving welfare stuck at $U(\bar{B}, \gamma, \bar{B})$ for $t \geq 1$. Adopting that equilibrium as a benchmark, we use trigger strategies to characterize the feasibility of the efficient outcome. When political economy motives really matter ($\lambda > \lambda_0^b$), the efficient policy can be implemented if there is some ($n \geq 2$), but not too much ($n \leq N^b(\beta, \lambda)$) political competition. By contrast, if political competition is very intense, the optimal policy is not sustainable. As a result, the economy becomes trapped in a bad equilibrium where welfare is $U(\bar{B}, \gamma, \bar{B})$ for $t \geq 1$. Hence, intense political competition can hurt social welfare considerably when governments have easy access to the public debt.

5 Political competition and debt limits

We have found that intense political competition encourages a political compromise when the government cannot borrow but discourages it when access to debt is unrestricted. We now show that there is a more general relationship between political competition, constraints on government borrowing, and the viability of efficient policies. We do so by generalizing the model of section 4 so that those of sections 3 and 4 become special cases. Specifically, we let the public debt be constrained by a legal ceiling B_L. If $B_L = 0$, we obtain the model without debt of section 3; if $B_L \geq \bar{B}$, we have the model of section 4.

We assume that the conditions that ensure that the spendthrift policy is an equilibrium outcome are satisfied. One can then extend the reasoning used in section 4 to establish that, for any $B_L \in (0, \bar{B})$, the *constrained spendthrift policy* (i.e., the spendthrift policy with B_L replacing $\bar{B}$) also is an equilibrium outcome. Defining $\Delta U_L \equiv U(0, g^*(0,0), 0) - U(B_L, G(B_L, B_L, \lambda), B_L)$ and $\Delta V_L \equiv V(0, G(0, B_L, \lambda), B_L) - V(0, g^*(0,0), 0)$, it follows that the efficient policy $(g^*(0,0),0)$ is an equilibrium outcome whenever the net gain from a

political compromise (NGC) is positive:

$$NGC \equiv \frac{\beta}{1-\beta}\left[\Delta U_L + \frac{\lambda}{n}(g^*(0,0) - G(B_L, B_L, \lambda))\right] - \Delta V_L \geq 0. \tag{51}$$

To study the conditions under which inequality (51) holds, define $N^L(\beta, \lambda, B_L)$ analogously to $N^0(\beta, \lambda)$ and $N^b(\beta, \lambda)$:

$$N^L(\beta, \lambda, B_L) \equiv \frac{\lambda[g^*(0,0) - G(B_L, B_L, \lambda)]}{\frac{1-\beta}{\beta}\Delta V_L - \Delta U_L}. \tag{52}$$

Now recall that $G(\bar{B}, \bar{B}, \lambda) = \gamma < g^*(0,0) < g^D = G(0,0,\lambda)$. Since $G(b, b, \lambda)$ is continuous in b, an appeal to the intermediate value theorem establishes that there exists a debt level $\hat{B} \in (0, \bar{B})$ with the property that $G(\hat{B}, \hat{B}, \lambda) = g^*(0,0)$. Since $G(b, b, \lambda)$ is strictly decreasing in b, $\hat{B}$ is unique. We then have that

$$sgn\left[B_L - \hat{B}\right] = sgn\left[g^*(0,0) - G(B_L, B_L, \lambda)\right].$$

Suppose first that $B_L > \hat{B}$. It follows from the approach of section 4 that if

$$\frac{\beta}{1-\beta}\Delta U_L \geq \Delta V_L, \tag{53}$$

then $(g^*(0,0), 0)$ is a symmetric political outcome for any n. If instead

$$\frac{\beta}{1-\beta}\Delta U_L < \Delta V_L, \tag{54}$$

then $(g^*(0,0), 0)$ is a symmetric political outcome if $n \leq N^L(\beta, \lambda, B_L)$.

Consider now the case in which $B_L < \hat{B}$. We use the approach of section 3 to conclude that if $[\beta/(1-\beta)]\Delta U_L \leq \Delta V_L$, then $(g^*(0,0), 0)$ cannot be a symmetric political outcome. Similarly, if $[\beta/(1-\beta)]\Delta U_L > \Delta V_L$, then $(g^*(0,0), 0)$ is a symmetric political outcome provided that $n \geq N^L(\beta, \lambda, B_L)$.

Finally, if $B_L = \hat{B}$, NGC does not depend on n and the efficient policy is an equilibrium outcome if (53) holds.

We can now study how a society can use B_L to improve economic policy. A simple example helps to fix ideas. Consider a society in which $N^b(\beta, \lambda) = 3$ and $N^0(\beta, \lambda) = 4$. Suppose that $n = 5$. Clearly, the efficient policy is feasible if $B_L = 0$, but may not be if debt is unrestricted. Now suppose that $n = 2$. If government borrowing is forbidden,

the efficient policy is not sustainable and the economy suffers as a result. However, if a legal reform removes the debt ceiling, then the parties will able to coordinate on the efficient policy. An analogous conclusion follows if we assume that the initial debt ceiling B_L belongs to $(0, \hat{B})$. If players have coordinated on the constrained spendthrift policy (so that $b_t = B_L$) and the debt ceiling is removed, then players will be able to coordinate on the efficient policy.[28] Hence, the removal of a debt ceiling can induce the political parties to forge a compromise that sustains the efficient policy by casting the shadow of an immiserating economic prospect.

This example illustrates a more general principle: a debt ceiling is deleterious whenever $n < \min\{N^0(\beta, \lambda), N^b(\beta, \lambda)\}$. On the other hand, a society can achieve better economic outcomes by placing a legal ceiling on the public debt if $n > \max\{N^0(\beta, \lambda), N^b(\beta, \lambda)\}$. In sum, the desirability of a legal debt limit hinges on the existing level of political competition. More generally, we have the following result.

Proposition 5 *The net gain from coordination (*NGC*) is submodular in the degree of political competition (n) and the ceiling on the public debt (B_L).*

To see this result, it suffices to note that

$$\frac{\partial^2 NGC}{\partial n \partial B_L} = \frac{\beta}{1-\beta}\frac{\lambda}{n^2}\left[G_b(B_L, B_L, \lambda) + G_{b'}(B_L, B_L, \lambda)\right] < 0.$$

It follows from the properties of submodular functions that the value of B_L that maximizes NGC is decreasing in n. The intuition is as follows. A tighter B_L lowers the short-run gain from not cooperating. It also influences the long-run gain from cooperation, affecting the payoffs of households and lowering the parties' expected future rent gain.[29] Now, the impact of B_L on the short-run gain from cooperation is independent of the degree of political competition, whereas its effect on the future reduction of rents is more important, the less intense political competition is. Thus, a tighter B_L is more likely to undermine an otherwise feasible political compromise when competition is limited. Conversely, it is more likely to promote an otherwise unfeasible compromise when political competition is intense.

At a more fundamental level, observe that a political compromise can both improve economic outcomes and preserve office rents. The latter effect can be critical to make the

[28] In this particular case the efficient policy will depend on B_L, since that will be the initial (i.e., before reform) value of the public debt.

[29] The future rent gain from cooperation falls from $\lambda(g^* - \gamma)$ when $B_L = \bar{B}$ to $\lambda(g^* - g^D)$ when $B_L = 0$, where $\lambda(g^* - g^D) < 0 < \lambda(g^* - \gamma)$.

compromise sustainable, and is present when constraints on the public debt are lax. Since preserving rents is more important when there is less political competition, constraints on political competition can improve economic outcomes when the public debt is relatively unconstrained. Conversely, a tight debt limit is advisable when political competition is intense. Put simply, the political feasibility of the efficient policy tends to require either a limit on political competition or a cap on the public debt, but not both.[30]

Figure 1 illustrates this result. It displays three curves: ΔV_L and $GC_L \equiv \frac{\beta}{1-\beta}[\Delta U_L + \frac{\lambda}{n}(g^*(0,0) - G(B_L, B_L, \lambda))]$ for $n = n_l$ and for $n = n_h$, $n_l < n_h$. Observe that ΔV_L is independent of n. The curves are drawn arbitrarily, except for the constraint imposed by Proposition 5, that the slope of GC_L in B_L must decrease with n. In the example of Figure 1, this implies that under intense political competition ($n = n_h$), the efficient policy is sustainable if the debt is sufficiently restricted ($B_L \leq B_1$). In turn, under weak political competition ($n = n_l$), the efficient policy is sustainable only if the debt is left relatively unrestricted ($B_L \geq B_2$). For moderate levels of the debt ceiling ($B_1 < B_L < B_2$), the policy g^* is unfeasible for either n_l or n_h.

6 Empirical implications

Our model sheds light on the relationship between political competition, debt limits and economic outcomes. Since the model is stylized, it does not lend itself directly to empirical scrutiny. However, its main message carries important empirical implications. The existence of legal constraints on government borrowing is often disregarded in studies of the effects of political competition. Similarly, measures of political competition are not regularly considered in studies of the effects of fiscal constraints.[31] Our findings indicate that such omissions can create systematic biases in empirical research. Moreover, simply controlling for the omitted factor is not enough. Rather, our analysis stresses the importance of the *interaction* between measures of political competition and debt restraints.

For example, suppose that we want to understand the role of fiscal and debt constraints on economic performance.[32] For the sake of the argument, assume that the constraints

[30] It is important to stress that although we have taken the level of political competition as given, in reality it is endogenous to the country's legislative and electoral rules. Morelli (2004), for example, shows how different electoral systems lead to different equilibrium numbers of political parties. Likewise, Callander (2005) shows how splitting the vote in heterogeneous districts affects the equilibrium level of political competition and the resulting policies.

[31] As noted in section 2, the empirical study of Grembi et al. (2014) is a notable exception.

