FUNDAÇÃO GETULIO VARGAS
ESCOLA DE ECONOMIA DE SÃO PAULO

DANIEL BATISTA DA SILVA

SEMIVOLATILITY MANAGED PORTFOLIOS

São Paulo

2021

DANIEL BATISTA DA SILVA

SEMIVOLATILITY MANAGED PORTFOLIOS

Dissertação apresentada à Escola de Economia de São Paulo como pré-requisito à obtenção de título de mestre em Economia de Empresas.

Orientador: Marcelo Fernandes.

São Paulo

2021

Silva, Daniel Batista da.
Semivolatility managed portfolios / Daniel Batista da Silva. - 2021.
39 f.

Orientador: Marcelo Fernandes.
Dissertação (mestrado CMEE) – Fundação Getulio Vargas, Escola de Economia de São Paulo.

1. Carteiras (Finanças) - Administração. 2. Administração de risco. 3. Volatilidade (Finanças). 4. Investimentos. I. Fernandes, Marcelo. II. Dissertação (mestrado CMEE) – Escola de Economia de São Paulo. III. Fundação Getulio Vargas. IV. Título.

CDU 336.767

Ficha Catalográfica elaborada por: Isabele Oliveira dos Santos Garcia CRB SP-010191/O
Biblioteca Karl A. Boedecker da Fundação Getulio Vargas - SP

DANIEL BATISTA DA SILVA

SEMIVOLATILITY MANAGED PORTFOLIOS

Dissertação apresentada à Escola de Economia de São Paulo como pré-requisito à obtenção de título de mestre em Economia de Empresas.

Data de aprovação: __/__/____

Banca examinadora:

____

Prof. Dr. Marcelo Fernandes
FGV-EESP

____

Prof. Dr. Marcelo Medeiros
PUC-Rio

____

Prof. Dr. Ruy Ribeiro
INSPER

____

Prof. Dr. Alan Moreira
University of Rochester

Agradecimentos

Este trabalho é dedicado à minha mãe.

Agradeço ao professor Marcelo Fernandes pela paciência, camaradagem e ensinamentos sem os quais este trabalho não seria possível. Extendo os agradecimentos também a todo o corpo docente da Escola de Economia de São Paulo, que me proporcionou a oportunidade de ter uma formação excepcional como pesquisador.

Agradeço também a todos os meus colegas e amigos da *Salinha* por tornarem menos pesados todos os fins de semana, feriados e madrugadas em claro nos quais estávamos juntos no estudo.

Por fim, agradeço aos professores que compõem a banca examinadora pelo tempo gentilmente cedido para ler e discutir este trabalho.

O presente trabalho foi realizado com apoio da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Código de Financiamento 001.

Quaisquer falhas e omissões são de minha inteira responsabilidade.

Resumo

Volatility management ainda é um tema de discussão relevante na literatura, com trabalhos argumentando tanto pela sua eficiência como pela sua ineficácia. Estudamos este tipo de estratégia e tentamos resolver críticas frequentes sobre a forma como estas são analisadas. Ao decompor a volatilidade em seus componentes positivos (*upside*) e negativos (*downside*), as nossas construções permitem um melhor controle de ambos os tipos de risco tal como são percebidos para o investidor comum. Alavancar em tempos de volatilidade “boa” e diminuir a exposição em tempos de volatilidade “ruim” parece funcionar geralmente melhor do que carteiras não geridas ou mesmo a tradicional gestão da volatilidade. Os resultados apontam para que as carteiras geridas utilizando semivolatilidade como medidas de risco sejam estratégias decentes na gestão ativa de carteiras, tanto num ponto de vista de média-variância como de média-semivariância.

Palavras-chave: gestão de carteiras, risco negativo, medidas realizadas, semivariâncias.

Abstract

Volatility management is still a relevant topic of discussion in the literature, with works arguing both for its efficiency or inefficacy. We study a new framework building on top of this kind of strategy and attempting to solve common critiques on the way they are analysed. By decomposing volatility into its upside and downside components, our constructions allow for better control of both types of risk as they are perceived for the common investor. Leveraging in times of “good” volatility and deleveraging in times of “bad” volatility seems to perform generally better than unmanaged portfolios or even the traditional volatility management. The results point to semivolatility managed portfolios being a fair strategy in active portfolio management, both in a mean-variance or mean-semivariance framework.

Keywords: portfolio management, downside volatility, realized measures, semivariances.

List of Figures

List of Tables

Contents

1 Introduction

There has been a lot of discussion in recent literature on whether timing volatility could lead to higher performance and utility gains for the investor. We aim to provide new evidence on the performance of volatility management, as well as expanding the discussion to a new framework that could be viewed as both more intuitive in a sense of risk perception by the investor and also more effective in terms of performance, namely semivolatility management.

The original underlying idea is that, since volatility is persistent and not immediately related to future returns, we can time our exposure to risky markets and achieve a higher Sharpe Ratio. However, if the goal is to avoid "bad" risk, it might make more sense to separate downside from upside volatility and control for them in different manners.

Therefore, I construct two different strategies, one analogous to the original volatility-managed portfolio strategy proposed in Moreira and Muir (2017) where we change the realized variance for the realized downside semivariance, and the other where we continue to deleverage in periods of high downside volatility but also increase our leverage in periods of high upside volatility, or "good" risk, essentially exploring the conditional skewness of the asset.

Whether it is on a portfolio theory application like Markowitz et al. (1993), Estrada (2007), Ballestero (2005) or in financial econometrics and risk modeling like Barndorff-Nielsen et al. (2008), Bollerslev et al. (2020) or Bollerslev et al. (2020a), the semivariance, or Downside Variance, has shown to be of great utility. The idea of using semivariance as a measure of risk in portfolio theory is not new and it is actually supported in Markowitz (1959), where the author states that "analysis based on semivariance tend to produce better portfolios than those based on variance". Since investors tend to worry more about underperforming than overperforming, the downside variance is a more natural and intuitive measure of risk as perceived by the representative agent in the market. In risk modeling, it was also shown in Bollerslev et al. (2020) that decomposing risk in downside and upside parts helps to forecast volatility as a whole. This means that downside and upside risks behave differently and there are real information gains we can benefit from in accounting for them separately. Bollerslev et al. (2020b) even manages to show that the risk premium is priced differently according to the returns' sign.

The idea of risk timing also has a long history both in the industry and academic literature. However, this most recent debate can be traced back to the work of Barroso and Santa-Clara (2015), where the authors demonstrate the remarkable performance of risk-timing for the momentum factor. Moreira and Muir (2017) then formalized and expanded this idea to multiple factors, testing robustness and trying to identify economic value in these volatility-managed strategies. In the paper, the authors present the volatility-managed portfolio as a robust alternative to Buy-and-Hold portfolios that seems to increase Sharpe Ratio and investor utility. Cederburg et al. (2020) then sheds doubt on these results by performing a more extensive analysis on the original construction, testing it more directly and on a larger sample. The paper found very little applicability of this strategy to real-time investors. The present work contributes to this literature by expanding the analysis of the performance of volatility-managed portfolios to a semivariance framework and at the same time building on top of the critiques found in Cederburg et al. (2020).

Among these many critiques, the authors bring attention to the fact that the spanning regression analysis performed in Moreira and Muir (2017) is not enough to claim real utility gains since, even if the analysis pointed that the new strategy expanded the mean-variance frontier, a significant alpha is not a sufficient condition for a higher Sharpe Ratio. Further, the optimal weights that lead to the new optimal market portfolio for this expanded frontier depend on the alpha itself, which can only be found once we have the full sample for the returns.

I address these arguments by performing direct comparisons beyond the spanning regressions. This allows us to compare one strategy against another objectively and also see if the new (semi)volatility strategy expands the mean-(semi)variance frontier. I also address minor problems pointed out in Cederburg et al. (2020) such as the ones generated by the unrestrained leverage factor of the volatility managed strategy, that could reach impracticable levels by imposing a simple and realistic restriction. I also consider an expanded sample as proposed in their paper in order to conduct a more robust analysis.

The portfolio constructions here naturally resembles that of Moreira and Muir (2017), but brought to a mean-semivariance framework. The results indicate that for most of the main factors used in the original paper, constructions that involve the variance decomposition into semivariances seem to outperform the Buy-and-Hold strategy and even the volatility-managed portfolios constructed as in Moreira and Muir (2017). This is observed from the results in both the spanning regressions and the direct comparisons, which involves statistical testing of the difference between strategies for parameters of interest such as the Sharpe Ratio and its analogous in the semivariance framework, the Sortino Ratio.

It is also worthwhile to mention studies that try to provide an economic intuition for the results finding some efficiency on volatility managing, since it implies that the price of risk goes down in periods of high volatility, which is not in line with financial theory. Moreira and Muir (2017) attempt to explain this saying that investors could be reacting slowly to volatility increase. That would cause a delay between increased volatility and higher expected returns, which in turn makes it possible for the volatility-managed strategy to achieve higher Sharpe Ratios. Barroso and Detzel (2021) analyses whether limits to arbitrage explain the abnormal returns of these strategies. They find the opposite, with results showing that the economic gains of volatility-timing increase in times of higher liquidity, where arbitrage should be easier. These results are also not consistent with the former explanation, but agree with models in which unsophisticated traders under-react to informed order flow when sentiment is high.

Finally, questions about the impact of transaction costs on the volatility management strategies are rather common, and were addressed in Moreira and Muir (2017), where the authors show that their strategy is profitable even after accounting for transaction costs, and in Barroso and Detzel (2021), where the authors show that profits from volatility timing are more concentrated on stocks that present lower idiosyncratic volatility (IV), which present lower transaction costs than stocks with higher IV.

This work is structured as follows: section 2 describes the constructions for the semivolatility managed portfolios, as well as the methodology to analyse them; section 3 presents and discusses the results; section 4 concludes.

2 Construction and methodology

I use the same monthly and daily data on factors as Moreira and Muir (2017) plus an Expected Growth factor and anomaly portfolios as described in Haddad et al. (2020).[1] Sample size varies among factors, with the smallest series ranging from February-1975 to December-2019 and the longest from July-1926 to September-2020. Figure 1 shows all monthly return series of these portfolios. Table 1 shows the description for each factor or anomaly and their descriptive statistics. Looking also at the cross-section, we can see that the average factor has an average monthly return of 0.40%, minimal and maximum monthly returns of -20.03% and 21.17%, standard deviation of 4.11% and downside deviation of 2.70%. They are also mostly (unconditionally) symmetric and have a positive excess kurtosis.

