Time- and State-Dependent Pricing: A Unified Framework

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May 25, 2011

Motivation

- Recent availability of vast amounts of micro price data (Bils and Klenow 2004 and followers).

- Renewed interest in price setting among macroeconomists

  - development of new microfounded models with explicit price-setting frictions

    - mostly menu-cost models: (Golosov and Lucas 2007, Gertler and Leahy 2008, Midrigan 2011, and Nakamura and Steinsson 2010).

    - but also recently implications of explicit informational frictions (Reis 2006, Woodford 2009, Maćkowiak and Wiederholt 2009).

Motivation - 2

- Type of pricing friction $\longrightarrow$ very different macroeconomic consequences:
  - menu cost $\longrightarrow$state-dependent rules that fit well microdata, but very little macro effects (Almeida and Bonomo 2002, Golosov and Lucas 2007).
  - information models $\longrightarrow$ more important macro effects (Reis 2006, Woodford 2009, Maćkowiak and Wiederholt 2009)
- Literature guided by dichotomy between time-dependent and state-dependent pricing rules

What we do

- Tractable unified framework for solving optimal time- and state-dependent price setting problems.

- Apply to models with adjustment ("menu") cost to changing prices (K) and various alternatives for the source and nature of infrequent information
  - Exogenously infrequent information
  - Costly information gathering and processing (F)

- Unified framework that can be applied to most of the price-setting problems analyzed previously in the literature

- The key to making our approach tractable is our choice of state variables

Features

- In our framework pricing rules are in general both time- and state-dependent..

- Features:
  - build up of unobserved information $\longrightarrow$ time dependency
  - inaction region widens with time elapsed since last information date - option value increases with time.
  - when information date is known $\longrightarrow$ it is never optimal to adjust immediately before the information date.
  - Firms may choose to make uninformed adjustments - those resemble indexation by trend inflation.

Outline

The framework

- p_t^*: frictionless optimal price (full information; no adjustment cost)

- Flow of profit loss from price discrepancy: $\propto (p_t - p_t^*)^2$

- Infrequent information:
  - "Information dates": dates when firm has full info about p_t^*
  - t_0 denotes "last information date"

The framework - 2

$$\begin{aligned} E_{t_0}(p_t - p_t^*)^2 &= (p_t - E_{t_0}p_t^*)^2 + E_{t_0}(p_t^* - E_{t_0}p_t^*)^2 \\ &= (p_t - E_{t_0}p_t^*)^2 + Var_{t_0}(p_t^*) \end{aligned}$$

$(p_t - E_{t_0}p_t^*)^2 =$ flow cost of deviating from the *expected* level of the frictionless optimal price

$E_{t_0}(p_t^* - E_{t_0}p_t^*)^2 = Var_{t_0}(p_t^*)$ =expected flow cost from not continuously entertaining information about p_t^*

- Firms' objective: minimize PDV of expected total costs
  - integral of flow profit losses
  - plus other costs that prevent the firm from charging p_t^* continuously

The framework - 3

- Assumption:
  - p_t^* follows a Markovian stochastic process
  - for any $\Delta t > 0$, distribution of $p_{t+\Delta t}^* | p_t^*$ depends only on Δt

- Define *time elapsed since the last information date*:

$$\tau = t - t_0$$

- And *expected price discrepancy*:

$$z_t = p_t - E_{t-\tau} p_t^*$$

- Thus:

$$\begin{aligned} E_{t_0}(p_t - p_t^*)^2 &= z_t^2 + Var_{t_0}(p_{t_0+\tau}^*) \\ &= f(z_t, \tau) \end{aligned}$$

The framework - 4

- $V(z_t, \tau)$: optimized value of the firm's dynamic cost-minimization problem

- Bellman equation in the (adjustment/information) inaction region:

$$V(z_t, \tau) = f(z_t, \tau)dt + e^{-\rho dt} E_t V(z_{t+dt}, \tau + dt) \tag{1}$$

  – holds for all cases - including the standard full-information case

- For each problem: rewrite (1) as either an ODE (in z_t or τ), or as a PDE (in z_t and τ)

- Boundary conditions pin down solution:

  – vary depending on the nature of informational frictions

Time dependency and information

- We examine the link between information and time dependency.

- For this purpose, we contrast:
  - a setting where innovations are infrequent, but become known immediately, as in Danzinger (1999) and Gertler and Leahy (2008).
    - implies a constant inaction range - a purely state-dependent pricing rule.
  - cases in which unobserved information builds up over time and is revealed infrequently.

Continuous information, infrequent innovations

$$dp_t^* = \mu dt - \sigma \varepsilon dq_t$$

where

q_t is Poisson arrival process with intensity λ

ε is $N(0,1)$

- In the inaction region z_t evolves according to:

$$dz_t = -\mu dt + \varepsilon \sigma dq_t$$

Continuous information, infrequent innovations - 2

- Continuous information:

$$t_0 = t \Longrightarrow \tau = 0$$

$$E_{t_0} p_t^* = p_t^* \Longrightarrow z_t = p_t - p_t^*$$

$$f(z_t, \tau) = z_t^2$$

Continuous information, infrequent innovations - 3

- Differential form of the Bellman equation:

$$V(z_t) = z_t^2 dt + e^{-\rho dt} E_t V(z_{t+dt}) \tag{2}$$

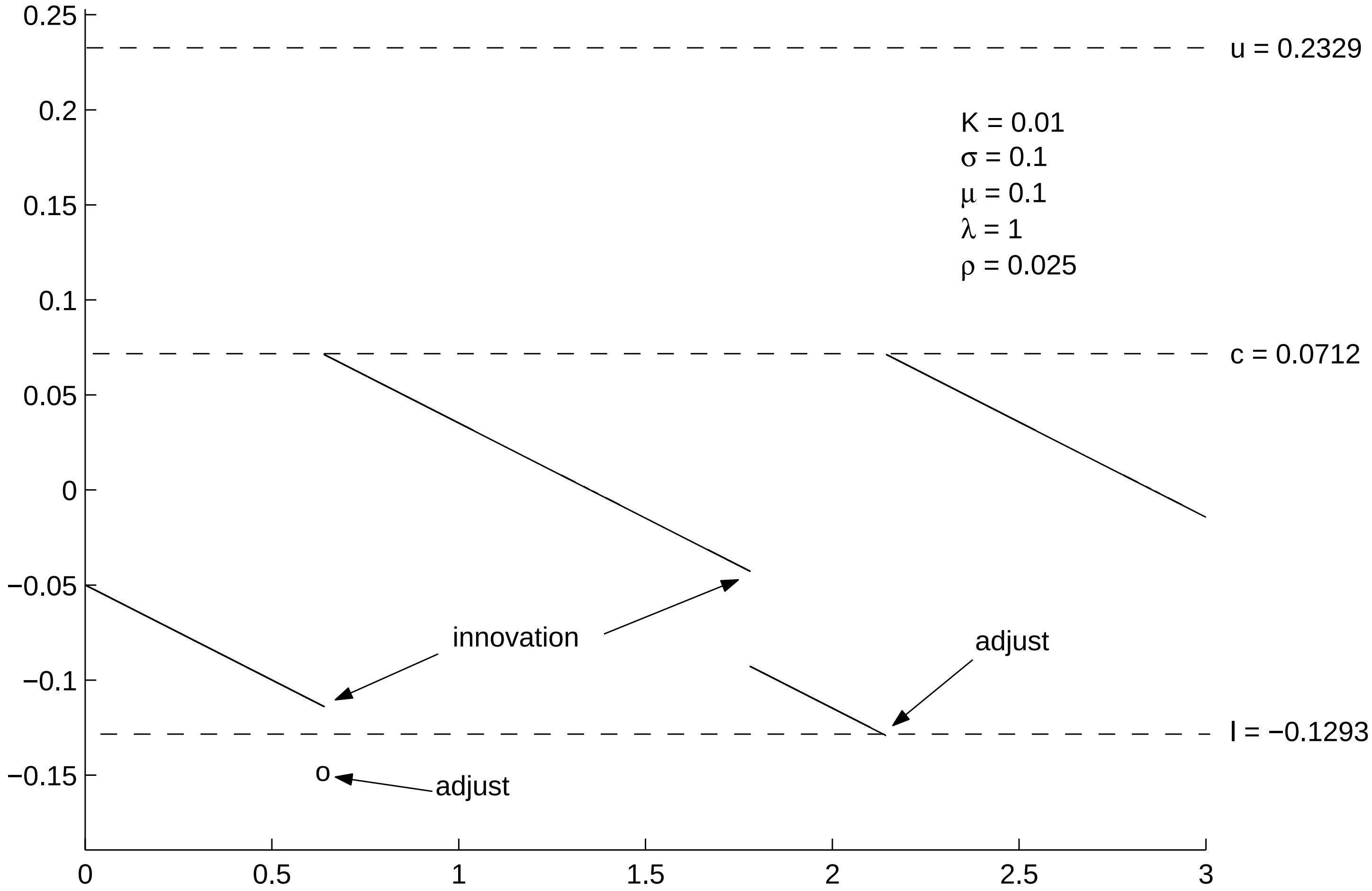
- Ito's lemma:

$$-\mu V_z(z) - (\rho + \lambda) V(z) + \lambda E\left[V(z + \sigma\varepsilon)\right] + z^2 = 0$$

- The optimal pricing rule is characterized by (l, c, u), which satisfy:

$$\begin{aligned} V_z(c) &= 0 \text{ (optimality of reset price)} \\ V(l) &= V(u) = V(c) + K \text{ (value matching)} \\ V_z(l) &= V_z(u) = 0 \text{ (smooth pasting optimality conditions)} \end{aligned}$$

Continuous information, infrequent innovations - 4



Build up of unobserved information

- Pricing problems in which information arrives infrequently for reasons outside the control of firms.
  - simpler than problems where information gathering and processing is costly;
  - highlight the importance of unobserved information for the nature of optimal pricing policies;
  - examples of exogenously infrequent information are pervasive:
    - $\longrightarrow$ economic developments often become news after having evolved unnoticed for some time;
    - $\longrightarrow$ data releases on prespecified dates usually reflect cumulative past information about the state of the economy.

Build up of unobserved information - 2

- Random information dates: continuous but unobserved innovations to the frictionless optimal (differs from the continuous-information infrequent-innovation specification).

  $\longrightarrow$ accumulate over time until they are fully revealed on the subsequent information date

  $\longrightarrow$ the boundaries of the inaction region depend on the time elapsed since the last information date: the pricing policy is both time- and state dependent.

  $\longrightarrow$ the inaction range widens as time elapses.

- Deterministic information dates.

  $\longrightarrow$ extreme form of inaction just prior to information dates: whatever the size of the mispricing, it is never optimal to adjust.

Random information dates

- p_t^* follows a Brownian motion with drift μ:

$$dp_t^* = \mu dt - \sigma dW_t \tag{3}$$

  where W_t is a standard Wiener process

- p_t^* is only observed at a random time, with a negative exponential distribution (essentially the sticky-information assumption of Mankiw and Reis 2002)

- z_t evolves according to:

$$dz_t = -\mu dt + \varepsilon\sigma\sqrt{\tau} dq_t \tag{4}$$

- On information dates, $z_t = p_t - E_{t_0} p_t^*$ jumps with the arrival of information about p_t^*

Random information dates - 2

- The flow deviation costs now depend on z_t and τ:

$$f(z_t, \tau) = z_t^2 + \sigma^2 \tau$$

- Barriers that determine the inaction region now depend on τ

- Pricing policy $\{l(\tau), c(\tau), u(\tau)\}_{0 \leq \tau < \infty}$

- Apply Ito's lemma to write the differential form of the Bellman equation as:

$$V_\tau(z, \tau) - \mu V_z(z, \tau) - (\rho + \lambda) V(z, \tau) + \lambda E\left[V\left(z + \sigma \varepsilon \sqrt{\tau}, 0\right)\right] + z^2 + \sigma^2 \tau = 0$$

Random information dates - 3

- Lump-sum adjustment costs:

$$c(\tau) = \arg\min_z V(z, \tau) \tag{5}$$

- Adjustment always an option:

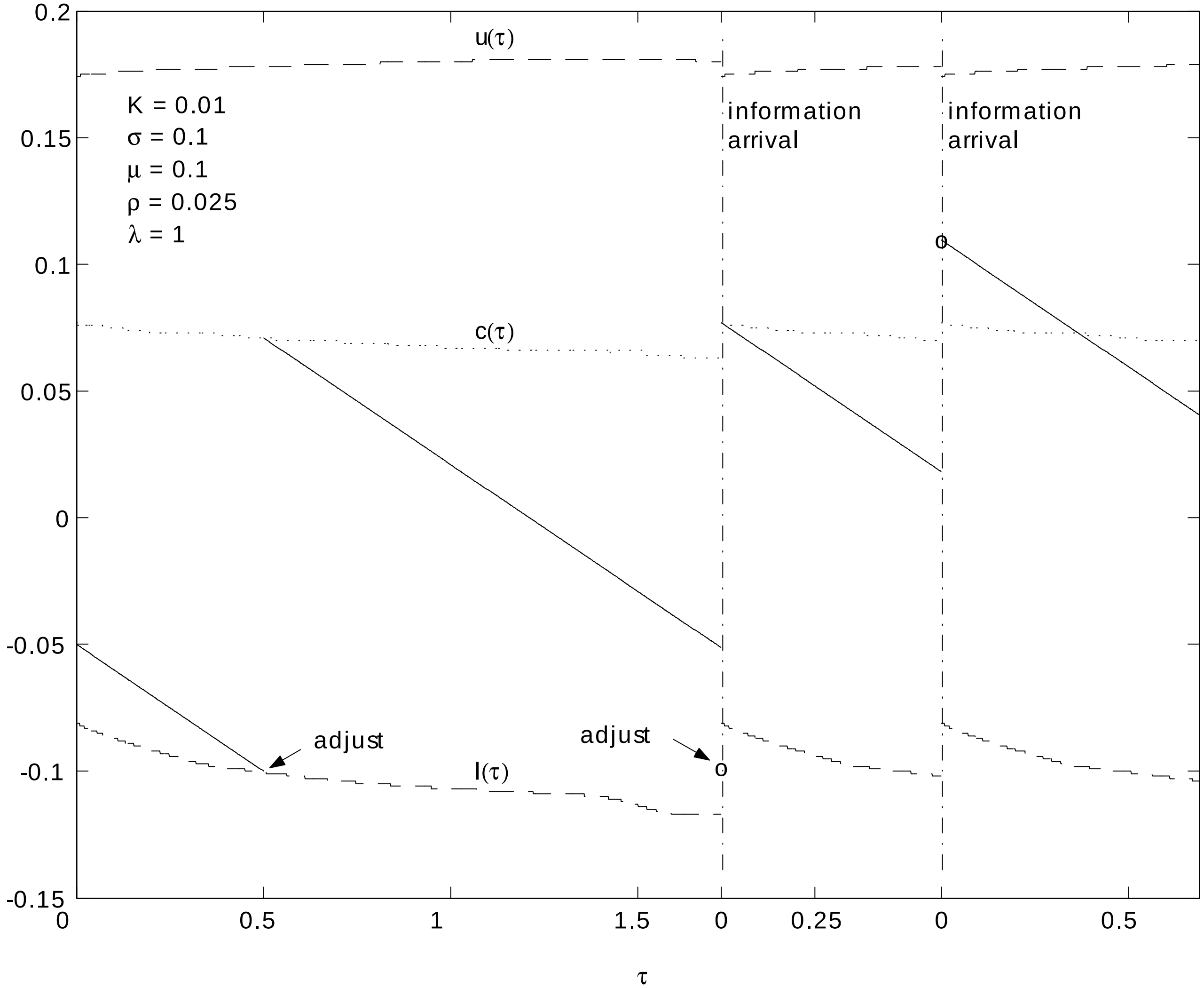
$$V(z, \tau) \leq V(c(\tau), \tau) + K \tag{6}$$