[32] In our model the only available fiscal constraint is on the level of the public debt. At a more general level, one may consider any rule that prevents a government from increasing either expenditures or debt

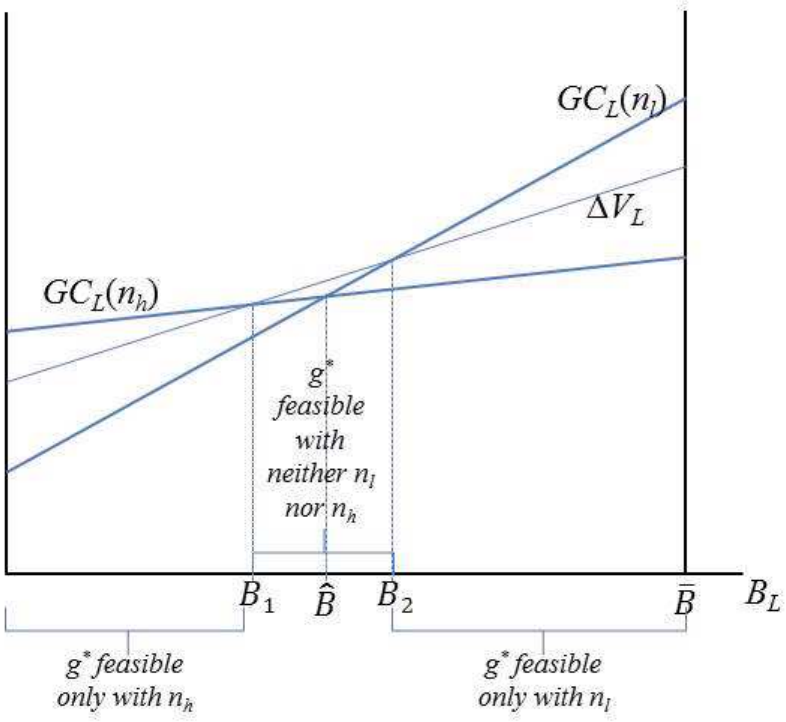

Figure 1: The tradeoff between political competition and debt limits

are exogenous—or more plausibly, that suitable instruments for the introduction of the constraints are available. One may then consider a panel regression of the form

$$y_{it} = \alpha_i + \alpha_t + \alpha_f FC_{it} + \boldsymbol{\alpha}_{\mathbf{x}}\mathbf{X}_{it} + \epsilon_{it},$$

where y_{it} is an economic outcome (such as income per capita), FC_{it} measures fiscal constraints and $\mathbf{X_{it}}$ is a vector of controls, with i and t indexing countries and years, respectively. α_i is a fixed effect to control for unobserved country characteristics that are constant (or change slowly) over time, whereas α_t is a year fixed effect to control for global changes in the variables of interest. Relating to our model, we can interpret FC_{it} as a proxy that is inversely related to constraints on the public debt ($B_{L_{it}}$). Our analysis indicates that the estimated α_f should be positive when the panel contains mostly cases with strong political competition (when n is high, a low B_L—which is equivalent to a tight fiscal constraint—facilitates a political compromise), but negative when it contains mostly cases where political competition is weak. If the panel is relatively balanced between the

as a real-world counterpart of our theoretical constraint.

two cases, the estimated α_f would tend to be statistically indistinguishable from zero. Thus, without explicitly considering the degree of political competition in the analysis, *any* estimate of α_f could be consistent with our model.

How can this ambiguity be fixed? The model stresses that a more appropriate specification would incorporate a measure of political competition (n_{it}) *and* its interaction with FC_{it}:[33]

$$y_{it} = \alpha_i + \alpha_t + \alpha_n n_{it} + \alpha_f FC_{it} + \alpha_{nf} n_{it} FC_{it} + \boldsymbol{\alpha}_{\mathbf{x}} \mathbf{X}_{it} + \epsilon_{it}. \tag{55}$$

The key prediction of our model is that $\alpha_{nf} > 0$, so that the impact of a fiscal constraint on economic performance should be greater the stronger political competition is. An analogous point applies to empirical analyses of the economic consequences of political competition.

Identifying the parameters of (55) empirically is a tall order, and it is beyond the scope of this paper to attempt such an investigation. The difficulties range from the measurement to the endogeneity of the key independent variables. Nevertheless, we can at least investigate whether the main variables of the analysis correlate in a manner consistent with the model. To do so, we proxy y_{it} with (the log of) GDP per capita, employ dummies to classify the existence of meaningful fiscal constraints, and use several proxies employed in the literature to measure the degree of political competition. All variables are at the country level.

Specifically, to represent political competition the definition that is closest in spirit to our model would be a measure of the number of political parties, like the first one below. As any such measures are imperfect, we also use three alternative definitions:[34]

1. Weighted number of political parties. Calculated as the inverse of a Herfindahl index of the number of political parties, with weights (s) given by the vote shares of each party: $(\sum_n s_i^2)^{-1}$.

2. Fractionalization. Calculated as the probability that two representatives picked at random from the legislature will be of different parties.

[33] Again, assume that fiscal constraints and the level of political competition are exogenous to the context of the analysis, or that appropriate instruments are available.

[34] These variables represent two of the key dimensions of political competition emphasized by Bartolini (1999, 2000): contestability and vulnerability. According to the Democracy Barometer, "vulnerability corresponds to the uncertainty of the electoral outcome, which is indicated by the closeness of election results as well as the degree of concentration of parliamentary or legislative seats," whereas "contestability refers to the stipulations that electoral competitors have to meet in order to be allowed to enter the race, [where] effective competition in elections is measured by the existence and the success of small parties" (http://www.democracybarometer.org/concept_en.html).

3. Winning margin. Calculated as 100 minus the percentage of votes obtained by the strongest party.

4. Seat difference. Calculated as 100 minus the difference between the largest and second largest lower house parties in percentage of all seats.

For each of those measures, a higher value indicates a more competitive political system. The first two measures are based on data from the World Bank's Political Institutions Database; the other two are from the Democracy Barometer. We take their logs to facilitate interpretation of the correlations.

To define the presence of meaningful fiscal constraints, we use the dataset compiled by and described in Schaechter et al. (2012), which covers national and supranational fiscal rules for IMF members. They distinguish between four types of rules: debt, budget balance, revenue and expenditure rules. For each of those, they codify 4 or 5 specific characteristics and aggregate them into sub-indices for each of the four types of rules. A global index can then be constructed from the four sub-indices. Simply adding up the four sub-indexes, which range from 0 to a maximum that varies from 0.8 for the revenue sub-index to 1.4 for the budget balance sub-index, the global index ranges from 0 to 4 in the sample. We define the presence of a 'strong' fiscal constraint ($FC_{it} = 1$) if the global index is 1.5 or higher. This yields a sample average of 0.15 for FC_{it}.

As the country fixed effects subsume all time-invariant country characteristics, in the vector of controls we include only the ratio of debt to GDP. Controlling for the debt-GDP ratio helps to avoid selection issues, as countries may adopt a fiscal rule precisely when that ratio has been increasing, or has reached a particularly high level.[35] Data come from the IMF's World Economic Outlook. There are available data for both fiscal restrictions and political competition measures for 69 countries when the World Bank measures are used, or for 47 countries when the Democracy Barometer variables are employed, for the period 1991-2012. We estimate equation (55) in five different ways; the results are presented in Table 1.

Observe that due to the country fixed effects, the parameters reflect partial correlations stemming only from within-country changes in policy and in the degree of political competition. For comparison purposes, in column (1) we omit the political variables. That specification might suggest that a fiscal constraint has no effect on economic performance.

[35] Moreover, authors such as Reinhart and Rogoff (2010) argue that the level of the debt itself can affect economic performance.

In each of the remaining four specifications, we proxy n_{it} by one of the four measures of political competition described above. Except for seat difference (column (5)), the interactions with the measures of political competition have a positive and statistically significant coefficient. Those correlations suggest that the benefits from adopting a tight fiscal constraint tend to increase with the degree of political competition, in line with the predictions of the model. For example, an increase of 10% in the measure for the number of parties is associated with a 0.6% increase in GDP per capita *if* there is a strong fiscal rule in place—and with a *decrease* of 0.5% in GDP per capita otherwise.[36]

Table 1: Fiscal Constraints, Political Competition and Economic Performance

Dep. var.: *log*(GDP per capita)	(1)	(2)	(3)	(4)	(5)
Fiscal Constraint$_{t-1}$	0.031	-0.081*	0.072**	-0.677**	0.127
	(0.062)	(0.041)	(0.033)	(0.257)	(0.192)
Nb of Parties$_{t-1}$		-0.054**			
		(0.026)			
FC$_{t-1}$×Nb of Parties$_{t-1}$		0.118***			
		(0.040)			
Fractionalization$_{t-1}$			-0.010		
			(0.025)		
FC$_{t-1}$×Fractionalization$_{t-1}$			0.209**		
			(0.095)		
Winning Margin$_{t-1}$				-0.118	
				(0.108)	
FC$_{t-1}$×Winning Margin$_{t-1}$				0.170**	
				(0.064)	
Seat Difference$_{t-1}$					0.056
					(0.046)
FC$_{t-1}$×Seat Difference$_{t-1}$					-0.023
					(0.045)
Debt/GDP$_{t-1}$	-0.146	-0.021	-0.011	-0.250***	-0.232***
	(0.141)	(0.023)	(0.024)	(0.042)	(0.048)
Country FE	yes	yes	yes	yes	yes
Year FE	yes	yes	yes	yes	yes
Number of obs	1,453	1,192	1,170	842	840
R-squared	0.181	0.673	0.670	0.229	0.225

***: significant at 1%; **: significant at 5%; *: significant at 10%
Robust standard errors in parentheses.

[36]Instead of using a dummy that captures all types of fiscal rules, we could define a similar dummy considering only debt constraints. In that case, except for fractionalization, the partial correlations remain qualitatively similar both in terms of magnitudes and of statistical significance. In contrast, if we use dummies representing a 'weak' fiscal or debt restriction—e.g., using criteria flexible enough to yield a dummy whose sample average is roughly 0.5—most of the coefficients become statistically indistinguishable from zero. This makes sense, as weak rules tend to make it especially easy for governments to circumvent the constraints, making them ineffective and hence blurring any statistical correlation with other variables.