I will also use the construction for volatility-managed portfolios as described in the paper, which is

$$f_t^\sigma = \frac{c}{\hat{\sigma}_{t-1}^2} f_t \tag{2.1}$$

Here $\hat{\sigma}_{t-1}^2$ is an estimator for the conditional variance at $t-1$, f_t is the Buy-and-Hold portfolio excess return (or unscaled portfolio) and c is the normalization constant that assures that f_t and f_t^σ have the same unconditional variance. My semivariance constructions will be the following

$$f_{SV,t}^1 = \frac{c^*}{\hat{\sigma}_{(-),t-1}^2} f_t \tag{2.2}$$

$$f_{SV,t}^2 = c^\dagger \frac{\hat{\sigma}_{(+),t-1}^2}{\hat{\sigma}_{(-),t-1}^2} f_t \tag{2.3}$$

[1] Data on MKT, SMB, HML, MOM, RMW, and CMA are from Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french. Data on BAB are from Andrea Frazzini's website at http://people.stern.nyu.edu/afrazzin. Data on IA, ROE and EG can be found at http://global-q.org/factors.html. Data on the anomaly portfolios can be found on Serhiy Kozak's website at https://www.serhiykozak.com/data.

Here $\hat{\sigma}^2_{(-),t} = \frac{\sum_{i=1}^{T} r^2_{i,t} \mathbb{1}\{r_{i,t}<0\}}{T}$, $\hat{\sigma}^2_{(+),t} = \frac{\sum_{i=1}^{T} r^2_{i,t} \mathbb{1}\{r_{i,t}>0\}}{T}$ and c^* and $c^\dagger$ are the normalization constants that assure that $f^1_{SV,t}$, $f^2_{SV,t}$ and f_t have the same unconditional downside variance. Naturally, these constants should not alter in any way the Sharpe or Sortino ratios of these strategies in their unrestrained form. However, when we impose a restriction on $\frac{c^*}{\hat{\sigma}^2_{(-),t-1}}$ (or $c^\dagger \frac{\hat{\sigma}^2_{(+),t-1}}{\hat{\sigma}^2_{(-),t-1}}$) in order to prevent unrealistic levels of leverage or a restriction on using future information by adapting the process to the natural filtration, the constants can impact the unconditional semivariance of the full sample strategy. These restrictions are imposed with intent to solve some of the critiques made by Cederburg et al. (2020), namely the feasibility of the leverage multiplier and the use of ex-post information within the process of implement the strategy.

In order to test if these strategies perform better than the Buy-and-Hold strategy, I perform a spanning regression analysis and a direct comparison between performance indices. The spanning regression analysis allows us to investigate if the strategy expands the mean-variance frontier in the sense that the investor could find an optimal portfolio that combines this strategy and the Buy-and-Hold strategy and achieve higher levels of utility. This is done by running the regression

$$f'_t = \alpha + \beta f_t + \varepsilon_t$$

where f' is the managed portfolio and f is the original portfolio, and testing

$$\begin{cases} H_0 : \alpha = 0 \\ H_1 : \alpha \neq 0 \end{cases}$$

If α is different from zero, then f' is said to expand the mean-variance frontier. However, as pointed out in Cederburg et al. (2020), the weights for this new optimal portfolio may require ex-post information since it depends on the spanning regression's alpha. That is, this information was not available to the investor in-sample. For this reason I also perform direct comparisons to investigate if one strategy is objectively better than another. This can be accomplished by testing the difference between Sharpe Ratios and between Sortino Ratios.

Jobson and Korkie (1981) present a statistical test to perform inference on the difference of Sharpe Ratios between two strategies. Memmel (2003) later corrected and simplified the test statistic. This is the test used in Cederburg et al. (2020) for measuring direct comparisons between the volatility-managed and the unscaled strategies. Although commonly employed in the literature, the test loses efficiency when returns are not i.i.d. or present high tail-risk. These are common financial data features, and thus this procedure might not be very reliable. Ledoit and Wolf (2008) propose a bootstrap-based alternative that accounts for serial correlation in returns and calculates a consistent estimator for the covariance matrix employed in the test statistic.

Given two strategies i and j where the bivariate distribution of the returns from these strategies is assumed stationary with

$$\mu = \begin{pmatrix} \mu_i \\ \mu_j \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \sigma_i^2 & \sigma_{ij} \\ \sigma_{ij} & \sigma_j^2 \end{pmatrix},$$

our parameter of interest is the difference between the two Sharpe ratios of strategies i and j:

$$\Delta = SR_i - SR_j = \frac{\mu_i}{\sigma_i} - \frac{\mu_j}{\sigma_j}$$

Our estimator is given by:

$$\hat{\Delta} = \widehat{SR}_i - \widehat{SR}_j = \frac{\hat{\mu}_i}{\hat{\sigma}_i} - \frac{\hat{\mu}_j}{\hat{\sigma}_j}$$

Where $\hat{\mu}_i$ and $\hat{\sigma}_i$ are the sample mean and sample standard deviation respectively. Letting $u = \left(\mu_i, \mu_j, \sigma_i^2, \sigma_j^2\right)'$, Jobson and Korkie (1981) and Memmel (2003) assume that returns are i.i.d. and follow a bivariate normal distribution and find that

$$\sqrt{T}(\hat{u} - u) \xrightarrow{d} N(0; \Omega) \quad \text{with} \quad \Omega = \begin{pmatrix} \sigma_i^2 & \sigma_{ij} & 0 & 0 \\ \sigma_{ij} & \sigma_j^2 & 0 & 0 \\ 0 & 0 & 2\sigma_i^4 & 2\sigma_{ij}^2 \\ 0 & 0 & 2\sigma_{ij}^2 & 2\sigma_j^4 \end{pmatrix}$$

Applying the delta method, they compute a standard deviation for $\hat{\Delta}$. However, these assumptions of i.i.d. and normal returns are rather strong and Ledoit and Wolf (2008) build on top of this, constructing via bootstrap a consistent estimator for Ω even in the presence of heavy tails and time series dependence. For the sake of comparison with Cederburg et al. (2020), we also report the Jobson-Korkie test results, even if it is valid only under i.i.d. Gaussian returns.

In order to perform inference on the difference of Sortino Ratios, however, it is necessary to expand these procedures. The Sortino ratio is given by

$$Sortino_i = \frac{\mu_i}{\sigma_i^-}$$

Let $v = (\mu_i, \mu_j, (\sigma_i^-)^2, (\sigma_j^-)^2)$. Let us assume that the assets i and j are normally distributed. We know from Barndorff-Nielsen et al. (2008) that for an asset i whose price process follows a Brownian Semimartingale given as

$$Y_t = \int_0^t a_s ds + \int_0^t \sigma_s dW_s$$

where a is a locally bounded predictable drift process and σ is a càdlàg volatility process, assuming that σ_s is constant we have that

$$\mathbb{V}((\sigma_i^-)^2) = \frac{5}{4}\sigma_i^4$$

.

Also, we know that

$$Cov(\sigma_i^2, \sigma_j^2) = Cov((\sigma_i^+)^2, (\sigma_j^+)^2) + Cov((\sigma_i^+)^2, (\sigma_j^-)^2) + Cov((\sigma_i^-)^2, (\sigma_j^+)^2) + Cov((\sigma_i^-)^2, (\sigma_j^-)^2)$$

and since the variables are assumed to be normally distributed, $(\sigma_i^-)^2 = (\sigma_i^+)^2$, which gives us

$$Cov((\sigma_i^-)^2, (\sigma_j^-)^2) = \frac{Cov(\sigma_i^2, \sigma_j^2)}{4} = \frac{2\sigma_{ij}^2}{4} = \frac{\sigma_{ij}^2}{2}$$

. Thus, we have that the covariance matrix for the vector v is given by

$$\Omega = \begin{pmatrix} \sigma_i^2 & \sigma_{ij} & 0 & 0 \\ \sigma_{ij} & \sigma_j^2 & 0 & 0 \\ 0 & 0 & \frac{5}{4}\sigma_i^4 & \frac{\sigma_{ij}^2}{2} \\ 0 & 0 & \frac{\sigma_{ij}^2}{2} & \frac{5}{4}\sigma_j^4 \end{pmatrix}$$

Our parameter of interest is given by

$$\Delta = f(v) \quad \text{with} \quad f(a,b,c,d) = \frac{a}{\sqrt{c}} - \frac{b}{\sqrt{d}}$$

which consists in the difference of Sortino Ratios between our portfolios. Knowing that

$$\nabla' f(a,b,c,d) = \left(\frac{1}{c^{\frac{1}{2}}}, -\frac{1}{d^{\frac{1}{2}}}, -\frac{a}{2c^{\frac{3}{2}}}, \frac{b}{2d^{\frac{3}{2}}} \right)$$

we can apply the Delta Method and deduce that

$$\begin{aligned} \mathbb{V}(f(v)) &= \frac{\nabla' f(v) \Omega \nabla f(v)}{T} \\ &= \frac{1}{T} \left(\frac{\sigma_i^2}{(\sigma_i^-)^2} - \frac{2\sigma_{ij}}{\sigma_i^- \sigma_j^-} + \frac{\sigma_j^2}{(\sigma_j^-)^2} + \frac{5\mu_i^2 \sigma_i^4}{16(\sigma_i^-)^6} - \frac{\mu_i \mu_j \sigma_{ij}^2}{4(\sigma_i^-)^3 (\sigma_j^-)^3} + \frac{5\mu_j^2 \sigma_j^4}{16(\sigma_j^-)^6} \right) \end{aligned}$$

The construction of the covariance matrix using the Ledoit and Wolf approach is analogous to the Sharpe problem.

Moreover, as mentioned previously, I also apply these tests in 3 scenarios: one unrestrained; one imposing a restriction on leverage to be less or equal than 2; and one imposing a restriction on leverage and on ex-post information. The restriction on ex-post information only means that the constant that is supposed to force the unconditional (semi)variances to be the same will be calculated only using past information. In practice this makes the strategy a process adapted to the natural filtration and therefore more realistic.