- Boundaries of inaction region satisfy:

$$\begin{aligned} V(l(\tau), \tau) &= V(c(\tau), \tau) + K \\ V(u(\tau), \tau) &= V(c(\tau), \tau) + K \end{aligned} \tag{7}$$

- Numerical solution using finite-difference algorithm

Random information dates - 4



Deterministic Information Dates

- Again we look for rules $\{l(\tau), c(\tau), u(\tau)\}_{0 \leq \tau \leq T}$, but now τ is bounded by T.

- In the absence of price changes and between information dates (for $0 < \tau < T$):

$$dz_t = -\mu dt.$$

Applying Ito's lemma to Bellman equation in the inaction region yields:

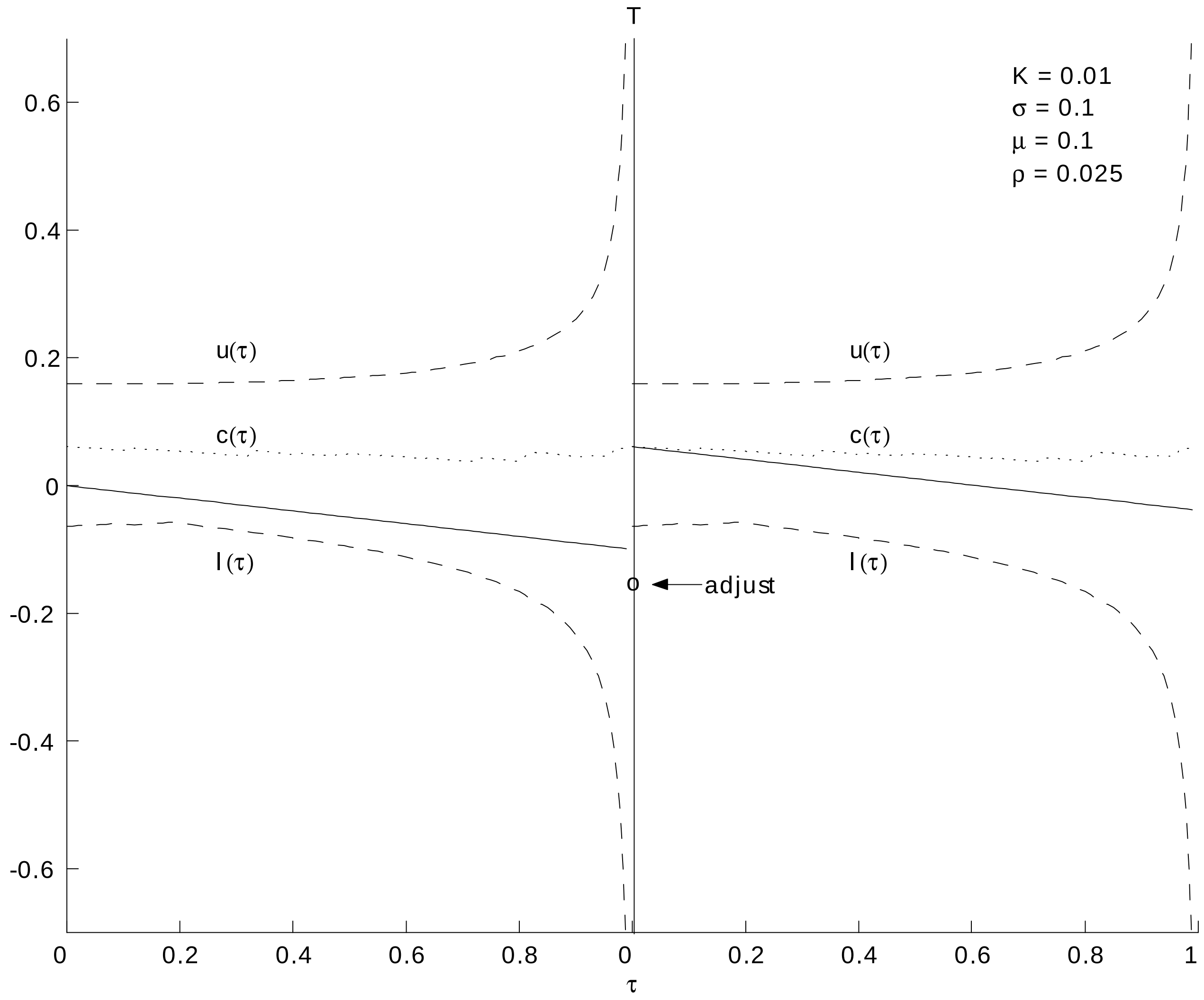
$$-\mu V_z(z,\tau) + V_\tau(z,\tau) - \rho V(z,\tau) + z^2 + \sigma^2 \tau = 0 \qquad (8)$$

- Conditions (5), (6), and (7) are still valid.

- When information arrives, the expected discrepancy receives a shock with distribution $N(0, \sigma^2 T)$, and τ is reset to zero. We thus have the following additional condition:

$$V(z,T) = E\left[V\left(z + \sigma\sqrt{T}\varepsilon, 0\right)\right], \qquad (9)$$

Deterministic Information Dates - 2



Lessons

- “Uninformed price adjustments” might be optimal

- Build up of unobserved information induces time dependency
  - Pricing rule is both time- and state-dependent
  - Inaction region widens as option value of waiting increases over time

- With deterministic information dates, extreme form of inaction:
  - Irrespective of the expected price gap, it is never optimal to adjust immediately before an information date

Costly information

- Even in contexts in which continuous access to information is possible, costs of gathering and processing such information might lead firms to incorporate it into pricing decisions only infrequently.