7 Concluding remarks

We study how ceilings on the public debt and the degree of political competition jointly affect the feasibility of efficient policies in a majoritarian political system. Due to political economy frictions, the efficient policy is implemented only if the political parties forge a compromise between them, a central facet of policymaking in democracies. We find that when the government faces tight limits on its ability to finance its expenditures through debt, efficient policies can be sustained only if political competition is sufficiently strong. The reason is that intense political competition reduces the probability that each party will hold power in the future. This lowers the value of future political rents, facilitating a compromise that curbs discretionary spending. Yet the reverse is true when the government is relatively free to finance its expenditures with debt. In that case, efficient policies *raise* future rents, by preventing equilibria where the economy becomes so indebted that future governments have no choice but to set public expenditures at an inefficiently low level. As a result, intense political competition makes efficient policies harder to sustain. In particular, when politicians are very profligate, efficient policies can be made feasible only under limited political competition and unrestricted access to the public debt.

We obtain those results by departing from the usual emphasis in the political economy literature on how competition allows voters to discipline politicians, to an environment where political parties can discipline *each other*. Similarly, the forces shaping the desirability of a debt limit are not the ones usually emphasized in the literature, where a debt ceiling curbs rent-seeking behavior but prevents the debt from fulfilling its tax smoothing purpose. Instead, here a debt limit affects the availability of future rents and, through that channel, the viability of an intertemporal compromise among political parties. This novel perspective allows us to highlight the difficulty of simultaneously having intense political competition, unrestricted access to the public debt, and efficient economic policies.

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Online Appendix (not intended for publication) for

The Limits of Political Compromise: Debt Ceilings and Political Competition

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In this appendix we (I) study the properties of an alternative equilibrium in which political parties coordinate on a policy that maximizes the lifetime payoff of the incumbent; (II) we show that the function $U(b, g, b')$ provides an exact measure of the lifetime payoff of the typical household in any of the equilibrium outcomes considered in the paper; (III) we establish that the spendthrift policy is an equilibrium outcome; and (IV) we present the proofs not included in the main text. We follow the numbering scheme of equations, propositions, lemmas, etc. of the main body of the paper.

I The politically optimal policy

In the main text we focus on the political feasibility of the efficient outcome, which maximizes household welfare. However, even if that outcome were sustainable, the political parties may want to coordinate on an alternative policy; in particular, politicians may prefer to coordinate on a policy that yields a higher lifetime payoff for *them*. This motivates the exercise carried out in this section. We present an equilibrium selection criterion based on politicians' incentives to maximize their payoffs and characterize the resulting equilibrium.

Without debt

Since the incumbent is the only player to implement an action at each date t, it is sensible to consider a stationary outcome where the incumbent proposes the (time-invariant) policy over which the parties coordinate. Due to the symmetry of the political parties, this proposed policy is independent of who holds office. Hence, in this equilibrium all political parties agree on implementing a stationary policy g^C that maximizes Ω in the universe of time-invariant policies, i.e., g^C maximizes (13). As it depends on β, λ and n, we denote it by $g^C(\beta, \lambda, n)$. We call this most cooperative policy the *politically optimal policy*. Note that among stationary policies, g^C is optimal from the perspective of the incumbent, whereas g^* is optimal from society's viewpoint.

Recall that g^D maximizes $V(g) = U(g) + \lambda g$. Since $0 < 1 - \beta + \beta/n < 1$, we can substitute $(1 - \beta + \beta/n)\lambda$ for λ in the definition of V to conclude that it is possible to apply our characterization of g^D to identify some of the properties of g^C. This reasoning establishes that $g^* < g^C(\beta, \lambda, n) \leq g^D$. Furthermore, if inequality

$$g^C(\beta, \lambda, n) < \Gamma \tag{56}$$

holds, $g^C(\beta, \lambda, n)$ is an interior optimum satisfying

$$U'(g^C(\beta, \lambda, n)) = -(1 - \beta + \beta/n)\lambda. \tag{57}$$

In that case, $g^C(\beta, \lambda, n) < g^D$, $g^C(\beta, \lambda, n)$ is strictly increasing in λ, and

$$\frac{\partial g^C}{\partial n} = \frac{\beta\lambda}{n^2 U''(g^C(\beta, \lambda, n))} < 0. \tag{58}$$

The intuition for the last result is simple. As n increases, the incumbent's future expected rents per period, $(\lambda/n)g$, decrease. Hence, its payoff is maximized at a lower level of g, which is closer to g^*.

Let $\Delta_0^C(n)$ denote the gain for the incumbent when all parties set $g = g^C$, relative to its payoff without cooperation:

$$\Delta_0^C(n) \equiv \Omega(g^C(\beta, \lambda, n), n) - \Omega(g^D, n),$$

where we emphasize that Ω depends on n both directly and indirectly, through g^C. Suppose that the political parties establish a political compromise to implement g^C. We have that if

political competition intensifies, both social welfare and politicians' gains from cooperation rise.

Proposition 6 *If (56) holds, then both household welfare and politicians' payoff under the politically optimal policy, respectively $U(g^C(\beta,\lambda,n))$ and $\Delta_0^C(n)$, are strictly increasing in the degree of political competition, n.*

Proof. Since $\frac{\partial U(g^C)}{\partial n} = U'(g^C)\frac{\partial g^C}{\partial n}$, combining (57) with (58) implies that $\frac{\partial U(g^C)}{\partial n} > 0$. Furthermore,

$$\frac{d\Delta_0^C}{dn} = \left[\frac{\partial\Omega(g^C,n)}{\partial g}\frac{\partial g^C}{\partial n} + \frac{\partial\Omega(g^C,n)}{\partial n}\right] - \left[\frac{\partial\Omega(g^D,n)}{\partial g}\frac{\partial g^D}{\partial n} + \frac{\partial\Omega(g^D,n)}{\partial n}\right].$$

However, $\frac{\partial\Omega(g^C,n)}{\partial g} = 0$ by the envelope theorem, $\frac{\partial g^D}{\partial n} = 0$ because the degree of political competition does not affect g^D, and $\frac{\partial\Omega(g,n)}{\partial n} = -\frac{\beta\lambda}{(1-\beta)n^2}g$. Therefore,

$$\frac{d\Delta_0^C}{dn} = -\frac{\beta\lambda}{(1-\beta)n^2}g^C + \frac{\beta\lambda}{(1-\beta)n^2}g^D = \frac{\beta\lambda}{(1-\beta)n^2}(g^D - g^C) > 0,$$

concluding the proof. ■

It is easy to grasp the intuition underlying Proposition 6. If g^C is an interior optimum, then the difference $\left[g^C(\beta,\lambda,n) - g^*\right]$ decreases as political competition intensifies, implying that $|U(g^C(\beta,\lambda,n)) - U(g^*)|$ also falls. Furthermore, by not implementing the dictatorial policy, the incumbent has an expected rent loss per period of $(\lambda/n)(g^D - g^C)$. A higher n decreases that loss.

We assume that coordination among the political parties is costless (and there are no collective action-like problems), but this may be unrealistic. For a moment, suppose instead that the parties would need to incur a fixed organizational cost $F > 0$ if they wanted to implement g^C. The feasibility of g^C in such a case would hinge on whether $\Delta_0^C(n) \geq F$. The flavor of our findings would nevertheless remain unchanged: because $d\Delta_0^C/dn > 0$ (Proposition 6), more intense political competition would make that condition more likely to hold (in the sense of increasing the range of parameters where it holds).[37]

As pointed out before, $g^C(\beta,\lambda,n) > g^*$, but g^C is weakly decreasing in n. This may tempt one to believe that $g^C(\beta,\lambda,n)$ would converge to g^*as n goes to ∞. That would be incorrect, however, since $g^C(\beta,\lambda,n)$ is bounded away from g^*.

Proposition 7 *Let (β,U,γ,Γ) be any economy. For every $\lambda > 0$, there exists a number $\underline{g}^C(\beta,\lambda)$ with the property that $g^* < \underline{g}^C(\beta,\lambda) \leq g^C(\beta,\lambda,n)$ for every n.*

Proof. Let (β,U,γ,Γ) be a generic economy and λ any real number. Since $g^C(\beta,\lambda,n)$ is bounded and weakly decreasing in n, $\lim_{n\to\infty} g^C(\beta,\lambda,n)$ is well defined. This allows us to set $\underline{g}^C(\beta,\lambda) = \lim_{n\to\infty} g^C(\beta,\lambda,n)$. Clearly, $\underline{g}^C(\beta,\lambda) \leq g^C(\beta,\lambda,n)$ for every n. If $g^C(\beta,\lambda,n) = \Gamma$ for all n, then $\underline{g}^C(\beta,\lambda) = \Gamma$. Since $g^* < \Gamma$, we conclude that $g^* < \underline{g}^C(\beta,\lambda)$.

[37] The exception is when the organization cost F increases with the number of political parties. In that case, one would need to compare the rate of increase of $F(n)$ and of $\Delta_0^C(n)$.