These procedures should then allow us to, first of all, check if these strategies expand the mean-variance frontier, meaning that it should be possible to create a new optimal portfolio that achieves a higher Sharpe Ratio than the previous optimal portfolio. We should also be able to say whether these volatility and semivolatility management strategies improved performance when compared directly to the buy-and-hold approach, and if so, in which factors. These answers could also bring new insights on the difference among the several facets of anomalies known to describe cross-sectional returns and how they relate with risk, be it as a whole or as best perceived by the investor.

3 Results

I use daily data to estimate monthly semivariances and monthly data to construct the portfolios. Table 2 shows the alphas from the spanning regressions estimated using Ordinary Least Squares,[1] as well as the Sharpe Ratio differences, Sortino Ratio differences and inferences on them using the Jobson and Korkie test statistic and the Ledoit and Wolf bootstrap-based methodology for the main factors presented in previous works, plus the Expected Growth factor from the augmented q-factors model. The differences for the performance ratios are constructed using the managed strategy against the Buy-and-Hold portfolio for the correspondent factor. We can observe a lot of unrealistic numbers, mainly within the spanning-regression alphas. This is due to the lack of restriction on the leverage multiplier, which can easily explode. Looking at the p-values for the Jobson-Korkie and Ledoit-Wolf test, results are pretty inconclusive, with only the performance of volatility-management on the Momentum factor being robust. Apart from this, the three strategies seem to perform in a similar, unimpressive fashion when compared to the Buy-and-Hold, with semivolatility management underperforming in a few cases.

Table 3 shows the same informations but for the second scenario, with limits to leverage. This means that the multiplier (e.g. $\frac{c}{\hat{\sigma}^2_{t-1}}$ for the volatility-managed) cannot be greater than 2. If at a given time the strategy suggests a multiplier greater than this value, we bound it to 2 in that period. The idea is to prevent unrealistic levels of leverage in short periods of time (one month in this case, since we recalibrate monthly). We can see that the alpha from the spanning regressions are relatively more tame than in the previous scenario. As in the previous scenario, the volatility-managed strategy only outperforms the Buy-and-Hold in the Momentum factor. My constructions begin to outperform more factors than the original volatility-management construction, and more interestingly, seem to cover other grounds in this sense. The second construction, for example, does not outperform in the momentum factor, but seems to work in other factors.

Finally, table 4 gives us the information for the last scenario, with the same restriction on leverage but also imposing a new restriction on ex-post information. In practice, this means that, for a given time t, the constants c^*, $c^\dagger$ and c only normalize

[1] Not to be mistaken with the alpha from the CAPM model.

unconditional downside variance (in the case of c^* and $c^\dagger$) and unconditional volatility (in the case of c) using information up to $t-1$. Here, even the performance of the volatility-managed strategy in the Momentum factor fades away, but my constructions stay robust, with the second construction improving on the first.

Overall, these findings are aligned to previous findings in this literature for the case of the original construction found in Moreira and Muir (2017). The results for the new constructions show their potential to stand out as a strategy for timing exposure to risk in a more intelligent way, attempting to escape the bad times but also to surf a little on the (volatile) good times, especially for the construction which accounts both for upside and downside volatility. The spanning regression analysis suggests that for most factors, the semivolatility-managed portfolios expand the mean-variance frontier, implying that the investor could find a new portfolio that combines these strategies and the Buy-and-Hold and achieve a higher level of utility. The alphas were tested using HAC standard errors.

Tables 5 through 7 complement with the number of factors and anomalies in the whole sample that presented alphas, Sharpe differences and Sortino differences significant at 5%. It becomes evident that analysing whether the strategies expand the maximum attainable Sharpe Ratio covers another ground than testing whether they actually performed individually better than its Buy-and-hold counterpart. We can see in table 8 that the volatility-management strategy only improves Sortino Ratio in 2 out of the 65 factors and anomalies tested, and those are two of the three momentum portfolios present in the sample. Looking at the real-time scenario, my constructions outperform the unscaled portfolios in more than 10% of the whole sample of factors and anomalies, more than twice as many as the volatility-management construction.[2]

3.1 Further analysis

Running an OLS regression for the monthly returns series using the level series of VIX gives us significant betas for 24 out of the 65 portfolios analyzed. When we compare all the betas against the respective portfolio's Sortino Ratio increase for each strategy f_σ, f^1_{SV} and f^2_{SV}, we get correlations of 7.75%, 0.87% and -4.38% respectively. If we interpret more negative betas as an indicator that the factor is more sensitive to the

[2] Here is worth noting that since managed constructions are of the form $f_{M,t} = \alpha f_t$, a significant negative difference between Sharpe (Sortino) ratios indicates that a portfolio consisting of $(1-\alpha)f_t$ has a strictly positive Sharpe (Sortino) ratio.

leverage effect, we can deduce that these factors that are more affected tend to benefit more from semivolatility management. This is something intuitive, yet very clarifying to think about. The more systemic volatility negatively affects returns, the more the asset's downside and upside volatility are informative about its conditional risk.

We know that volatility management in theory should work well if volatility is persistent and does not positively relate with future returns. This is shown to be true for the momentum factor, as shown in figure 2. In fact, realized volatility negatively correlates with future returns in this particular case. However, this might not tell the whole story, since by this logic volatility management should also work nicely on other factors that present this behavior, and this does not always happen, as shows figure 3. The same logic applies for the downside variance management, but for the construction involving upside and downside variances it should be the case that $\frac{\sigma_t^+}{\sigma_t^-}$ should not be negatively correlated with future returns. Indeed, this is true for 4 out of the 5 factors in which this construction outperforms the Buy-and-Hold significantly in the Sortino Ratio (with the BAB factor being the only exception). All semivariances present serial persistence.

Figure 4 show plots for the two principal components of the original, unscaled monthly return series. Graph A shows every factor and anomaly in the sample, while graphs B, C and D show only those present in tables 7 through 9. Grossly speaking, Principal Component 1 seems to be related to the size and age of the companies, while Principal Component 2 seems to be related with how well the company is doing in the market. We can visually see that the first semivariance construction seems to expand the area where timing risk works relative to the original volatility management, while the second semivariance construction does not improve so much on the first.

All of this serves as evidence of the importance of accounting for the signal of the risk, and not only its magnitude. This is aligned with recent discoveries on the literature of semivariance measures and their relationship with risk precification within the market.[3]

[3] See Bollerslev et al. (2020b) for an example.

4 Conclusions

It is still subject to an ongoing debate on the finance literature what are the benefits, if there are any, of volatility timing on active portfolio management. This work aims to provide new results that might bring more information to this discussion, as well as to expand recent works on this area by comprising a larger class of possible constructions involving risk exposure timing.

My results point to the already expected result that the original construction of volatility managed portfolios presented in Moreira and Muir (2017) only significantly outperform the unmanaged strategies in momentum factors. That said, the strategy still seemed to bring something new to the table, expanding the mean-variance frontier in a number of factors.

The constructions I presented as alternatives using semivariances on the other hand performed consistently better. Even when restrained with leverage and post-information constraints and tested using a more robust approach via bootstrap, they still outperformed their unmanaged counterparts in more than twice as many factors as the original construction. The mechanism and logic of why the strategies work on what they work is still unclear. However, decomposing risk into its two signed components seems to give us information on how it could be better understood.

Given these results, future work following this idea could focus on exploring new and more sophisticated constructions that better separate each kind of risk, accounting for possible conditional symmetry and incorporating more aspects of the risk perception from the individual in the strategy. There is also space to further expand the asymptotic theory and modelling of the underlying price processes, possibly allowing for jumps and other financial data features, and to investigate whether they allow for better risk control.

Bibliography

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *The Journal of Finance 61*(1), 259–299.

Asness, C. and A. Frazzini (2013). The devil in hml's details. *The Journal of Portfolio Management 39*(4), 49–68.

Ballestero, E. (2005). Mean-semivariance efficient frontier: A downside risk model for portfolio selection. *Applied Mathematical Finance 12*(1), 1–15.

Barbee Jr, W. C., S. Mukherji, and G. A. Raines (1996). Do sales–price and debt–equity explain stock returns better than book–market and firm size? *Financial Analysts Journal 52*(2), 56–60.

Barndorff-Nielsen, O. E., S. Kinnebrock, and N. Shephard (2008). Measuring downside risk-realised semivariance. *CREATES Research Paper* (2008-42).

Barroso, P. and A. Detzel (2021). Do limits to arbitrage explain the benefits of volatility-managed portfolios? *Journal of Financial Economics*.

Barroso, P. and P. Santa-Clara (2015). Momentum has its moments. *Journal of Financial Economics 116*(1), 111–120.

Barry, C. B. and S. J. Brown (1984). Differential information and the small firm effect. *Journal of financial economics 13*(2), 283–294.

Basu, S. (1977). Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. *The journal of Finance 32*(3), 663–682.

Bhandari, L. C. (1988). Debt/equity ratio and expected common stock returns: Empirical evidence. *The journal of finance 43*(2), 507–528.

Blume, M. E. and F. Husic (1973). Price, beta, and exchange listing. *The Journal of Finance 28*(2), 283–299.

Bollerslev, T., J. Li, A. J. Patton, and R. Quaedvlieg (2020). Realized semicovariances. *Econometrica 88*(4), 1515–1551.

Bollerslev, T., A. J. Patton, and R. Quaedvlieg (2020a). Multivariate leverage effects and realized semicovariance garch models. *Journal of Econometrics*.

Bollerslev, T., A. J. Patton, and R. Quaedvlieg (2020b). Realized semibetas: Signs of things to come. *Available at SSRN 3528276*.

Cederburg, S., M. S. O'Doherty, F. Wang, and X. S. Yan (2020). On the performance of volatility-managed portfolios. *Journal of Financial Economics*.

Chen, L., R. Novy-Marx, and L. Zhang (2011). An alternative three-factor model. *Available at SSRN 1418117*.

Cooper, M. J., H. Gulen, and M. J. Schill (2008). Asset growth and the cross-section of stock returns. *the Journal of Finance 63*(4), 1609–1651.