- In the paper:

  – Information cost with no adjustment cost (Caballero 1989, Reis 2006)

  – Joint information/adjustment cost (Bonomo and Carvalho 2004, 2010)

  – Model with dissociated information and adjustment costs

Dissociated information and adjustment costs

- The differential equation for V is still given by (8).

- Separate lump-sum costs for adjustment (K) and information gathering/processing (F)

- The option to incur F and entertain information implies:

$$\forall (z, \tau), \quad V(z, \tau) \leq E\left[V\left(z + \sigma\sqrt{\tau}\varepsilon, 0\right)\right] + F$$

- As before, the option to pay the adjustment cost K and reset z to a chosen expected discrepancy $c(\tau)$ implies

$$\forall (z, \tau), \quad V(z, \tau) \leq V(c(\tau), \tau) + K$$

where:

$$c(\tau) = \arg\min_{z} V(z, \tau)$$

Dissociated information and adjustment costs - 2

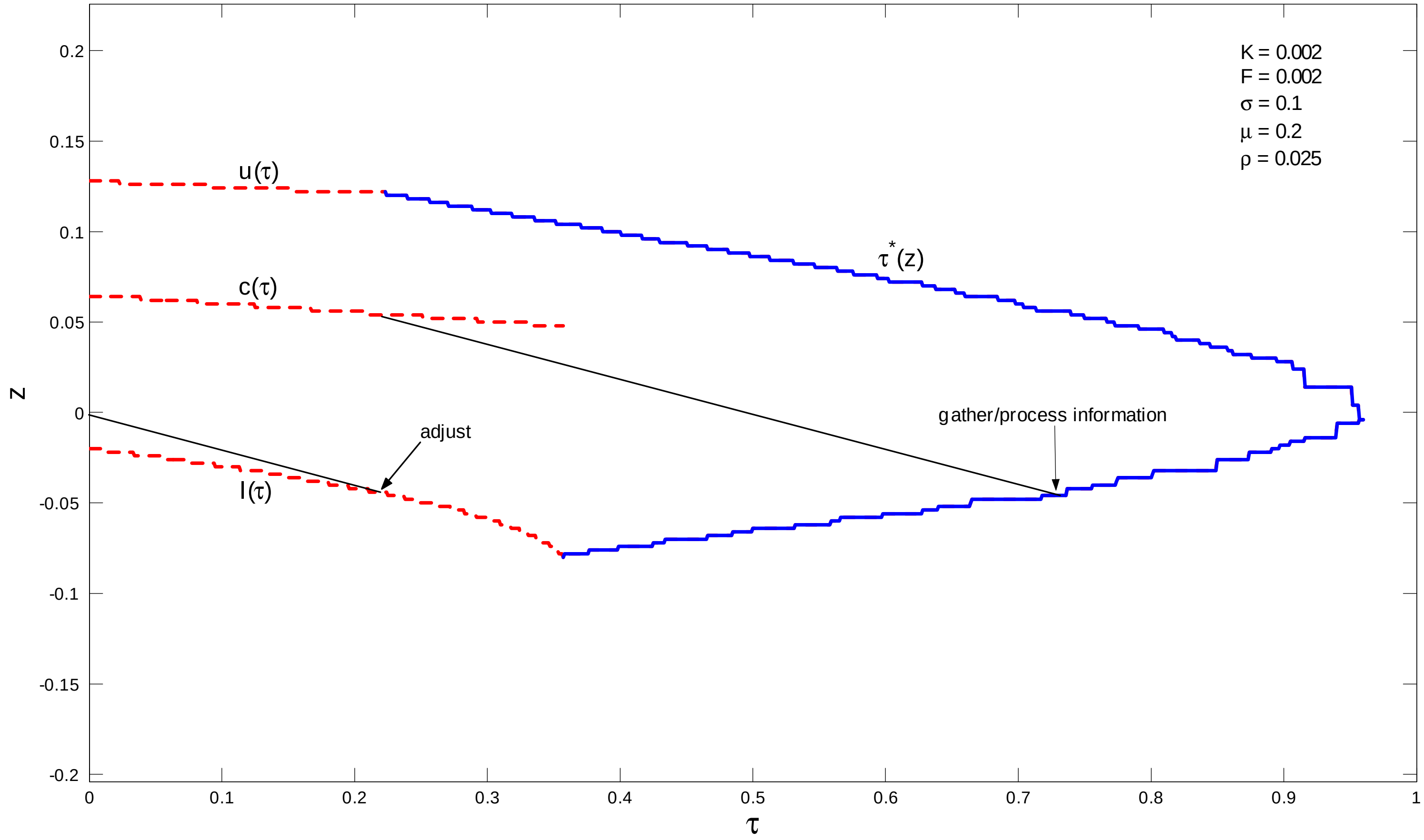
- The adjustment barriers at each point in (elapsed) time $l(\tau), u(\tau)$ satisfy:

$$V\left(l\left(\tau\right),\tau\right) = V\left(c\left(\tau\right),\tau\right) + K$$
$$V\left(u\left(\tau\right),\tau\right) = V\left(c\left(\tau\right),\tau\right) + K$$

- The boundary of the information inaction region is define by $\tau^*\left(z\right)$, which satisfies

$$V\left(z,\tau^*\left(z\right)\right) = E\left[V\left(z+\sigma\sqrt{\tau^*\left(z\right)}\varepsilon,0\right)\right] + F$$

Dissociated information and adjustment costs - 3



Lessons

- Optimal policy is both time- and state-dependent

- Uninformed price adjustments might be optimal

- As in the case with deterministically exogenous information, it is never optimal to adjust just before an information date
  - Rather than incurring the menu cost to make an uninformed adjustment and then immediately incurring the information gathering/processing cost, it is always better to reverse the order of these actions and keep the option to adjust (to be "exercised" or not depending on the new information)

Partial Information

- $dp^*_{it} = \mu dt - \sigma_i dW_{it} - \sigma_a dW_{at}$, where W_{it} and W_{at} are independent standard Wiener processes

- Information about W_{it} is continuously and freely available, and costless to process