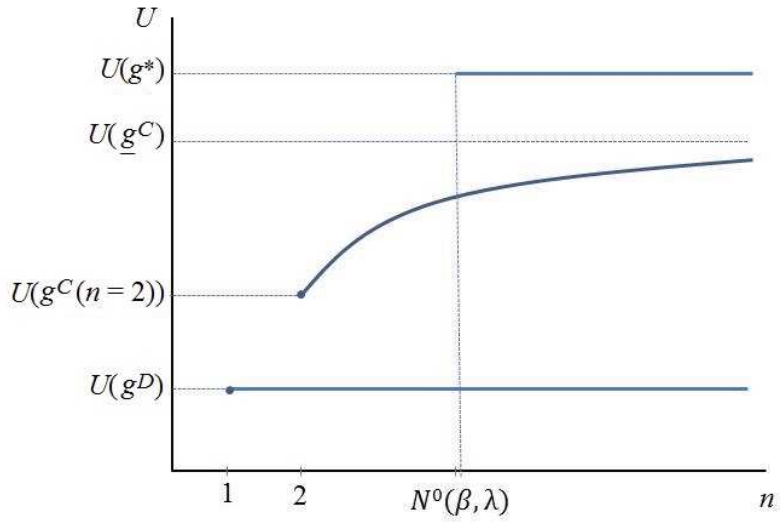

Figure 2: The welfare impact of political competition; no public debt, $\lambda < \lambda_0$

It remains to consider the case in which $g^C < \Gamma$ for some n. Hence, assume that $g^C(\beta, \lambda, n_0) < \Gamma$ for some number n_0. Use the fact that g^C is weakly decreasing in n to conclude that $g^C(\beta, \lambda, n) < \Gamma$ for every $n \geq n_0$. Hence, (57) holds for every such n. Then, make n goes to ∞ to conclude that $U'(\underline{g}^C(\beta, \lambda)) = -(1-\beta)\lambda < 0 = U'(g^*)$. Since U' is strictly decreasing, $\underline{g}^C(\beta, \lambda) > g^*$. ■

We close this section by relating the findings of this extension with the results from section 3 of the main text. Those combined results are represented in Figure 2 for $\lambda < \lambda_0$. We consider a repeated political game in which the players are competing political parties, all of which with an inherent penchant for allocating more resources to public expenditures than is socially optimal when in power. If politicians are very profligate ($\lambda > \lambda_0$), the efficient outcome is politically infeasible, regardless of the degree of political competition. In such a case, the welfare level $U(g^*)$ is unachievable. Otherwise, sufficiently strong competition among political parties can support a political compromise that yields the efficient outcome, as Figure 2 illustrates.

The political parties may choose instead to coordinate on a policy that maximizes the incumbent's present value payoff at each date. By construction, such a policy is an equilibrium outcome. Hence, a welfare level of at least $U(g^C(n = 2))$ is always politically feasible. This policy also has the property that an increase in the degree of political competition leads to higher politicians' payoff and higher aggregate welfare (converging to $U(\underline{g}^c)$ as $n \to \infty$), thus making politicians' and society's interests increasingly aligned with each other.

With unrestricted debt

Again, if the parties are able to forge a compromise in which they coordinate policies intertemporally, they will presumably choose such policies optimally. We focus on the static policy that maximizes the payoff of the date-0 incumbent. By construction, it also maximizes (among static policies) the payoff of the date-t incumbent for each date t.

To characterize this cooperative equilibrium for a given b, we redefine g^C as the value of g that maximizes $\Omega(g,b)$ subject to $g \leq f^g(b,b)$. If that constraint does not bind, the necessary and sufficient first-order condition is $\Omega_g(g^C,b)=0$, which is equivalent to

$$U_g(b,g^C(b,\beta,\lambda,n),b) = -\left(1-\beta+\beta/n\right)\lambda. \tag{59}$$

Observe the similarity between this condition and the first-order condition for g^C in the model without debt. Furthermore, if the politicians coordinate on a static policy, the cooperative level of the public debt b^C must be equal to b_0. Since $b_0=0$, we have that $b^C=0$. Therefore, the same properties of g^C discussed in the previous section also apply to $g^C(b) \equiv g^C(b,\beta,\lambda,n)$.

There are, however, important differences in the payoff implications of $g^C(b)$. Define the incumbent's gain from implementing the cooperative instead of the spendthrift policy:

$$\begin{aligned}\Delta_b^C(n) &\equiv \Omega(g^C(0),0) - \Omega_0(\{\tilde{g}_t,\tilde{b}_{t+1}\}_{t=0}^{\infty}) \\ &= \Omega(g^C(0),0) - \Omega(\gamma,\bar{B}) + V(\bar{B},\gamma,\bar{B}) - V(0,G(0,\bar{B},\lambda),\bar{B}).\end{aligned}$$

We have the following, henceforth writing $\Omega(g,b,n)$ instead of $\Omega(g,b)$ to emphasize that the expression that defines Ω depends on n.

Proposition 8 *If $g^C(0,\beta,\lambda,n) < f^g(0,0)$, then $U(0,g^C(0,\beta,\lambda,n),0)$ is strictly increasing in n but $\Delta_b^C(n)$ is strictly decreasing in n.*

Proof. Recall that $U_g(0,g^C(0,\beta,\lambda,n),0)<0$. Furthermore, $\frac{\partial g^C}{\partial n}<0$. Thus, $\frac{\partial U(0,g^C,0)}{\partial n} = U_g(0,g^C,0)\frac{\partial g^C}{\partial n}>0$. Now observe that

$$\frac{d\Delta_b^C}{dn} = \left[\frac{\partial\Omega(g^C(0,\beta,\lambda,n),0,n)}{\partial g}\frac{\partial g^C}{\partial n} + \frac{\partial\Omega(g^C(0,\beta,\lambda,n),0,n)}{\partial n}\right] - \frac{\partial\Omega(\gamma,\bar{B},n)}{\partial n} + \frac{\partial V(\bar{B},\gamma,\bar{B})}{\partial n} - \frac{\partial V(0,G(0,\bar{B},\lambda),\bar{B})}{\partial n}.$$

Since $\frac{\partial\Omega(g^C,0,n)}{\partial g}=0$ from the definition of g^C, $\frac{\partial\Omega(g,b,n)}{\partial n} = -\frac{\beta\lambda}{(1-\beta)n^2}g$ from (46), $\frac{\partial V(\bar{B},\gamma,\bar{B})}{\partial n}=0$ and $\frac{\partial V(0,G(0,\bar{B},\lambda),\bar{B})}{\partial n}=0$, we have

$$\frac{d\Delta_b^C}{dn} = -\frac{\beta\lambda}{(1-\beta)n^2}g^C(0,\beta,\lambda,n) + \frac{\beta\lambda}{(1-\beta)n^2}\gamma = -\frac{\beta\lambda}{(1-\beta)n^2}[g^C(0,\beta,\lambda,n)-\gamma] < 0,$$

concluding the proof. ■

Intuitively, as n increases, $g^C(0,\beta,\lambda,n)$ moves toward $g^*(0,0)$. As a consequence, the payoff of the representative household under the cooperative policy approaches its efficient

level—although, as in Proposition 7, one can show that $g^C(0,\beta,\lambda,n)$ is bounded away from $g^*(0,0)$. On the other hand, by implementing the politically optimal policy the incumbent earns in each future period an expected rent gain of $(\lambda/n)[g^C(0,\beta,\lambda,n)-\gamma]$. This rent clearly decreases with n.

By construction, $\Omega(g^C(0),0,n)\geq\Omega(g,0,n)$ for all g, so $(g^C(0),0)$ is the static policy that allows for cooperation under the broadest set of parameters. Thus, if a static policy $(g,0)$ is a symmetric political outcome, so is $(g^C(0),0)$. Since the efficient policy is static, if it is an equilibrium outcome, then so is the politically optimal policy. We can then apply Proposition 4 to conclude that if a society $(\beta,U,\gamma,f^g,f^b,\bar{B},\lambda,n)$ satisfies (49) and $n\leq N^b(\beta,\lambda)$, then the politically optimal policy is a symmetric political outcome.

Conversely, if $(g^C(0),0)$ is not an equilibrium policy, then no static policy will be. And in contrast to the $b\equiv 0$ case, $(g^C(0),0)$ need not be an equilibrium outcome. If we evaluate the left-hand side of (45) at $(g^C(0),0)$, that inequality becomes

$$\Omega(g^C(0),0,n)\geq\Omega_0(\{\tilde{g}_t,\tilde{b}_{t+1}\}_{t=0}^{\infty},n). \tag{60}$$

We cannot be sure that this inequality holds, since the policy in the right-hand side is not static. In fact, if λ is large, then n has to be sufficiently small for $(g^C(0),0)$ to be a symmetric equilibrium outcome.

Proposition 9 *Let $(\beta,U,\gamma,f^g,f^b,\bar{B})$ be a generic economy. There are numbers λ_1^b and $N^C(\beta,\lambda)$ with the property that, for every polity (λ,n) satisfying $\lambda>\lambda_1^b$, the politically optimal policy $(g^C(0,\beta,\lambda,n),0)$ is a symmetric political outcome if and only if $n\leq N^C(\beta,\lambda)$. Furthermore, $N^C(\beta,\lambda)>N^b(\beta,\lambda)$.*

Proof. From Lemma 3, $G(0,\bar{B},\lambda)=f^g(0,\bar{B})$ for a sufficiently large λ. Furthermore, $f^g(0,0)\geq g^C(0,\beta,\lambda,2)$. Therefore,

$$G(0,\bar{B},\lambda)-g^C(0,\beta,\lambda,2)\geq f^g(0,\bar{B})-f^g(0,0)>0$$

for a large λ. Hence, there exists a number λ_1^b (that does not depend on n) such that if $\lambda>\lambda_1^b$, then

$$U(0,f^g(0,\bar{B}),\bar{B})-U(0,g^*(0,0),0)+\lambda[G(0,\bar{B},\lambda)-g^C(0,\beta,\lambda,2)]>\frac{\beta}{1-\beta}\left[U(0,g^*(0,0),0)-U(\bar{B},\gamma,\bar{B})\right].$$

We use the facts that

$$U(0,f^g(0,\bar{B}),\bar{B})-U(0,g^*(0,0),0)+\lambda[G(0,\bar{B},\lambda)-g^C(0,\beta,\lambda,2)]=[U(0,f^g(0,\bar{B}),\bar{B})+\lambda G(0,\bar{B},\lambda)]-[U(0,g^*(0,0),0)+\lambda g^C(0,\beta,\lambda,2)]$$

and $G(0,\bar{B},\lambda) = f^g(0,\bar{B})$ to conclude that

$$[U(0,G(0,\bar{B},\lambda),\bar{B}) + \lambda G(0,\bar{B},\lambda)] - [U(0,g^*(0,0),0) + \lambda g^C(0,\beta,\lambda,2)] > \frac{\beta}{1-\beta}\left[U(0,g^*(0,0),0) - U(\bar{B},\gamma,\bar{B})\right] \geq \frac{\beta}{1-\beta}\left[U(0,g^C(0,\beta,\lambda,n),0) - U(\bar{B},\gamma,\bar{B})\right]$$

for every n. These inequalities imply that there is a number $k(\beta,\lambda)$ with the property that if $n \geq k(\beta,\lambda)$, then

$$[U(0,G(0,\bar{B},\lambda),\bar{B}) + \lambda G(0,\bar{B},\lambda)] - [U(0,g^*(0,0),0) + \lambda g^C(0,\beta,\lambda,2)] > \frac{\beta}{1-\beta}\left[U(0,g^C(0,\beta,\lambda,n),0) - U(\bar{B},\gamma,\bar{B})\right] + \frac{1}{n}\frac{\beta}{1-\beta}\lambda[f^g(0,0) - \gamma] \ .$$