Da, Z., Q. Liu, and E. Schaumburg (2014). A closer look at the short-term return reversal. *Management science 60*(3), 658–674.

Daniel, K. and S. Titman (2006). Market reactions to tangible and intangible information. *The Journal of Finance 61*(4), 1605–1643.

Datar, V. T., N. Y. Naik, and R. Radcliffe (1998). Liquidity and stock returns: An alternative test. *Journal of financial markets 1*(2), 203–219.

De Bondt, W. F. and R. Thaler (1985). Does the stock market overreact? *The Journal of finance 40*(3), 793–805.

Dechow, P. M., S. P. Kothari, and R. L. Watts (1998). The relation between earnings and cash flows. *Journal of accounting and Economics 25*(2), 133–168.

Estrada, J. (2007). Mean-semivariance behavior: Downside risk and capital asset pricing. *International Review of Economics & Finance 16*(2), 169–185.

Fairfield, P. M., J. S. Whisenant, and T. L. Yohn (2003). Accrued earnings and growth: Implications for future profitability and market mispricing. *The accounting review 78*(1), 353–371.

Foster, G., C. Olsen, and T. Shevlin (1984). Earnings releases, anomalies, and the behavior of security returns. *Accounting Review*, 574–603.

Haddad, V., S. Kozak, and S. Santosh (2020). Factor timing. *The Review of Financial Studies 33*(5), 1980–2018.

Haugen, R. A. and N. L. Baker (1996). Commonality in the determinants of expected stock returns. *Journal of financial economics 41*(3), 401–439.

Heston, S. L. and R. Sadka (2008). Seasonality in the cross-section of stock returns. *Journal of Financial Economics 87*(2), 418–445.

Hirshleifer, D., K. Hou, S. H. Teoh, and Y. Zhang (2004). Do investors overvalue firms with bloated balance sheets? *Journal of Accounting and Economics 38*, 297–331.

Ikenberry, D., J. Lakonishok, and T. Vermaelen (1995). Market underreaction to open market share repurchases. *Journal of financial economics 39*(2-3), 181–208.

Jegadeesh, N. (1990). Evidence of predictable behavior of security returns. *The Journal of finance 45*(3), 881–898.

Jegadeesh, N. and S. Titman (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance 48*(1), 65–91.

Jobson, J. D. and B. M. Korkie (1981). Performance hypothesis testing with the sharpe and treynor measures. *Journal of Finance*, 889–908.

Lakonishok, J., A. Shleifer, and R. W. Vishny (1994). Contrarian investment, extrapolation, and risk. *The journal of finance 49*(5), 1541–1578.

Ledoit, O. and M. Wolf (2008). Robust performance hypothesis testing with the sharpe ratio. *Journal of Empirical Finance 15*(5), 850–859.

Markowitz, H., P. Todd, G. Xu, and Y. Yamane (1993). Computation of mean-semivariance efficient sets by the critical line algorithm. *Annals of Operations Research 45*(1), 307–317.

Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. Yale University Press.

Memmel, C. (2003). Performance hypothesis testing with the sharpe ratio. *Finance Letters 1*(1).

Moreira, A. and T. Muir (2017). Volatility-managed portfolios. *The Journal of Finance 72*(4), 1611–1644.

Moskowitz, T. J. and M. Grinblatt (1999). Do industries explain momentum? *The Journal of finance 54*(4), 1249–1290.

Naranjo, A., M. Nimalendran, and M. Ryngaert (1998). Stock returns, dividend yields, and taxes. *The Journal of Finance 53*(6), 2029–2057.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of financial economics 108*(1), 1–28.

Piotroski, J. D. (2000). Value investing: The use of historical financial statement information to separate winners from losers. *Journal of Accounting Research*, 1–41.

Pontiff, J. and A. Woodgate (2008). Share issuance and cross-sectional returns. *The Journal of Finance 63*(2), 921–945.

Sloan, R. G. (1996). Do stock prices fully reflect information in accruals and cash flows about future earnings? *Accounting review*, 289–315.

Soliman, M. T. (2008). The use of dupont analysis by market participants. *The Accounting Review 83*(3), 823–853.

Spiess, D. K. and J. Affleck-Graves (1999). The long-run performance of stock returns following debt offerings. *Journal of Financial Economics 54*(1), 45–73.

Xing, Y. (2008). Interpreting the value effect through the q-theory: An empirical investigation. *The Review of Financial Studies 21*(4), 1767–1795.

Appendix

APPENDIX A – Tables

Table 1 – Descriptive statistics for the whole sample of factors and anomalies used. Statistics were calculated using monthly returns.

Factor	Description	Mean	Min	Max	Standard Deviation	Downside Deviation	Skewness	Kurtosis
MKT	Fama-French Excess Market Returns factor	0.69%	-29.13%	38.85%	5.34%	3.54%	0.17	7.60
SMB	Fama-French Small Minus Big factor	0.20%	-16.82%	36.70%	3.18%	1.90%	1.88	18.82
HML	Fama-French High Minus Low (value minus growth) factor	0.34%	-13.96%	35.46%	3.51%	1.99%	2.07	18.24
MOM	Momentum Factor	0.64%	-52.27%	18.36%	4.71%	3.60%	-2.96	26.82
RMW	Fama-French Robust Minus Weak factor	0.26%	-18.48%	13.38%	2.18%	1.42%	-0.3	11.95
CMA	Fama-French Conservative Minus Aggresive factor	0.26%	-6.86%	9.56%	2.00%	1.21%	0.32	1.53
BAB	Betting-Against-Beta factor	0.68%	-21.95%	18.65%	3.26%	2.15%	-0.68	7.02
IA	q-factors model Investment factor	0.33%	-7.16%	9.24%	1.89%	1.13%	0.15	1.22
ROE	q-factors model Return On Equity factor	0.51%	-14.46%	10.38%	2.56%	1.73%	-0.91	5.72
EG	Expected Growth factor	0.80%	-9.72%	11.51%	1.98%	1.01%	0.11	3.95
accruals	Accruals anomaly.Sloan (1996)	0.40%	-12.42%	13.81%	3.16%	1.95%	0.24	1.93
age	The number of months that a firm has been listed in the CRSP database. Barry and Brown (1984)	0.03%	-26.39%	27.03%	4.15%	2.99%	-0.17	10.98
aturnover	Sales to total assets. Soliman (2008)	0.41%	-16.50%	14.11%	3.76%	2.41%	0.09	1.30
betaarb	Beta with respect to the CRSP equal-weighted return index. Cooper et al. (2008)	0.11%	-23.59%	24.36%	6.15%	4.37%	-0.11	1.40
cfp	Net income plus depreciation and amortization	0.36%	-18.23%	15.98%	4.35%	2.84%	0.07	1.81
ciss	Composite Equity Issuance anomaly. Daniel and Titman (2006)	0.50%	-15.02%	14.06%	3.27%	2.14%	-0.33	2.14
debtiss	Binary variable equal to one if long-term debt issuance indicated in statement of cash flow. Spiess and Affleck-Graves (1999)	0.12%	-9.45%	12.94%	1.79%	1.21%	0.27	6.55
divg	Dividend Growth anomaly	-0.03%	-12.16%	26.76%	3.31%	2.18%	1.07	7.82
divp	Dividend scaled by price. Naranjo et al. (1998)	0.14%	-25.72%	20.46%	5.12%	3.54%	-0.09	2.91
dur	Present value of expected cashflows	0.37%	-22.62%	19.37%	4.83%	3.19%	0.01	1.25
ep	Net income scaled by market value of equity. Basu (1977)	0.54%	-28.54%	21.69%	4.74%	3.09%	-0.1	4.59
exchsw	Exchange Switch anomaly	0.23%	-16.54%	15.92%	4.10%	2.80%	-0.03	1.56
fscore	Piotroski's F-score. Piotroski (2000)	0.08%	-5.67%	6.31%	1.63%	1.11%	0.08	1.50
gltnoa	Growth in Net Operating Assets minus Accruals. Fairfield et al. (2003)	-0.03%	-15.06%	10.09%	2.95%	2.21%	-0.56	2.80
gmargins	Gross profits to Total revenue. Novy-Marx (2013)	0.05%	-11.22%	13.06%	3.35%	2.35%	-0.02	0.80
growth	Asset growth anomaly. Cooper et al. (2008)	0.26%	-12.92%	13.43%	3.46%	2.26%	0.18	1.03
igrowth	Investment growth anomaly. Xing (2008)	0.34%	-10.26%	12.82%	2.74%	1.71%	0.21	1.02
indmom	Industry momentum anomaly. Moskowitz and Grinblatt (1999)	0.44%	-24.47%	31.01%	6.15%	4.19%	-0.04	3.27
indmomrev	Industry momentum reversal anomaly. Moskowitz and Grinblatt (1999)	1.10%	-17.72%	16.24%	3.47%	1.97%	-0.15	3.29
indrrev	Industry Relative Reversals. Da et al. (2014)	0.95%	-16.24%	26.69%	4.05%	2.23%	0.75	5.82
indrrevlv	Industry Relative Reversals (Low volatility). Da et al. (2014)	1.28%	-13.82%	23.71%	3.04%	1.48%	0.55	5.75
inv	Investment anomaly. Chen et al. (2011)	0.45%	-8.93%	10.64%	3.07%	1.89%	0.12	0.27
invaci	Abnormal Corporate Investment anomaly	0.10%	-28.13%	40.19%	5.68%	3.72%	0.7	5.88
invcap	Investment to capital anomaly. Xing (2008)	0.07%	-28.62%	29.21%	4.92%	3.52%	-0.12	5.56
ipo	Initial public offering anomaly	-0.04%	-22.98%	26.90%	4.77%	3.43%	-0.01	5.54
ivol	Idiossincratic volatility. Ang et al. (2006)	0.57%	-30.83%	34.34%	7.02%	4.82%	-0.21	3.48
lev	Market leverage anomaly. Bhandari (1988)	0.21%	-23.84%	17.74%	4.60%	3.13%	-0.01	2.25
lrrev	Long-term reversals anomaly. De Bondt and Thaler (1985)	0.30%	-18.97%	32.48%	5.08%	3.11%	0.89	4.00
	-0.07%	0.37%	-52.50%	24.38%	6.24%	4.74%	-1.44	9.89
mom	6-month momentum anomaly. Jegadeesh and Titman (1993)	1.26%	-45.32%	22.30%	6.88%	4.87%	-1.4	7.48
mom12	12-month momentum anomaly. Jegadeesh and Titman (1993)	0.41%	-16.34%	21.78%	4.77%	2.92%	0.64	3.07
momrev	Momentum-reversal anomaly. Jegadeesh and Titman (1993)	0.70%	-9.64%	16.07%	2.89%	1.56%	0.57	2.41
nissa	Annual share issuance anomaly. Pontiff and Woodgate (2008)	0.59%	-14.04%	19.14%	3.20%	1.89%	0.36	3.92
nissm	Monthly share issuance anomaly. Pontiff and Woodgate (2008)	0.41%	-15.29%	13.88%	3.21%	2.03%	0.09	2.84
noa	Net Operating Assets anomaly. Hirshleifer et al. (2004)	0.07%	-57.54%	17.55%	6.68%	5.38%	-2.04	11.73
price	Log of stock price anomaly. Blume and Husic (1973)	0.39%	-13.39%	11.44%	3.37%	2.16%	-0.03	0.97
prof	Gross profitability anomaly. Novy-Marx (2013)	0.13%	-12.84%	9.31%	1.73%	1.11%	0.14	8.19
repurch	Binary variable equal to one if repurchase of common or preferred shares indicated in statement of cash flow. Ikenberry et al. (1995)	0.55%	-20.18%	23.89%	4.77%	3.12%	0.06	3.05
roa	Quarterly return on assets anomaly. Chen et al. (2011)	0.22%	-16.54%	18.04%	4.07%	2.70%	0.19	1.88
roaa	Annual return on assets anomaly. Chen et al. (2011)	0.63%	-20.77%	29.74%	4.83%	2.91%	0.67	4.87
roe	Quarterly return on book equity anomaly. Chen et al. (2011)	0.10%	-20.87%	21.73%	4.37%	3.03%	0.02	2.54
roea	Annual return on book equity anomaly. Haugen and Baker (1996)	1.12%	-18.97%	25.60%	4.95%	2.82%	0.32	2.89
rome	Quarterly return on market equity anomaly. Chen et al. (2011)	0.78%	-17.95%	19.07%	3.91%	2.35%	0.13	2.79
season	Seasonality anomaly. Heston and Sadka (2008)	-0.08%	-16.21%	11.54%	3.68%	2.72%	-0.28	1.45
sgrowth	Sales growth anomaly. Lakonishok et al. (1994)	-0.04%	-18.71%	17.72%	4.16%	2.96%	0.04	1.89
shortint	Ratio of shares shorted anomaly. Dechow et al. (1998)	-0.03%	-29.96%	26.49%	5.91%	4.25%	-0.13	2.64
shvol	Share volume anomaly. Datar et al. (1998)	0.21%	-20.60%	32.07%	4.73%	2.98%	0.76	4.30
size	The CRSP end of June price times shares outstanding	0.44%	-16.25%	22.44%	4.22%	2.57%	0.61	2.99
sp	Sales-to-price anomaly. Barbee Jr et al. (1996)	0.38%	-24.66%	26.23%	5.21%	3.40%	0.22	3.82
strev	Short-term reversal anomaly. Jegadeesh (1990)	0.51%	-18.98%	16.11%	4.08%	2.70%	-0.3	2.93
sue	Standardized Unexpected Earnings anomaly. Foster et al. (1984)	0.47%	-23.55%	24.11%	4.98%	3.39%	-0.24	3.38
valmom	Sum of ranks in univariate sorts on book-to-market and momentum. Novy-Marx (2013)	0.81%	-22.16%	35.67%	4.78%	2.95%	0.41	5.90
valmomprof	Sum of ranks in univariate sorts on book-to-market	0.70%	-16.33%	20.90%	3.82%	2.27%	0.24	2.39
valprof	Sum of ranks in univariate sorts on book-to-market and profitability. Novy-Marx (2013)	0.36%	-17.76%	21.08%	4.57%	2.94%	0.25	1.26
value	Annual book-to-market anomaly	0.30%	-17.00%	54.42%	5.85%	3.35%	2.31	17.39
valuem	Monthly book-to-market anomaly. Asness and Frazzini (2013)	-0.08%	-17.96%	54.41%	5.85%	3.55%	2.33	17.55