- Firms face infrequent information about W_{at}

  - Exogenously infrequent

  - Endogenously infrequent

Partial Information - 2

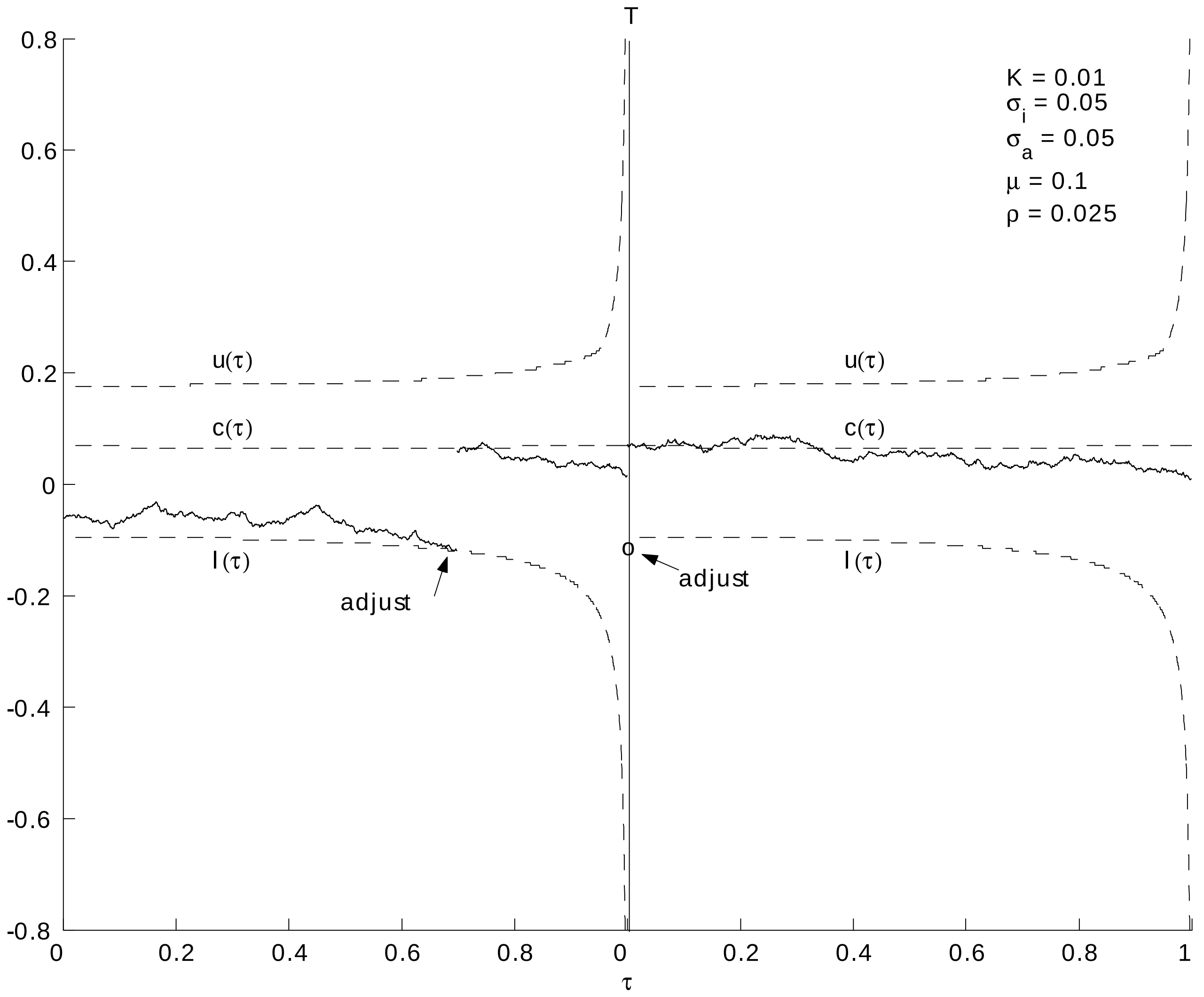
- $dz_t = -\mu dt + \sigma_i dW_{it}$.

- $f(z_t, \tau) = z_t^2 + \sigma_a^2 \tau$.

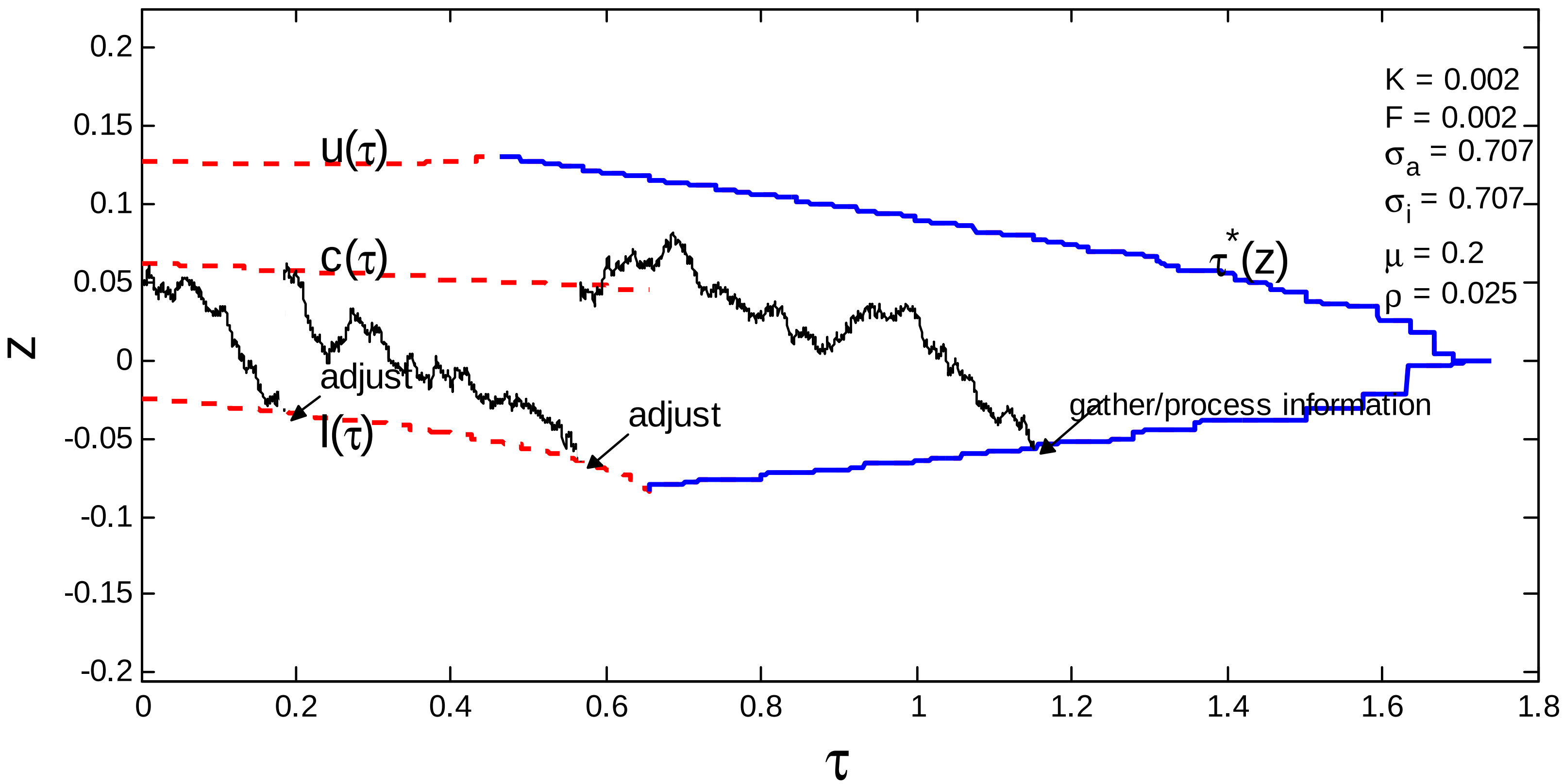
- The differential form of the Bellman equation (1) is now written as:

$$\frac{1}{2}\sigma_i^2 V_{zz}(z,\tau) - V_z(z,\tau)\mu + V_\tau(z,\tau) - \rho V(z,\tau) + z^2 + \sigma_a^2 \tau = 0. \quad (10)$$

Partial Information - 3



Partial Information - 4



Lessons

- Even when there is a continuous flow of relevant information, there should never be adjustments before an important information announcement.