Combine the last inequality with $U(0,g^*(0,0),0) \geq U(0,g^C(0,\beta,\lambda,n),0)$ and $f^g(0,0) \geq g^C(0,\beta,\lambda,2) \geq g^C(0,\beta,\lambda,n)$ to conclude that

$$[U(0,G(0,\bar{B},\lambda),\bar{B}) + \lambda G(0,\bar{B},\lambda)] - [U(0,g^C(0,\beta,\lambda,n),0) + \lambda g^C(0,\beta,\lambda,n)] > \frac{\beta}{1-\beta}\left[U(0,g^C(0,\beta,\lambda,n),0) - U(\bar{B},\gamma,\bar{B})\right] + \frac{1}{n}\frac{\beta}{1-\beta}\lambda[g^C(0,\beta,\lambda,n) - \gamma] \ .$$

This inequality is equivalent to $\Omega_0(\{\tilde{g}_t,\tilde{b}_{t+1}\}_{t=0}^{\infty},n) > \Omega(g^C(0,\beta,\lambda,n),0,n)$. Thus, if $\lambda > \lambda_1^b$ and $n \geq k(\beta,\lambda)$, then $(g^C(0,\beta,\lambda,n),0)$ is not a symmetric political outcome.

Consider inequality (45). We use it to conclude that the politically optimal policy is an equilibrium outcome if and only if

$$\Delta_b^C(n) = \Omega(g^C(0,\beta,\lambda,n),0,n) - \Omega_0(\{\tilde{g}_t,\tilde{b}_{t+1}\}_{t=0}^{\infty},n) \geq 0.$$

Now, observe that $\Omega(g^C(0,\beta,\lambda,N^b(\beta,\lambda)),0,N^b(\beta,\lambda)) > \Omega(g^*(0,0),0,N^b(\beta,\lambda))$. Furthermore, $\Omega(g^*(0,0),0,N^b(\beta,\lambda)) = \Omega_0(\{\tilde{g}_t,\tilde{b}_{t+1}\}_{t=0}^{\infty},N^b(\beta,\lambda))$. Thus, $\Delta_b^C(N^b(\beta,\lambda)) > 0$. On the other hand, $\Delta_b^C(k(\beta,\lambda)) < 0$. Hence, the intermediate value theorem implies that there exists a number $N^C(\beta,\lambda)$ satisfying $\Delta_b^C(N^C(\beta,\lambda)) = 0$.

It remains to show that (i) $(g^C(0,\beta,\lambda,n),0)$ is a symmetric political outcome if and only if $n \leq N^C(\beta,\lambda)$ and (ii) $N^C(\beta,\lambda) > N^b(\beta,\lambda)$. We start by (i). Regardless of whether the constraint $g^C(0,\beta,\lambda,n) \leq f^g(0,0)$ binds or not, it is possible to show that

$$\frac{d\Delta_b^C(n)}{dn} = -\frac{\beta\lambda}{(1-\beta)n^2}[g^C(b,\beta,\lambda,n) - \gamma] < 0.$$

Hence, $\Delta_b^C(n) \geq 0$ if and only if $n \leq N^C(\beta,\lambda)$. Concerning (ii), $\Delta_b^C(N^b(\beta,\lambda)) > 0 = \Delta_b^C(N^C(\beta,\lambda))$; we then use the fact that $\Delta_b^C(n)$ is strictly decreasing to obtain the desired result. ■

The intuition for this result is simple. The larger λ is, the higher is the incumbent's incentive to implement the spendthrift policy. We also know, from Proposition 8, that the politicians' gain from implementing the politically optimal policy decreases with n. There-

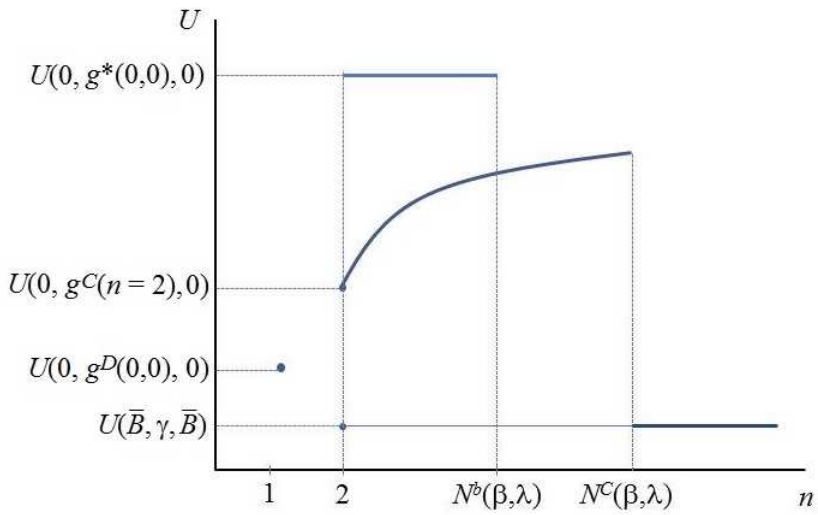

Figure 3: The welfare impact of political competition; public debt, $\lambda > \max\{\widetilde{\lambda}, \lambda_0^b, \lambda_1^b\}$

fore, the combination of large values for both λ and n suffices to rule out $(g^C(0, \beta, \lambda, n), 0)$ as an equilibrium outcome. Hence, for a sufficiently high degree of party profligacy, intense political competition hinders the implementation of the politically optimal policy.

Observe, however, that the payoff of the representative household in the politically optimal equilibrium is strictly increasing in n. Thus, when $\lambda > \lambda_1^b$ and politicians seek to implement that policy, household welfare would be maximized when $n = N^C(\beta, \lambda)$. Still, if there were a fixed cost $F > 0$ from coordinating on a policy, the maximum n under which g^C would be an equilibrium would be lower than $N^C(\beta, \lambda)$. And if such a cost increases with the number of parties—as would be plausible from a collective action perspective—then the possibility of sustaining g^C would be further reduced, as the gains would decrease while the costs increase when political competition intensifies.

Putting together the analysis of this section together with the analysis of section 4 of the main text, the main results for the case in which the government can issue bonds are summarized in Figure 3 for $\lambda > \max\{\widetilde{\lambda}, \lambda_0^b, \lambda_1^b\}$. If political competition is very intense, neither the efficient nor the politically optimal policies are sustainable. As a result, the economy becomes trapped in a bad equilibrium that leaves welfare stuck at $U(\bar{B}, \gamma, \bar{B})$ for $t \geq 1$. There is, however, a range of n where the efficient policy is infeasible but where the politically optimal policy can be sustained. If there is some $(n \geq 2)$, but not too much $(n \leq N^b(\beta, \lambda))$ political competition, then both the efficient policy and the politically optimal policy can be implemented.

II The accuracy of the function $U(b, g, b')$

Let (g, b) be any attainable steady state. We use (28) to conclude that

$$\begin{aligned} U(b, g, b) &= \mathcal{U}(b, \{g, b\}) - \beta\mathcal{U}(b, \{g, b\}) = (1-\beta)\mathcal{U}(b, \{g, b\}) \\ &\Rightarrow \sum_{t=0}^{\infty} \beta^t U(b, g, b) = \mathcal{U}(b, \{g, b\}). \end{aligned}$$

Therefore, we have a perfect measurement of the household's lifetime utility in any steady state. Since the efficient policy is static, U properly measures its corresponding payoff.

It remains to show that the same holds for the household payoff under the spendthrift policy. Define X according to

$$X \equiv U\left(b_0, G(b_0, \bar{B}, \lambda), \bar{B}\right) + \frac{\beta}{1-\beta} U\left(\bar{B}, \gamma, \bar{B}\right).$$

X corresponds to household payoff under the metric provided by the function U. The value of $U\left(\bar{B}, \gamma, \bar{B}\right)$ is given by (28), while $U\left(b_0, G(b_0, \bar{B}, \lambda), \bar{B}\right)$ follows (29). Hence,

$$\begin{aligned} X &= \mathcal{U}\left(b_0, [(\tilde{g}(b_0, \bar{B}), \bar{B}), \{\gamma, \bar{B}\}]\right) - \beta\mathcal{U}\left(\bar{B}, \{\gamma, \bar{B}\}\right) + \frac{\beta}{1-\beta}(1-\beta)\mathcal{U}\left(\bar{B}, \{\gamma, \bar{B}\}\right) \\ &\Rightarrow X = \mathcal{U}\left(b_0, [(\tilde{g}(b_0, \bar{B}), \bar{B}), \{\gamma, \bar{B}\}]\right). \end{aligned}$$

From the forthcoming Lemma 3, $G(b_0, \bar{B}, \lambda)$ is equal to the maximum attainable value for g_0 when λ is large. Hence, $G(b_0, \bar{B}, \lambda) = \tilde{g}(b_0, \bar{B})$. Therefore,

$$X = \mathcal{U}\left(b_0, [(G(b_0, \bar{B}, \lambda), \bar{B}), \{\gamma, \bar{B}\}]\right),$$

as we wanted to show.

III The spendthrift equilibrium

We establish here that the spendthrift policy is an equilibrium outcome. As this is a long exercise, we proceed in steps before proving the main result. Those interim steps consist of:

1. Proving that the function G is strictly decreasing in b, strictly increasing in b', and weakly increasing in λ.

2. Showing that there exists a number λ_2 that does not depend on (b, b') with the property that $\lambda > \lambda_2 \Rightarrow G(b, b', \lambda) = f^g(b, b')$ for every (b, b').