Table 2 – Results for the unrestrained constructions. P-values in brackets. Values statistically significant at 5% are in bold.

	Factor									
	MKT	SMB	HML	MOM	RMW	CMA	BAB	IA	ROE	EG
Spanning-regression Alphas										
f_σ	**4.56***	-0.51	1.64	**12.08***	**2.53***	0.28	**6.97***	**1.43***	**5.39***	**4.11***
	(0.0068)	(0.6202)	(0.1636)	(0.0000)	(0.0040)	(0.6279)	(0.0000)	(0.0345)	(0.0000)	(0.0000)
f_{SV}^1	**6.85***	**4.54***	**8.77***	**25.21***	**9.35***	-0.52	**72.50***	1.04	**18.05***	**7.17**
	(0.0028)	(0.0075)	(0.0002)	(0.0097)	(0.0007)	(0.4273)	(0.0238)	(0.1429)	(0.0073)	(0.0000)
f_{SV}^2	**5.78***	2.17	**10.06***	**7.77***	**9.55***	-0.49	**59.48***	1.14	**9.42***	2.40
	(0.0078)	(0.0746)	(0.0008)	(0.0474)	(0.0005)	(0.4505)	(0.0224)	(0.1033)	(0.0008)	(0.0747)
Jobson and Korkie Test for Difference of Sharpe Ratios										
f_σ	0.07	-0.13	-0.01	**0.49***	0.17	-0.10	**0.31***	0.04	**0.36***	0.17
	(0.4372)	(0.1430)	(0.9176)	(0.0000)	(0.1680)	(0.3223)	(0.0016)	(0.6993)	(0.0023)	(0.1368)
f_{SV}^1	-0.01	0.12	0.18	-0.14	0.20	**-0.53***	**-0.47***	-0.15	-0.17	**-0.70***
	(0.9357)	(0.3160)	(0.1331)	(0.3035)	(0.1910)	(0.0027)	(0.0010)	(0.3107)	(0.3185)	(0.0000)
f_{SV}^2	0.00	0.02	0.03	-0.18	0.20	**-0.51***	**-0.46***	-0.09	-0.06	**-1.02***
	(0.9744)	(0.8452)	(0.7956)	(0.1663)	(0.1677)	(0.0032)	(0.0013)	(0.4918)	(0.6783)	(0.0000)
Ledoit and Wolf Test for Difference of Sharpe Ratios										
f_σ	0.07	-0.13	-0.01	**0.49***	0.17	-0.10	0.31	0.04	0.36	0.17
	(0.5263)	(0.2405)	(0.9480)	(0.0133)	(0.3165)	(0.3458)	(0.6509)	(0.6962)	(0.6409)	(1.0000)
f_{SV}^1	-0.01	0.12	0.18	-0.14	0.20	**-0.53***	-0.47	-0.15	-0.17	-0.7
	(0.9374)	(0.2665)	(0.3091)	(0.4237)	(0.3338)	(0.0273)	(0.0713)	(0.5989)	(0.9227)	(0.0786)
f_{SV}^2	0.00	0.02	0.03	-0.18	0.20	**-0.51***	-0.46	-0.09	-0.06	**-1.02***
	(0.9760)	(0.8374)	(0.8368)	(0.2472)	(0.2305)	(0.0426)	(0.0893)	(0.7069)	(0.8008)	(0.0007)
Jobson and Korkie Test for Difference of Sortino Ratios										
f_σ	0.15	-0.24	-0.08	**1.29***	0.42	-0.17	**1.03***	0.13	**1.23***	**1.24***
	(0.3027)	(0.0847)	(0.6360)	(0.0000)	(0.0517)	(0.3340)	(0.0000)	(0.5177)	(0.0001)	(0.0442)
f_{SV}^1	0.19	0.49	**1.07***	1.71	1.81	**-0.83***	11.09	-0.38	3.08	2.39
	(0.3768)	(0.0784)	(0.0131)	(0.4138)	(0.0605)	(0.0008)	(0.9816)	(0.0943)	(0.6010)	(0.6910)
f_{SV}^2	0.41	0.10	1.88	0.32	2.04	**-0.81***	9.68	-0.25	1.57	1.19
	(0.0722)	(0.6548)	(0.3439)	(0.3854)	(0.0906)	(0.0008)	(0.9739)	(0.2568)	(0.1620)	(0.8924)
Ledoit and Wolf Test for Difference of Sortino Ratios										
f_σ	0.15	-0.24	-0.08	**1.29***	0.42	-0.17	1.03	0.13	1.23	1.24
	(0.4930)	(0.3624)	(0.8294)	(0.0073)	(0.2452)	(0.4350)	(0.1559)	(0.6262)	(0.2132)	(0.9294)
f_{SV}^1	0.19	0.49	**1.07***	1.71	**1.81***	**-0.83***	11.09	-0.38	3.08	2.39
	(0.5350)	(0.2905)	(0.0466)	(0.2079)	(0.0460)	(0.0153)	(0.1646)	(0.4790)	(0.1706)	(0.1599)
f_{SV}^2	0.41	0.10	1.88	0.32	**2.04***	**-0.81***	9.68	-0.25	1.57	1.19
	(0.2012)	(0.7755)	(0.0500)	(0.5576)	(0.0253)	(0.0140)	(0.1472)	(0.6955)	(0.1093)	(0.5643)

Table 3 – Results for the constructions with restriction to leverage only. P-values in brackets. Values statistically significant at 5% are in bold.