- When partial information is freely observed, one should use it in interim adjustments between information dates. If the free observation is the price index, one can rationalize interim adjustments by cumulative past inflation.

- When half of the source of variation in p_t^* is continuously observed, the firm is willing to wait much longer to collect the remaining part of the information.

Illustration

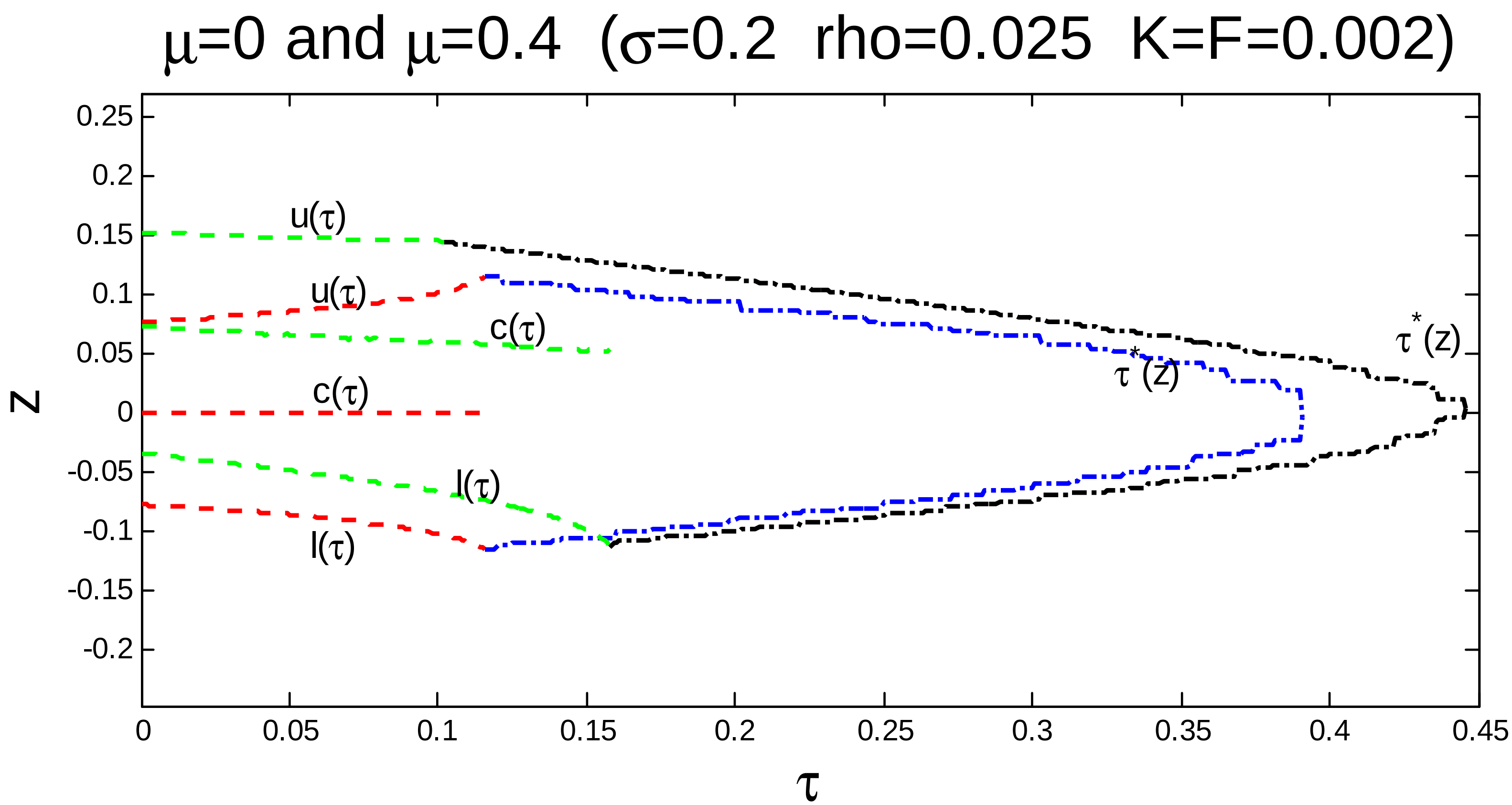


Illustration - 2

$K = F = 0.002$ and $\rho = 0.025$

μ	σ	Total fr. of info	Fr.of info without adj.	Fr.of info with adj.	Fr.of uninf. adj.	Total fr. of adj.
0	0.1	1.32	0.60	0.72	0	0.72
0	0.2	2.69	1.2312	1.46	0	1.46
0.4	0.1	1.41	0.3355	1.08	1.49	2.56
0.4	0.2	3.11	1.1013	2.00	0.08	2.08