3. Showing that the partial derivatives of G are bounded.

4. Establishing a technical condition that is equivalent to the intuitive constraint corresponding to inequality (42) of the main text, $-G_{b'}(b_t, b_{t+1}, \lambda)/G_b(b_{t+1}, b_{t+2}, \lambda) > \beta/2$.

5. Characterizing the part of the expected payoff of the date-t incumbent that depends on that player's actions and showing that it is strictly increasing in b_{t+1} for every t.

Before we start, we define some notation. We denote the solution of the unconstrained version of (35) by G^u. That is, $G^u(b, b', \lambda)$ is the maximizer of $U(b, g, b') + \lambda g$. Since $U(b, \gamma, b') \geq 0$ and U is strictly concave in g, $G^u(b, b', \lambda) \geq \gamma$. We also denote the maximum of f^g by $f^g_{\max}$; such a maximum is well defined because f^g is continuous and defined in the compact set $[-\bar{b}, \bar{B}] \times [-\bar{b}, \bar{B}]$.

Lemma 2 *The function G is strictly decreasing in b, strictly increasing in b', and weakly increasing in λ.*

Proof. Let G^u_b, $G^u_{b'}$, and G^u_λ denote the partial derivatives of G^u. We adopt similar notation for the partial derivatives of G and f^g. The differentiation of (37) when it holds with equality establishes that

$$G^u_b = -\frac{U_{bg}}{U_{gg}},\ G^u_{b'} = -\frac{U_{gb'}}{U_{gg}},\ \text{and}\ G^u_\lambda = -\frac{1}{U_{gg}}\ .$$

Recall that $U_{gg} < 0$. Therefore, $G^u_\lambda(b, b', \lambda) > 0$. Then, combine the former inequality with (30) to conclude that $G^u_b(b, b', \lambda) < 0$ and $G^u_{b'}(b, b', \lambda) > 0$.

The function G may fail to be differentiable exactly when $G^u(b, b', \lambda) = f^g(b, b')$. However, G is differentiable whenever $G^u(b, b', \lambda) \neq f^g(b, b')$. Suppose that $G^u(b, b', \lambda) < f^g(b, b')$; thus, $G(b, b', \lambda) = G^u(b, b', \lambda)$ and $G_b = G^u_b < 0$, $G_{b'} = G^u_{b'} > 0$, and $G_\lambda = G^u_\lambda > 0$. If $G^u(b, b', \lambda) > f^g(b, b')$, then $G(b, b', \lambda) = f^g(b, b')$; as a consequence, $G_b = f^g_b < 0$, $G_{b'} = f^g_{b'} > 0$, and $G_\lambda = f^g_\lambda = 0$.

Let G^-_b and G^+_b denote, respectively, the left and right derivatives of G with respect to b. We use analogous notation for the side derivatives with respect to b' and λ. It should be clear from the previous paragraph that G^-_b is equal to G^u_b or f^g_b. Similarly, $G^+_b = G^u_b$ or $G^+_b = f^g_b$. The same reasoning applies to the side derivatives with respect to b' and λ. Therefore,

$$G^-_b < 0,\ G^+_b < 0,\ G^-_{b'} > 0,\ G^+_{b'} > 0,\ G^-_\lambda \geq 0,\ \text{and}\ G^+_\lambda \geq 0\ .$$

Even if G is not differentiable when $G^u(b, b', \lambda) = f^g(b, b')$, the inequalities above allow us to conclude that G is strictly decreasing in b, strictly increasing in b' and weakly increasing in λ. Consider the variable b. At a point in which G_b is not defined, both the left G^-_b and the right G^+_b partial derivatives are negative. Since G is continuous, we can be sure that its value decreases as b increases. Similar reasoning applies to b' and λ. ■

Lemma 3 *There exists a number λ_2 that does not depend on (b, b') with the property that, if $\lambda > \lambda_2$, then $G(b, b', \lambda) = f^g(b, b')$ for every (b, b').*

Proof. The definition of G^u implies that $U_g(\bar{B}, G^u(\bar{B}, -\bar{b}, \lambda), -\bar{b}) = -\lambda$. Therefore, $\lim_{\lambda \to \infty} U_g(\bar{B}, G^u(\bar{B}, -\bar{b}, \lambda), -\bar{b}) = -\infty$. Since $U_g(\bar{B}, f^g_{\max}, -\bar{b}) > -\infty$, it must exist a number λ_2 with the property that if $\lambda > \lambda_2$, then

$$f^g_{\max} \leq G^u(\bar{B}, -\bar{b}, \lambda)\ . \tag{61}$$

Now, observe that both b and b' belong to $[-\bar{b}, \bar{B}]$. Hence, $b \leq \bar{B}$ and $b' \geq -\bar{b}$. Use the fact that $G^u_b < 0$ and $G^u_{b'} > 0$ to conclude that

$$G^u(\bar{B}, -\bar{b}, \lambda) \leq G^u(b, b', \lambda)$$

for every (b, b'). Combine the last inequality with (61) to conclude that if $\lambda > \lambda_2$, then $f^g_{\max} \leq G^u(b, b', \lambda)$ for every (b, b'). However, $f^g(b, b') \leq f^g_{\max}$. Hence, $G^u(b, b', \lambda) \geq f^g(b, b')$ whenever $\lambda > \lambda_2$. Thus, $G(b, b', \lambda) = f^g(b, b')$ for every $\lambda > \lambda_2$. ■

Our next step consists in showing that some of the partial derivatives of G are bounded. Taking into account that G_b may be undefined at some points, we need to establish that

$$\sup_{(b,b',\lambda)} \left[\max\{|G^-_b(b, b', \lambda)|, |G^+_b(b, b', \lambda)|\}\right] < \infty \ . \tag{62}$$

Observe that if G_b is defined everywhere, then $G^-_b = G^+_b$. Furthermore, (62) is equivalent to $\sup_{(b,b',\lambda)} |G_b(b, b', \lambda)| < \infty$. In a similar fashion, we have to prove that

$$\sup_{(b,b',\lambda)} \left[\max\{|G^-_{b'}(b, b', \lambda)|, |G^+_{b'}(b, b', \lambda)|\}\right] < \infty \ . \tag{63}$$

Lemma 4 *The partial derivatives of G satisfy (62) and (63).*

Proof. Since $(b, b') \in [-\bar{b}, \bar{B}]^2$, we can conclude that $\sup_{(b,b')} |f^g_b(b, b')| < \infty$. Now, take any λ larger than λ_2. Lemma 3 implies that G_b is well defined and equal to f^g_b. Therefore, (62) holds if we impose the extra condition that $\lambda > \lambda_2$. If $\lambda \leq \lambda_2$, then (b, b', λ) lies in a compact set; hence, $\sup_{(b,b',\lambda)} |G^u_b(b, b', \lambda)| < \infty$. Moreover, G^-_b is equal to G^u_b or f^g_b; the same applies to G^+_b. Thus, both G^-_b and G^+_b are bounded for $\lambda \leq \lambda_2$. Hence, (62) holds if we impose the extra condition that $\lambda \leq \lambda_2$. Since (62) holds for $\lambda > \lambda_2$ and $\lambda \leq \lambda_2$, it clearly holds if we do not place any constraint on λ. Similar reasoning establishes that (63) holds. ■

We now lay out a technical condition that is equivalent to the intuitive constraint (42) on the partial derivatives of G_b and $G_{b'}$. For a moment, assume that those derivatives are well defined. Use the fact that $G_b < 0$ to rewrite (42) as

$$G_{b'}(b, b', \lambda) - \frac{\beta}{2}|G_b(b', b'', \lambda)| > 0,$$

where b'' denotes the public debt two dates ahead. For technical reasons, we need the left-hand side of that inequality to be bounded away from zero. That is,

$$G_{b'}(b, b', \lambda) - \frac{\beta}{2}|G_b(b', b'', \lambda)| \geq \varepsilon$$

for some positive ε. After we take into consideration that G_b and $G_{b'}$ may be undefined at some points, the last inequality has to be replaced by[38]

$$G^-_{b'}(b, b', \lambda) - \frac{\beta}{2}|G^-_b(b', b'', \lambda)| \geq \varepsilon \tag{64}$$

and

$$G^+_{b'}(b, b', \lambda) - \frac{\beta}{2}|G^+_b(b', b'', \lambda)| \geq \varepsilon. \tag{65}$$

[38] Observe that the derivatives of the function G brought forth by the functions (43) and (44) in the example we provide in the main text satisfy (64) and (65), provided that $a_3 > \beta \max\{a_4, a_1 a_2 e^{a_2 \bar{b}}/(2-\beta)\}$ and $a_5 > \beta \max\{a_4, (a_3 + a_1 a_2 e^{a_2 \bar{b}})/2\}$.

For our purposes, it is possible to replace inequalities (64) and (65) by two much weaker conditions. It suffices to assume that there exist numbers $\tilde{\lambda}_0 > 1$, $\varepsilon > 0$ and $\alpha \in [0, 1)$ such that, if $\lambda \geq \tilde{\lambda}_0$, then

$$G^-_{b'}(b, b', \lambda) - \frac{\beta}{2}|G^-_b(b', b'', \lambda)| \geq \varepsilon/\lambda^\alpha \tag{66}$$

and

$$G^+_{b'}(b, b', \lambda) - \frac{\beta}{2}|G^+_b(b', b'', \lambda)| \geq \varepsilon/\lambda^\alpha \tag{67}$$

for every (b, b', b''). Observe that the left-hand side of (64) is bounded away from zero, while the left-hand side of (66) may fall to zero as λ goes to ∞, provided that such a fall does not happen too fast. Similar remark applies to (65) and (67).

Our next step consists of characterizing the part of the expected payoff of the date-t incumbent that depends on that player's actions. Given that each incumbent faces a problem similar to the ones faced by its predecessors and successors, it suffices to carry out that task for party p_0 when the initial public debt assumes a generic value b_0.