	Factor									
	MKT	SMB	HML	MOM	RMW	CMA	BAB	IA	ROE	EG
	Spanning-regression Alphas									
f_σ	**3.42***	-0.23	1.51	**7.67***	**1.62***	0.17	**4.52***	**1.05***	**4.42***	**3.45***
	(0.0072)	(0.7435)	(0.0575)	(0.0000)	(0.0232)	(0.7156)	(0.0000)	(0.0429)	(0.0000)	(0.0000)
f_{SV}^1	**3.53***	**1.23***	**3.19***	**5.48***	**2.66***	0.03	**6.52***	**1.26***	**4.81***	**3.12***
	(0.0010)	(0.0222)	(0.0000)	(0.0000)	(0.0003)	(0.8122)	(0.0000)	(0.0012)	(0.0000)	(0.0001)
f_{SV}^2	**2.56***	0.90	**3.35***	1.07	**3.11***	0.06	**5.11***	**1.29***	**2.86***	0.37
	(0.0114)	(0.0948)	(0.0000)	(0.2799)	(0.0001)	(0.6637)	(0.0000)	(0.0005)	(0.0002)	(0.5425)
	Jobson and Korkie Test for Difference of Sharpe Ratios									
f_σ	0.09	-0.09	0.04	**0.53***	0.12	-0.09	**0.27***	0.04	**0.39***	0.18
	(0.2595)	(0.2429)	(0.6322)	(0.0000)	(0.2832)	(0.3495)	(0.0026)	(0.6823)	(0.0003)	(0.0861)
f_{SV}^1	0.12	0.09	**0.22***	**0.35***	**0.27***	-0.27	**0.38***	0.13	**0.43***	0.04
	(0.1837)	(0.3352)	(0.0118)	(0.0006)	(0.0183)	(0.0736)	(0.0000)	(0.2611)	(0.0002)	(0.7371)
f_{SV}^2	0.03	0.05	**0.18***	-0.06	**0.31***	-0.21	**0.28***	0.12	0.17	**-0.27***
	(0.6550)	(0.6179)	(0.0051)	(0.5060)	(0.0005)	(0.1437)	(0.0000)	(0.2713)	(0.0855)	(0.0057)
	Ledoit and Wolf Test for Difference of Sharpe Ratios									
f_σ	0.09	-0.09	0.04	**0.53***	0.12	-0.09	0.27	0.04	0.39	0.18
	(0.3578)	(0.2965)	(0.7628)	(0.0120)	(0.4464)	(0.3684)	(0.8075)	(0.7149)	(0.5943)	(0.9987)
f_{SV}^1	0.12	0.09	0.22	**0.35***	0.27	-0.27	0.38	0.13	0.43	0.04
	(0.3225)	(0.3691)	(0.0759)	(0.0433)	(0.1046)	(0.2652)	(0.4410)	(0.3225)	(0.5283)	(1.0000)
f_{SV}^2	0.03	0.05	**0.18***	-0.06	**0.31***	-0.21	**0.28***	0.12	0.17	-0.27
	(0.6729)	(0.7155)	(0.0386)	(0.6982)	(0.0260)	(0.4370)	(0.0140)	(0.2885)	(0.1792)	(0.1259)
	Jobson and Korkie Test for Difference of Sortino Ratios									
f_σ	0.15	-0.18	0.04	**1.21***	0.26	-0.17	**0.63***	0.08	**1.15***	**1.06***
	(0.2305)	(0.1567)	(0.8125)	(0.0000)	(0.1622)	(0.3011)	(0.0002)	(0.6376)	(0.0000)	(0.0467)
f_{SV}^1	0.25	0.19	**0.50***	**0.90***	**0.65***	**-0.53***	**1.13***	0.48	**1.35***	0.76
	(0.1064)	(0.2571)	(0.0054)	(0.0000)	(0.0031)	(0.0214)	(0.0000)	(0.0640)	(0.0000)	(0.1493)
f_{SV}^2	0.29	0.01	**0.75***	-0.01	**0.95***	**-0.44***	**0.89***	**0.59***	**0.72***	-0.13
	(0.0513)	(0.9348)	(0.0005)	(0.9535)	(0.0001)	(0.0410)	(0.0000)	(0.0326)	(0.0028)	(0.7327)
	Ledoit and Wolf Test for Difference of Sortino Ratios									
f_σ	0.15	-0.18	0.04	**1.21***	0.26	-0.17	0.63	0.08	1.15	1.06
	(0.4310)	(0.2672)	(0.9101)	(0.0013)	(0.3991)	(0.4250)	(0.5889)	(0.7082)	(0.1979)	(0.9214)
f_{SV}^1	0.25	0.19	**0.50***	0.90	**0.65***	**-0.53***	1.13	0.48	1.35	0.76
	(0.5350)	(0.2905)	(0.0466)	(0.2079)	(0.0460)	(0.0153)	(0.1646)	(0.4790)	(0.1706)	(0.1599)
f_{SV}^2	0.29	0.01	0.75	-0.01	**0.95***	**-0.44***	0.89	0.59	0.72	-0.13
	(0.2012)	(0.7755)	(0.0500)	(0.5576)	(0.0253)	(0.0140)	(0.1472)	(0.6955)	(0.1093)	(0.5643)

Table 4 – Results for the constructions with restrictions on leverage and ex-post information. P-values in brackets. Values statistically significant at 5% are in bold.

	Factor									
	MKT	SMB	HML	MOM	RMW	CMA	BAB	IA	ROE	EG
Spanning-regression Alphas										
f_σ	**3.20***	-0.07	**1.79***	**7.00***	**1.38***	0.17	**3.40**	**1.18***	**3.78***	**2.90***
	(0.0247)	(0.9224)	(0.0366)	(0.0000)	(0.0097)	(0.6964)	(0.0008)	(0.0191)	(0.0000)	(0.0000)
f^1_{SV}	**2.91***	**1.32***	**3.37***	**4.38***	**2.48***	0.02	**5.36***	0.51	**4.05***	**2.86***
	(0.0288)	(0.0447)	(0.0001)	(0.0107)	(0.0001)	(0.9153)	(0.0000)	(0.1061)	(0.0000)	(0.0001)
f^2_{SV}	1.86	1.26	**3.13***	0.56	**2.84***	0.14	**4.64***	0.46	**2.34***	0.52
	(0.0876)	(0.0776)	(0.0000)	(0.7478)	(0.0005)	(0.4633)	(0.0000)	(0.1358)	(0.0004)	(0.3666)
Jobson and Korkie Test for Difference of Sharpe Ratios										
f_σ	0.07	-0.07	0.05	**0.39***	0.16	-0.09	0.12	0.07	**0.37***	0.14
	(0.3847)	(0.3667)	(0.5228)	(0.0000)	(0.1657)	(0.3481)	(0.1178)	(0.4588)	(0.0008)	(0.1849)
f^1_{SV}	0.05	0.08	**0.20***	0.13	**0.31***	-0.30	**0.26***	-0.07	**0.41***	0.01
	(0.5130)	(0.3724)	(0.0082)	(0.1498)	(0.0094)	(0.0511)	(0.0007)	(0.5900)	(0.0009)	(0.9638)
f^2_{SV}	0.00	0.06	**0.15***	-0.13	**0.28***	-0.22	**0.23***	-0.06	0.14	**-0.26***
	(0.9941)	(0.4296)	(0.0155)	(0.1525)	(0.0019)	(0.1397)	(0.0001)	(0.6002)	(0.1908)	(0.0103)
Ledoit and Wolf Test for Difference of Sharpe Ratios										
f_σ	0.07	-0.07	0.05	**0.39***	0.16	-0.09	0.12	0.07	0.37	0.14
	(0.5217)	(0.4177)	(0.7215)	(0.0247)	(0.2918)	(0.3764)	(0.2512)	(0.4963)	(0.1193)	(0.9987)
f^1_{SV}	0.05	0.08	0.20	0.13	0.31	-0.30	**0.26***	-0.07	0.41	0.01
	(0.6895)	(0.3911)	(0.1159)	(0.4777)	(0.0873)	(0.0799)	(0.0280)	(0.6436)	(0.6889)	(0.9880)
f^2_{SV}	0.00	0.06	0.15	-0.13	**0.28***	-0.22	**0.23***	-0.06	0.14	-0.26
	(1.0000)	(0.5296)	(0.0646)	(0.4977)	(0.0460)	(0.8881)	(0.0100)	(0.6289)	(0.5750)	(0.2159)
Jobson and Korkie Test for Difference of Sortino Ratios										
f_σ	0.09	-0.14	0.10	**0.74***	0.35	-0.17	0.22	0.15	**1.10***	0.98
	(0.4688)	(0.2647)	(0.5096)	(0.0000)	(0.0737)	(0.3094)	(0.0998)	(0.3923)	(0.0000)	(0.0650)
f^1_{SV}	0.07	0.16	**0.45***	0.24	**0.78***	**-0.51***	**0.65***	-0.05	**1.51***	0.77
	(0.5785)	(0.3125)	(0.0039)	(0.0615)	(0.0011)	(0.0405)	(0.0000)	(0.8419)	(0.0001)	(0.1676)
f^2_{SV}	0.16	0.13	**0.68***	-0.16	**0.87***	-0.34	**0.67***	0.04	**0.67***	-0.17
	(0.2124)	(0.3624)	(0.0013)	(0.1567)	(0.0002)	(0.1872)	(0.0000)	(0.8729)	(0.0073)	(0.6660)
Ledoit and Wolf Test for Difference of Sortino Ratios										
f_σ	0.09	-0.14	0.10	0.74	0.35	-0.17	0.22	0.15	1.10	0.98
	(0.6835)	(0.3804)	(0.7275)	(0.0553)	(0.2998)	(0.4117)	(0.3438)	(0.4917)	(0.0566)	(0.9567)
f^1_{SV}	0.07	0.16	0.45	0.24	**0.78***	-0.51	**0.65***	-0.05	1.51	0.77
	(0.7695)	(0.4290)	(0.1093)	(0.4943)	(0.0466)	(0.1552)	(0.0213)	(0.9067)	(0.1692)	(0.5516)
f^2_{SV}	0.16	0.13	**0.68***	-0.16	**0.87***	-0.34	**0.67***	0.04	0.67	-0.17
	(0.9554)	(0.6069)	(0.0120)	(0.5716)	(0.0087)	(0.8421)	(0.0067)	(0.9167)	(0.1912)	(0.8688)

Table 5 – Number of significant factors at 5% by strategy and statistical test employed. — Unconstrained Constructions