Illustration - 3

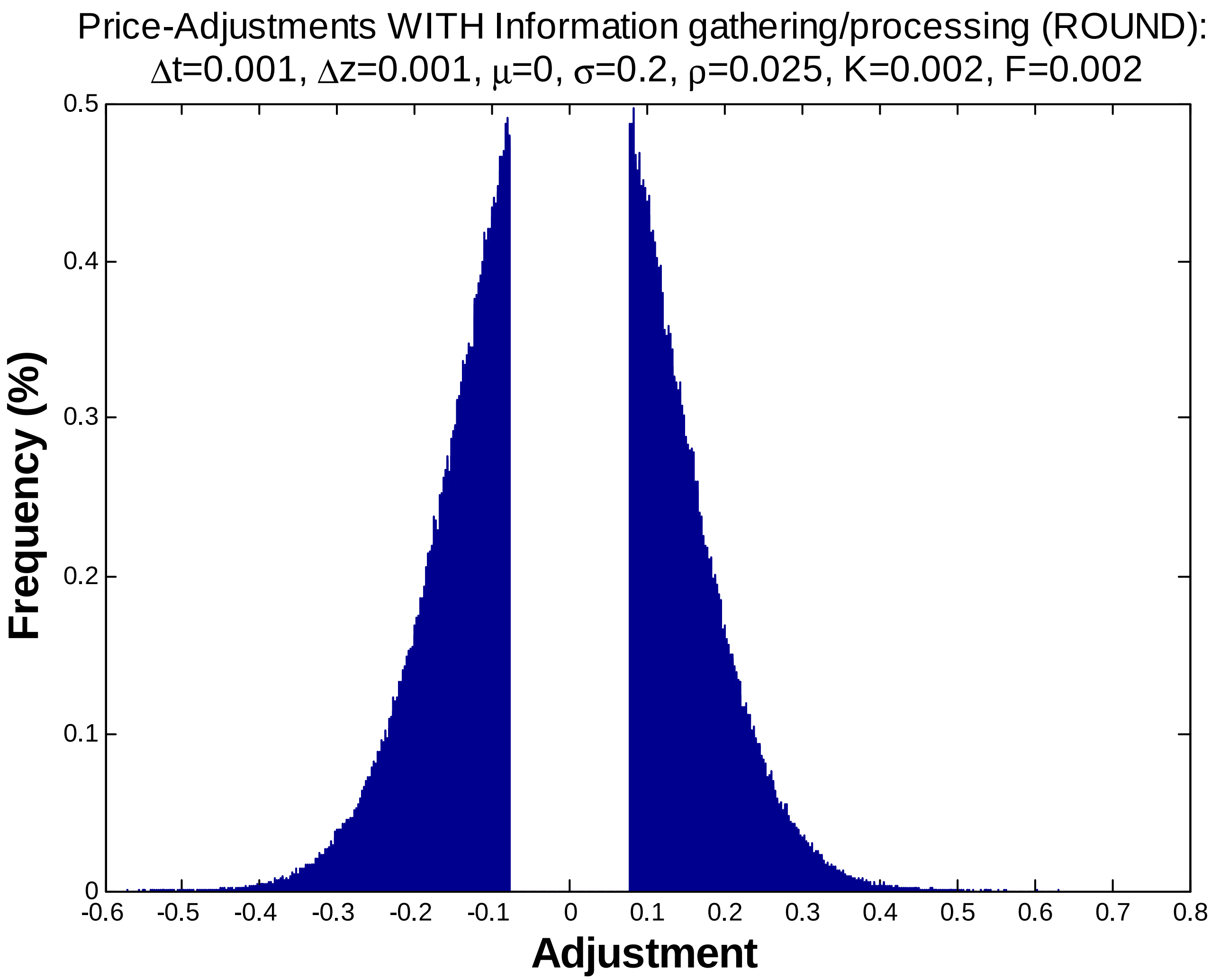


Illustration - 4

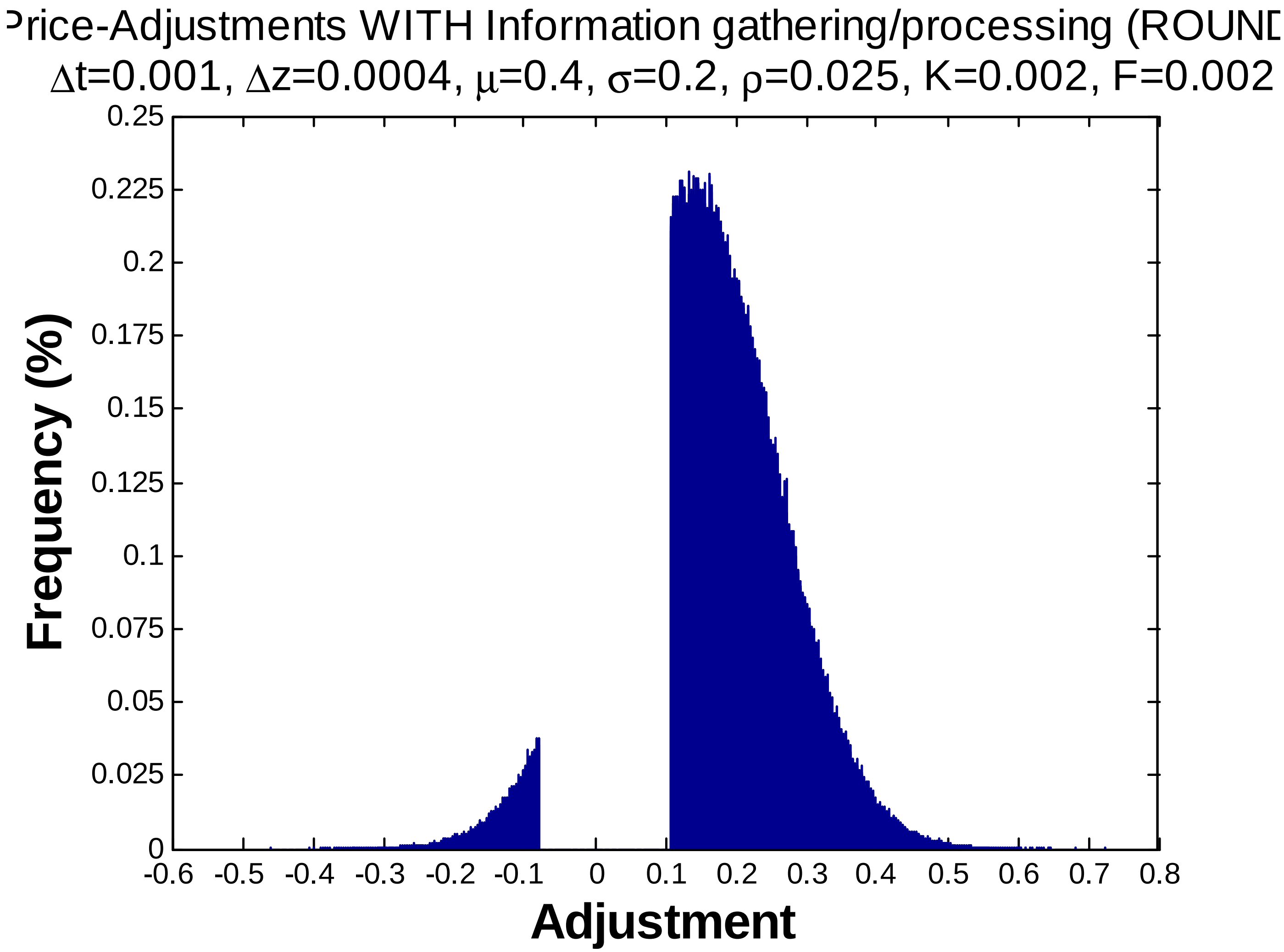


Illustration - 5

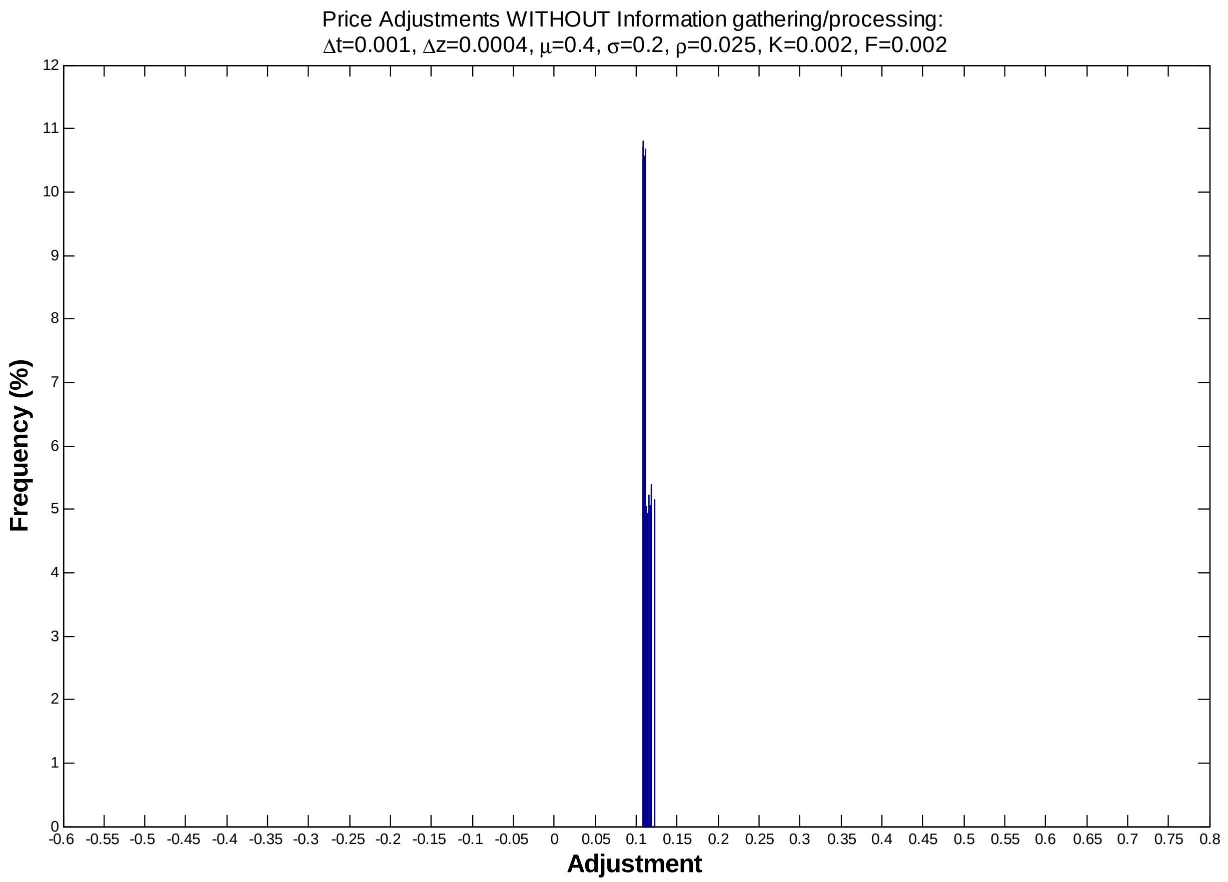
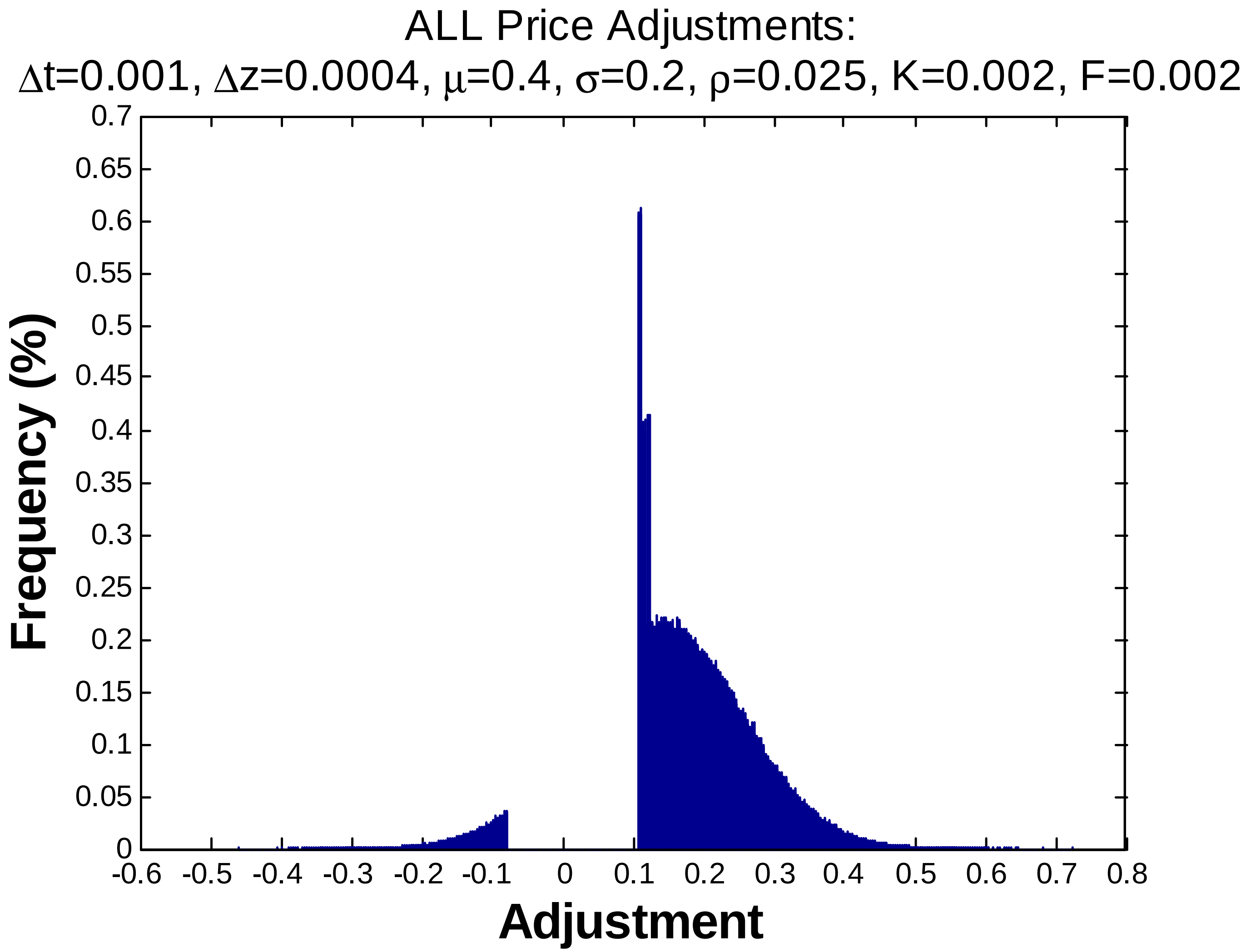


Illustration - 6



Lessons

- The option to make uninformed adjustments and to collect information without adjusting introduce richer responses to change in the environment.

- Frequency of adjustments increase with higher uncertainty and larger inflation, but

  - larger inflation increases the frequency of uninformed adjustments and reduce the frequency of information collection without adjustments

  - larger uncertainty increases the fraction of information collection without adjustments and reduces the fraction of uninformed adjustments

- Distribution of price changes when price adjustments and information are costly are in general bimodal, having an empty region close to zero, but a large inflation may alter the distribution substantially:

  - the presence of uninformed adjustments may introduce spikes concentrated in the lower part of the non-empty region

Summing up

- Unified framework for optimal price setting under infrequent information
- Time dependency arises when unobserved information builds up over time
- Uninformed price adjustments might be optimal
- Never optimal to adjust just prior to an information date
- Different implications for frequency of informed and uniformed adjustments and distribution of the magnitude of price changes.
- Applications