Let Ω_{p_0} denote the expected payoff of the date-zero incumbent and ω_{p_0} be the part of Ω_{p_0} that depends on that player's actions. We define $\omega_{p_0,t}$ as the undiscounted date-t part of ω_{p_0}. Thus, $\omega_{p_0} = \sum_{t=0}^{\infty} \beta^t \omega_{p_0,t}$. To assess ω_{p_0}, we evaluate each of the factors $\omega_{p_0,t}$. At date zero, party p_0 chooses b_1 and its date-zero period payoff is equal to $U(b_0, G(b_0, b_1, \lambda), b_1) + \lambda G(b_0, b_1, \lambda)$. Hence, $\omega_{p_0,0}$ is equal to that expression.

With respect to date 1, if the date-zero incumbent p_0 were again in office, then its period payoff is $U(b_1, G(b_1, b_2, \lambda), b_2) + \lambda G(b_1, b_2, \lambda)$; otherwise, the party in office will leave the debt $\bar{B}$ and the period payoff of party p_0 is $U(b_1, G(b_1, \bar{B}, \lambda), \bar{B})$. Hence,

$$\omega_{p_0,1} = \frac{1}{n}\left[U(b_1, G(b_1, b_2, \lambda), b_2) + \lambda G(b_1, b_2, \lambda)\right] + \frac{n-1}{n} U(b_1, G(b_1, \bar{B}, \lambda), \bar{B}).$$

At date 2, suppose that p_0 were in office at date 1. If it were again in office at $t = 2$, then its payoff is $U(b_2, G(b_2, b_3, \lambda), b_3) + \lambda G(b_2, b_3, \lambda)$; otherwise, its period payoff is $U(b_2, G(b_2, \bar{B}, \lambda), \bar{B})$. Hence, the term

$$\frac{1}{n}\left\{\frac{1}{n}\left[U(b_2, G(b_2, b_3, \lambda), b_3) + \lambda G(b_2, b_3, \lambda)\right] + \frac{n-1}{n} U(b_2, G(b_2, \bar{B}, \lambda), \bar{B})\right\} \tag{68}$$

must be a component of $\omega_{p_0,2}$. Suppose now that party p_0 were not in office at date 1; its period payoff is $U(\bar{B}, G(\bar{B}, \bar{B}, \lambda), \bar{B}) + \lambda G(\bar{B}, \bar{B}, \lambda)$ if it were in office at date 2 and $U(\bar{B}, G(\bar{B}, \bar{B}, \lambda), \bar{B})$ otherwise. Since these last expressions do not depend on the choices of party p_0, we conclude that $\omega_{p_0,2}$ is equal to the expression in (68).

We now apply this reasoning to a generic date $t \geq 2$. Suppose that p_0 were in office at all previous dates. If it were again in office, then its period payoff would be equal to $U(b_t, G(b_t, b_{t+1}, \lambda), b_{t+1}) + \lambda G(b_t, b_{t+1}, \lambda)$; otherwise, its period payoff would be equal to $U(b_t, G(b_t, \bar{B}, \lambda), \bar{B})$. If p_0 were not in office in at least one of the previous dates, then its period payoff is $U(\bar{B}, \bar{B}, \lambda) + \lambda G(\bar{B}, \bar{B}, \lambda)$ if it were in office at date t

and $U(\bar{B}, G(\bar{B}, \bar{B}, \lambda), \bar{B})$ otherwise. Therefore,

$$\begin{aligned} \omega_{p_0,t} &= \left(\frac{1}{n}\right)^{t-1} \left\{ \frac{1}{n}[U(b_t, G(b_t, b_{t+1}, \lambda), b_{t+1}) + \right. \\ &\left. \lambda G(b_t, b_{t+1}, \lambda)] + \frac{n-1}{n} U(b_t, G(b_t, \bar{B}, \lambda), \bar{B}) \right\}. \end{aligned} \tag{69}$$

We conclude that

$$\begin{aligned} \omega_{p_0} &= U(b_0, G(b_0, b_1, \lambda), b_1) + \lambda G(b_0, b_1, \lambda) \\ &\beta \left\{ \frac{1}{n} \left[U(b_1, G(b_1, b_2, \lambda), b_2) + \lambda G(b_1, b_2, \lambda) \right] + \right. \\ &\left. \frac{n-1}{n} U(b_1, G(b_1, \bar{B}, \lambda), \bar{B}) \right\} + \sum_{t=2}^{\infty} \beta^t \omega_{p_0,t}. \end{aligned} \tag{70}$$

For future reference, we point out that $\sum_{t=2}^{\infty} \beta^t \omega_{p_0,t}$ does not depend on b_1.

Lemma 5 *Suppose that (66) and (67) hold. Then, there exists a real number $\tilde{\lambda}$ with the property that, if $\lambda \geq \tilde{\lambda}$, then ω_{p_0} is strictly increasing in b_{t+1} for every t.*

Proof. Let $\frac{\partial \omega_{p_0}}{\partial b_1^-}$ and $\frac{\partial \omega_{p_0}}{\partial b_1^+}$ denote the left- and right-side partial derivatives of ω_{p_0} with respect to b_1. Hence,

$$\begin{aligned} \frac{\partial \omega_{p_0}}{\partial b_1^-} &= U_{b'}(b_0, G(b_0, b_1, \lambda), b_1) + U_g(b_0, G(b_0, b_1, \lambda), b_1) G_{b'}^-(b_0, b_1, \lambda) + \\ &\lambda G_{b'}^-(b_0, b_1, \lambda) + \beta \frac{1}{n} [U_b(b_1, G(b_1, b_2, \lambda), b_2) + \\ &U_g(b_1, G(b_1, b_2, \lambda), b_2) G_b^-(b_1, b_2, \lambda) + \lambda G_b^-(b_1, b_2, \lambda)] + \\ &\beta \frac{n-1}{n} [U_b(b_1, G(b_1, \bar{B}, \lambda), \bar{B}) + U_g(b_1, G(b_1, \bar{B}, \lambda), \bar{B}) G_b^-(b_1, \bar{B}, \lambda)]. \end{aligned} \tag{71}$$

Use the fact that $G_b^- \leq 0$ and $U_{b'} \geq 0$ to conclude that

$$\begin{aligned} \frac{\partial \omega_{p_0}}{\partial b_1^-} &\geq U_g(b_0, G(b_0, b_1, \lambda), b_1) G_{b'}^-(b_0, b_1, \lambda) + \\ &\beta \frac{1}{n} U_b(b_1, G(b_1, b_2, \lambda), b_2) + \beta \frac{n-1}{n} U_b(b_1, G(b_1, \bar{B}, \lambda), \bar{B}) + \\ &\beta \frac{1}{n} U_g(b_1, G(b_1, b_2, \lambda), b_2) G_b^-(b_1, b_2, \lambda) + \\ &\beta \frac{n-1}{n} U_g(b_1, G(b_1, \bar{B}, \lambda), \bar{B}) G_b^-(b_1, \bar{B}, \lambda) + \\ &\lambda \left[G_{b'}^-(b_0, b_1, \lambda) - \beta \frac{1}{n} |G_b^-(b_1, b_2, \lambda)| \right]. \end{aligned}$$

Now, observe that $U_{gg} \leq 0$, $G^-_{b'} \geq 0$, $U_{bg} \leq 0$. Therefore,

$$\begin{aligned}\frac{\partial \omega_{p_0}}{\partial b_1^-} \;\geq\; & U_g(b_0, f^g_{\max}, b_1) G^-_{b'}(b_0, b_1, \lambda) + \\ & \beta \left\{ \frac{1}{n} U_b(b_1, f^g_{\max}, b_2) + \frac{n-1}{n} U_b(b_1, f^g_{\max}, \bar{B}) \right\} + \\ & \beta \left\{ \frac{1}{n} U_g(b_1, \gamma, b_2) G^-_b(b_1, b_2, \lambda) + \frac{n-1}{n} U_g(b_1, \gamma, \bar{B}) G^-_b(b_1, \bar{B}, \lambda) \right\} + \\ & \lambda \left[G^-_{b'}(b_0, b_1, \lambda) - \beta \frac{1}{n} |G^-_b(b_1, b_2, \lambda)| \right].\end{aligned}$$

The last expression implies that

$$\begin{aligned}\frac{\partial \omega_{p_0}}{\partial b_1^-} \geq \left[\min_{(b,b')} U_g(b, f^g_{\max}, b') \right] G^-_{b'}(b_0, b_1, \lambda) + \beta \min_{(b,b')} U_b(b, f^g_{\max}, b') + \\ \beta \left\{ \frac{1}{n} \left[\max_{(b,b')} U_g(b, \gamma, b') \right] G^-_b(b_1, b_2, \lambda) + \frac{n-1}{n} \left[\max_{(b,b')} U_g(b, \gamma, b') \right] G^-_b(b_1, \bar{B}, \lambda) \right\} + \\ \lambda \left[G^-_{b'}(b_0, b_1, \lambda) - \beta \frac{1}{n} |G^-_b(b_1, b_2, \lambda)| \right] > -\infty.\end{aligned}$$

Since $n \geq 2$, $U_g(b, f^g_{\max}, b') < 0$, $G^-_{b'} \geq 0$, and $U_g(b, \gamma, b') > 0$, we have

$$\frac{\partial \omega_{p_0}}{\partial b_1^-} \geq A^- + \lambda \left[G^-_{b'}(b_0, b_1, \lambda) - \beta \frac{1}{2} |G^-_b(b_1, b_2, \lambda)| \right], \tag{72}$$

where

$$\begin{aligned}A^- \;=\; & \left[\min_{(b,b')} U_g(b, f^g_{\max}, b') \right] \left[\sup_{(b,b',\lambda)} G^-_{b'}(b, b', \lambda) \right] + \beta \min_{(b,b')} U_b(b, f^g_{\max}, b') - \\ & \beta \left[\max_{(b,b')} U_g(b, \gamma, b') \right] \left[\sup_{(b,b',\lambda)} |G^-_b(b, b', \lambda)| \right].\end{aligned} \tag{73}$$

Now use the fact that b and b' belong to $[-\bar{b}, \bar{B}]$ and that the partial derivatives of U are continuous to conclude that $\min_{(b,b')} U_g(b, f^g_{\max}, b') > -\infty$, $\min_{(b,b')} U_b(b, f^g_{\max}, b') > -\infty$, and $\max_{(b,b')} U_g(b, \gamma, b') < \infty$. Therefore, (62) and (63) imply that $A^- > -\infty$.