	Total	Number of Factors	
		Spanning-regression alpha - Signif. [$\alpha > 0$]	
f_σ	65	23 [22]	
f^1_{SV}	65	38 [38]	
f^2_{SV}	65	23 [23]	
		Signif. [$\Delta > 0$] - Jobson and Korkie	Signif. [$\Delta > 0$] - Ledoit and Wolf
		Sharpe Ratio Difference	
f_σ	65	5 [5]	3 [3]
f^1_{SV}	65	13 [6]	6 [3]
f^2_{SV}	65	14 [2]	10 [2]
		Sortino Ratio Difference	
f_σ	65	8 [8]	4 [4]
f^1_{SV}	65	13 [11]	4 [3]
f^2_{SV}	65	12 [6]	4 [2]

Table 6 – Number of significant factors at 5% by strategy and statistical test employed. — Leverage Constrained Constructions

	Total	Number of Factors	
		Spanning-regression alpha - Signif. [$\alpha > 0$]	
f_σ	65	21 [21]	
f^1_{SV}	65	42 [42]	
f^2_{SV}	65	36 [36]	
		Signif. [$\Delta > 0$] - Jobson and Korkie	Signif. [$\Delta > 0$] - Ledoit and Wolf
		Sharpe Ratio Difference	
f_σ	65	8 [8]	4 [4]
f^1_{SV}	65	21 [21]	6 [6]
f^2_{SV}	65	19 [16]	8 [8]
		Sortino Ratio Difference	
f_σ	65	11 [11]	3 [3]
f^1_{SV}	65	29 [28]	7 [7]
f^2_{SV}	65	28 [25]	8 [8]

Table 7 – Number of significant factors at 5% by strategy and statistical test employed. — Real Time Constructions

	Total	Number of Factors	
		Spanning-regression alpha - Signif. [$\alpha > 0$]	
f_σ	65	19 [19]	
f_{SV}^1	65	39 [39]	
f_{SV}^2	65	32 [32]	
		Signif. [$\Delta > 0$] - Jobson and Korkie	Signif. [$\Delta > 0$] - Ledoit and Wolf
		Sharpe Ratio Difference	
f_σ	65	5 [5]	3 [3]
f_{SV}^1	65	20 [19]	8 [7]
f_{SV}^2	65	17 [13]	7 [7]
		Sortino Ratio Difference	
f_σ	65	8 [7]	2 [2]
f_{SV}^1	65	23 [22]	8 [8]
f_{SV}^2	65	23 [21]	5 [5]

Table 8 – Significant factors at 5% (Sortino differences using Ledoit and Wolf) for f_σ.

Factor/Anomaly	Description
mom	Cumulated past performance in the previous 6 months by skipping the most recent month
mom12	Cumulated past performance in the previous year by skipping the most recent month

Table 9 – Significant factors at 5% (Sortino differences using Ledoit and Wolf) for f^1_{SV}.

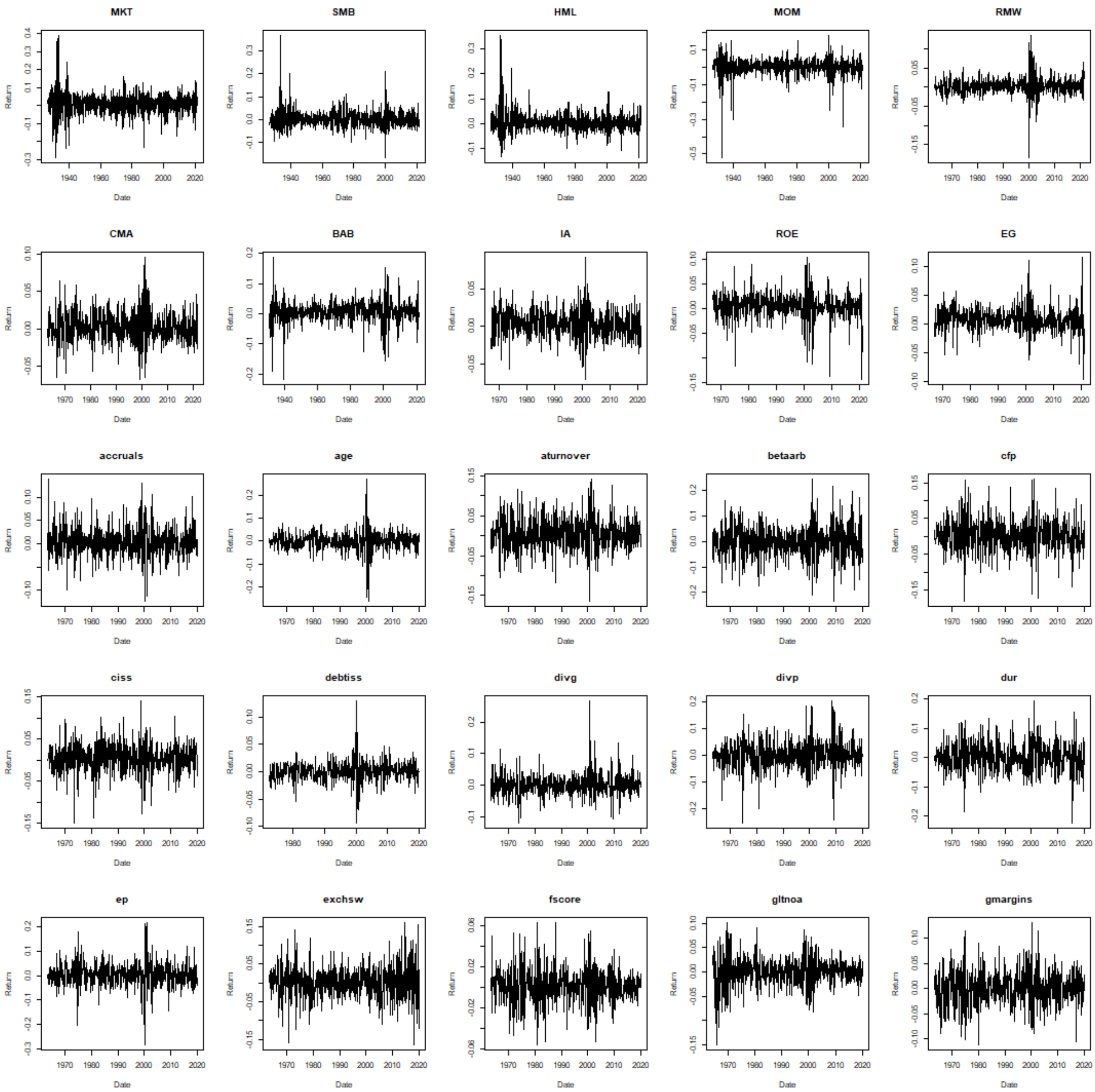
Factor/Anomaly	Description
age	The number of months that a firm has been listed in the CRSP database
BAB	Betting-Against-Beta factor
roaa	Annual return on assets anomaly
mom12	Cumulated past performance in the previous year by skipping the mostrecent m
lev	Market leverage anomaly
roea	Annual return on book equity anomaly
valmomprof	Sum of ranks in univariate sorts on book-to-market, profitability, and momentu
RMW	Fama-French Robust Minus Weak factor

Table 10 – Significant factors at 5% (Sortino differences using Ledoit and Wolf) for f^2_{SV}.

Factor/Anomaly	Description
BAB	Betting-Against-Beta factor
RMW	Fama-French Robust Minus Weak factor
roea	Annual return on book equity anomaly
HML	Fama-French High Minus Low (value minus growth) factor
roaa	Annual return on assets anomaly

APPENDIX B – Figures

Figure 1 – Returns series



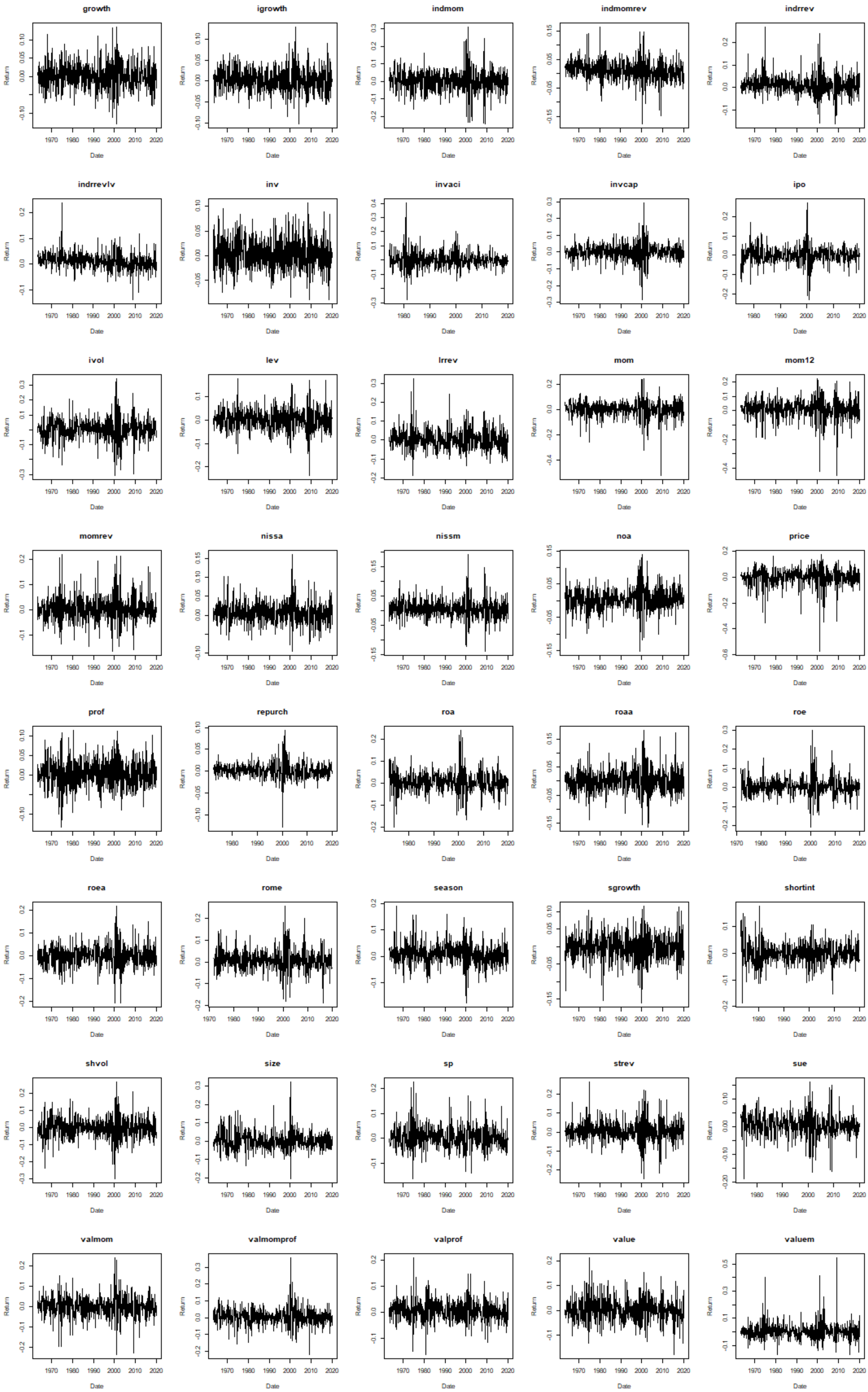

growth
igrowth
indmom
indmomrev
indrrev
indrrevlv
inv
invaci
invcap
ipo
ivol
lev
lrrev
mom
mom12
momrev
nissa
nissm
noa
price
prof
repurch
roa
roaa
roe
roea
rome
season
sgrowth
shortint
shvol
size
sp
strev
sue
valmom
valmomprof
valprof
value
valuem
Return
Date

Figure 2 – Autocorrelation Function for the realized variance of the Momentum Factor (top) and Cross-correlation Function between its realized variance and returns (bottom). Negative lags indicate correlation between past volatility and future returns.

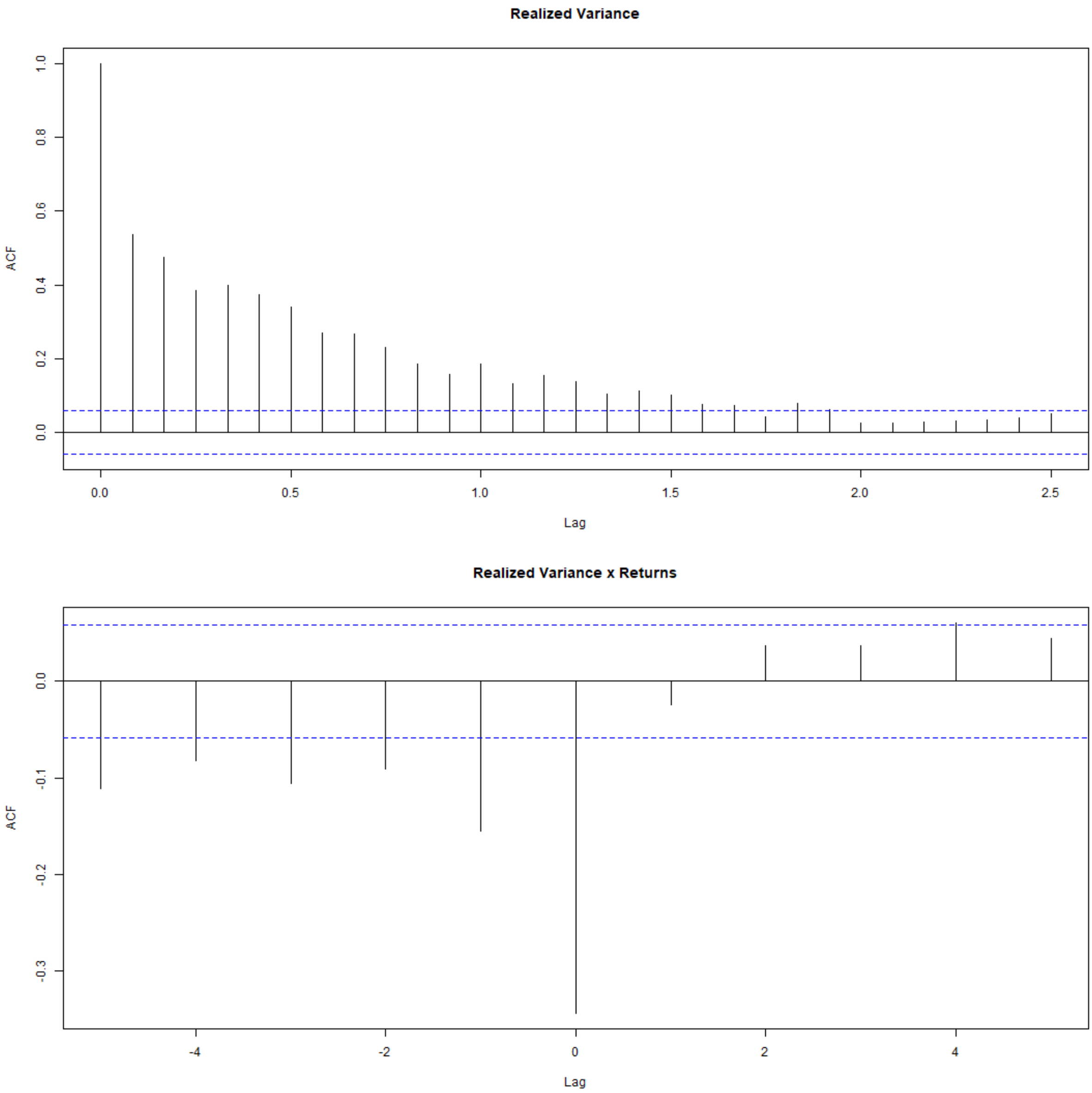


Figure 3 – Autocorrelation Function for the realized variance of the *price* anomaly (top) and Cross-correlation Function between its realized variance and returns (bottom). Negative lags indicate correlation between past volatility and future returns.

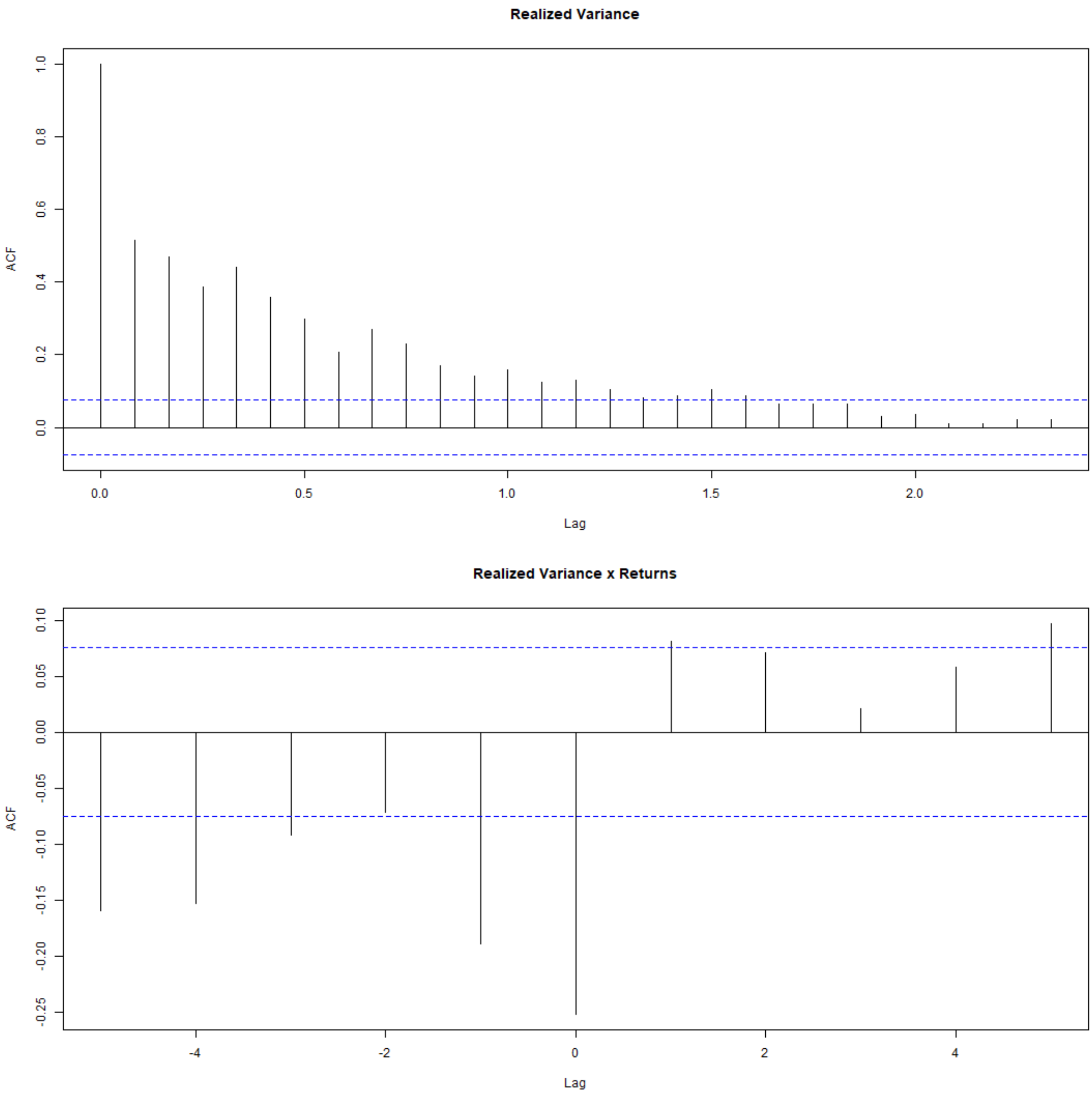


Figure 4 – Principal Component Analysis graphs of the monthly unscaled returns, with Principal Component 1 in the X-axis and Principal Component 2 in the Y-axis. Combined they explain 32.3% of the variation within the sample. Graph A shows every factor and anomaly in the sample. Graph B, C and D illustrate only the significant factors (Sortino Ratio, Ledoit-Wolf test) for the volatility managed, semivolatility managed construction 1 and semivolatility managed construction 2 respectively.