Combine (72) with (66) to conclude that, if $\lambda > \tilde{\lambda}_0$, then

$$\frac{\partial \omega_{p_0}}{\partial b_1^-} \geq A^- + \lambda^{1-\alpha} \varepsilon > -\infty.$$

Similar reasoning establishes

$$\frac{\partial \omega_{p_0}}{\partial b_1^+} \geq A^+ + \lambda^{1-\alpha} \varepsilon > -\infty,$$

where A^+ is defined exactly as A^-, except that $G^+_{b'}$ and G^+_b replace their left-sided counterparts in (73). Thus, there exists a number $\tilde{\lambda} \geq \tilde{\lambda}_0$ with the property that if $\lambda > \tilde{\lambda}$, then $\frac{\partial \omega_{p_0}}{\partial b_1^-} > 0$ and $\frac{\partial \omega_{p_0}}{\partial b_1^+} > 0$. Since ω_{p_0} is a continuous function of b_1, we conclude that ω_{p_0} is strictly increasing in b_1.

We still have to show that ω_{p_0} is strictly increasing in b_{t+1} for a generic date t. From (70) we conclude that

$$\frac{\partial \omega_{p_0}}{\partial b_{t+1}^-} = \beta^t \frac{\partial \omega_{p_{0,t}}}{\partial b_{t+1}^-} + \beta^{t+1} \frac{\partial \omega_{p_{0,t+1}}}{\partial b_{t+1}^-}.$$

Combine this expression with (69) to obtain

$$\begin{aligned}\frac{\partial \omega_{p_0}}{\partial b_{t+1}^-} = \left(\frac{\beta}{n}\right)^t [U_{b'}(b_t, G(b_t, b_{t+1}, \lambda), b_{t+1}) + U_g(b_t, G(b_t, b_{t+1}, \lambda), b_{t+1}) G^-_{b'}(b_t, b_{t+1}, \lambda) + \\ \lambda G^-_{b'}(b_t, b_{t+1}, \lambda)] + \left(\frac{\beta}{n}\right)^t \left\{ \beta \frac{1}{n} [U_b(b_{t+1}, G(b_{t+1}, b_{t+2}, \lambda), b_{t+2}) + \right. \\ U_g(b_{t+1}, G(b_{t+1}, b_{t+2}, \lambda), b_{t+2}) G^-_b(b_{t+1}, b_{t+2}, \lambda) + \lambda G^-_b(b_{t+1}, b_{t+2}, \lambda)] + \\ \left. \beta \frac{n-1}{n} [U_b(b_{t+1}, G(b_{t+1}, \bar{B}, \lambda), \bar{B}) + U_g(b_{t+1}, G(b_{t+1}, \bar{B}, \lambda), \bar{B}) G^-_b(b_{t+1}, \bar{B}, \lambda)] \right\}.\end{aligned}$$

If we follow the reasoning used after obtaining equality (71), we conclude that

$$\frac{\partial \omega_{p_0}}{\partial b_{t+1}^-} \geq \left(\frac{\beta}{n}\right)^t (A^- + \lambda^{1-\alpha} \varepsilon) > -\infty$$

and

$$\frac{\partial \omega_{p_0}}{\partial b_{t+1}^+} \geq \left(\frac{\beta}{n}\right)^t (A^+ + \lambda^{1-\alpha} \varepsilon) > -\infty.$$

Hence, $\frac{\partial \omega_{p_0}}{\partial b_{t+1}^-} > 0$ and $\frac{\partial \omega_{p_0}}{\partial b_{t+1}^+} > 0$ for $\lambda > \tilde{\lambda}$. An appeal to continuity establishes that ω_{p_0} is strictly increasing in b_{t+1}. ■

We can now finally establish the main result of this section.

Proposition 10 *Suppose that (66) and (67) hold. If $\lambda \geq \tilde{\lambda}$, then the spendthrift policy plan $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$ is a symmetric political equilibrium.*

Proof. Let t be any date. We have to show that if party p_t believes that the other parties follow the strategy $\{\tilde{\sigma}_s\}_{s=0}^{\infty}$, then $\{\tilde{\sigma}_s\}_{s=0}^{\infty}$ is an optimal choice for p_t. It is enough to consider the situation of party p_0 when the initial public debt has a generic value b_0. The problem of party p_0 consists in selecting a sequence $\{b_{t+1}\}_{t=0}^{\infty}$ that maximizes ω_{p_0} subject to

$$\begin{aligned} b_{t+1} &\leq \bar{B}, & (74) \\ f^b(b_t) &\leq b_{t+1}. & (75) \end{aligned}$$

Let $\{\hat{b}_{t+1}\}_{t=0}^{\infty}$ be any sequence that satisfies (74) and (75) with the property that $\hat{b}_1 < \bar{B}$. We will show that such a sequence cannot solve the problem of party p_0 by constructing a sequence $\{b_{t+1}\}_{t=0}^{\infty}$ that satisfies these constraints and yields a higher payoff.

Let b_1 be any debt level that satisfies $\hat{b}_1 < b_1 \leq \bar{B}$. Define the debt level at the other dates recursively according to

$$b_{t+1} = \max\{f^b(b_t), \hat{b}_{t+1}\} \tag{76}$$

Therefore, $\{b_{t+1}\}_{t=0}^{\infty}$ satisfies (75).

We next show that $\{b_{t+1}\}_{t=0}^{\infty}$ satisfies (74). Recall that f^b is strictly increasing and $f^b(\bar{B}) = \bar{B}$. Thus, the inequality $b_1 \leq \bar{B}$ implies that $f^b(b_1) \leq f^b(\bar{B}) = \bar{B}$. Since $\hat{b}_2 \leq \bar{B}$, we conclude that $\max\{f^b(b_1), \hat{b}_2\} \leq \bar{B}$. Thus, $b_2 \leq \bar{B}$. Apply this reasoning recursively to conclude that $\{b_{t+1}\}_{t=0}^{\infty}$ satisfies (74).

To conclude the proof, observe that (76) implies that $b_{t+1} \geq \hat{b}_{t+1}$. Therefore, an appeal to Lemma 5 establishes that $\{b_{t+1}\}_{t=0}^{\infty}$ yields a higher payoff than $\{\hat{b}_{t+1}\}_{t=0}^{\infty}$. Hence, the optimal action for party p_0 entails setting b_1 equals to $\bar{B}$. Therefore, $\{\tilde{\sigma}_t\}_{t=0}^{\infty}$ is an optimal strategy for the date-zero incumbent. ■

IV Proofs

Proof of Proposition 1. Take an economy $(\beta, U, \gamma, \Gamma)$ and let λ be any positive real number. Define

$$\hat{\beta}(\lambda) \equiv \left[1 + \frac{U(g^*) - U(g^D)}{V(g^D) - V(g^*)}\right]^{-1} .$$

$\hat{\beta}(\lambda)$ corresponds to the maximum value of β that satisfies (16). Since the ratio $\beta/(1-\beta)$ is a strictly increasing function of β, any value for β below $\hat{\beta}(\lambda)$ satisfies (16). Note also that $0 < \hat{\beta}(\lambda) < 1$. Observe now that

$$V(g^D) - V(g^*) = U(g^D) - U(g^*) + \lambda(g^D - g^*) \geq U(\Gamma) - U(g^*) + \lambda(g^D - g^*) .$$

Therefore, $\lim_{\lambda\to\infty}\left[V(g^D) - V(g^*)\right] = \infty$. Since $0 < U(g^*) - U(g^D) \leq U(g^*) - U(\Gamma)$, $\lim_{\lambda\to\infty}\hat{\beta}(\lambda) = 1$. Thus, there exists a λ_0 (that does not depend on n) with the property that, if $\lambda \geq \lambda_0$, then $\hat{\beta}(\lambda) \geq \beta$ and inequality (16) holds. Since $g^* - g^D < 0$, condition (15) is not satisfied and the policy g^* cannot be implemented with the revert-to-dictatorship strategy.

Proof of Lemma 1. Let $(\beta, U, \gamma, f_b, f_g, \bar{B})$ be a generic economy and λ a positive real number. Define $\hat{\beta}(\lambda)$ according to

$$\hat{\beta}(\lambda) = \left[1 + \frac{\Delta U}{\Delta V}\right]^{-1} . \tag{77}$$

Hence, $0 < \hat{\beta}(\lambda) < 1$ and (48) holds with equality only when $\beta = \hat{\beta}(\lambda)$. Moreover,

$$\begin{aligned}
&\Delta V = U(0, G(0, \bar{B}, \lambda), \bar{B}) - U(0, g^*(0,0), 0) + \lambda[G(0, \bar{B}, \lambda) - g^*(0,0)] \Rightarrow \\
&\Delta V \geq U(0, f^g_{\max}, \bar{B}) - U(0, g^*(0,0), 0) + \lambda[G(0, \bar{B}, \lambda) - g^*(0,0)] .
\end{aligned}$$

Lemma 2 implies that $G(0, \bar{B}, \lambda) > G(0, 0, \lambda)$. Thus,

$$\Delta V \geq U(0, f^g_{\max}, \bar{B}) - U(0, g^*(0,0), 0) + \lambda[G(0,0,\lambda) - g^*(0,0)] \ .$$

Furthermore, the difference $G(0,0,\lambda) - g^*(0,0)$ is positive and weakly increasing in λ. Hence, $\lim_{\lambda\to\infty} \Delta V = \infty$. Since ΔU does not depend on λ, $\lim_{\lambda\to\infty} \hat{\beta}(\lambda) = 1$. Thus, there exists a λ_0 (that does not depend on n) with the property that if $\lambda > \lambda_0$, then $\beta < \hat{\beta}(\lambda)$. The fact that $\beta/(1-\beta)$ is strictly increasing in β concludes the proof. ■