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*ON MEASURING SOCIAL TENSIONS: with Applications to Brazil[1]

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ABSTRACT

There are a number of different types of social tensions that can generate social unrest. Starting from standard inequality and poverty concerns to the ones related to temporal fluctuations in living standards including both systemic and idiosyncratic sources of risk. These social tensions may also include social groups immobility, polarization and middle class related considerations. This paper provides a common methodology to model different sources of social tensions and applies it to the recent Brazilian experience.

In all cases, we start from explicit normative values to each particular social tension, paying special attention to specifying their respective social welfare functions as in Atkinson (1970). This approach allows rigorous treatment to the consequences of normative values initially assumed in different dimensions. For example, we demonstrate for an entire class of social tensions associated with poverty that they will always decrease when there is a positive inequality neutral growth rate. Thus inequality neutral growth reduces poverty tensions but by definition has no impact on inequality tension.

While a social welfare function based approach is well established in the inequality and in the poverty literatures, other elements are not equally developed. In particular, a social welfare function based approach has not been attempted to polarization and associated middle class issues. This is a particular innovation proposed in this paper. We also address social tensions related to macroeconomic volatility and relative social immobility.

The overall contribution of the paper is to model and measure, side by side, different sources of social tension and to discuss their interrelations found in practice. We calculate various social tensions levels and trends in Brazil in the recent period using microdata from national household surveys. The recent Brazilian experience turns out to be an interesting case of study because it combines improvements in stability, equity and growth of household incomes whose influences on society climate should be sorted out. First across the last two decades there was increasing stability for individual and aggregate income fluctuations. On top of that, there was in the last decade the resumption of growth, especially in household incomes with increasing income equality both between individuals and socio-economic groups. As a result the relationship between these primitive causes behind social welfare improvement and other by products such as poverty emancipation and the rise of an emerging middle class can be better understood.

[1] A preliminary version of this paper was presented in the 43rd Meeting of the Brazilian Economic Association held in Florianopolis, Brazil during 2015. See https://www.anpec.org.br/encontro/2015/submissao/files_I/i12-f128f309820b08ed93dac61a9ab3bc01.pdf

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6. Middle Class; 7. Volatility; 8. Social Mobility

Resumo

Existem diferentes tipos de tensões sociais que podem gerar manifestações populares. Seja por preocupações acerca do nível de desigualdade e pobreza, como também por flutuações temporais no padrão de qualidade de vida através de fontes de riscos sistêmicos ou idiossincráticos. Estas tensões sociais também podem ser geradas por conta da imobilidade observada entre os grupos sociais, polarização e questões relacionadas às chamadas classes médias. Este artigo utiliza uma metodologia comum para modelar diferentes fontes de tensões sociais, aplicando-a sobre a experiência recente brasileira.

Em todos os casos, nós começamos apresentando valores normativos explícitos associados a cada tensão social, dando ênfase para a especificação das respectivas funções de bem-estar social como em Atkinson (1970). Essa abordagem permite um tratamento rigoroso para as consequências dos valores normativos assumidos inicialmente nas diferentes dimensões. Por exemplo, nós demonstramos para uma classe inteira de tensões sociais associadas à pobreza que elas irão sempre diminuir caso haja uma taxa de crescimento positivo com desigualdade em situação neutra. Assim, o crescimento com desigualdade neutra reduz as tensões provenientes da pobreza, mas por definição não tem qualquer impacto sobre a tensão proveniente da desigualdade.

Enquanto a abordagem baseada na função de bem-estar social está bem estabelecida na literatura sobre desigualdade, e em menor medida em pobreza ela ainda não está igualmente desenvolvida em diversos outros casos. Em particular, uma abordagem baseada na função de bem-estar social ainda não foi implementada para analisar questões sobre polarização e também associadas as definições de classe média. Esta é uma inovação proposta neste *artigo*. Nós também endereçamos as tensões sociais relacionadas à volatilidade macroeconômica e à imobilidade social relativa.

A principal contribuição deste trabalho é modelar e mensurar, lado a lado, diferentes fontes de tensões sociais e discutir suas implicações na prática. Nós calculamos vários níveis e tendências de tensões sociais no país entre 1992 e 2014 utilizando microdados obtidos em pesquisas domiciliares nacionais. A recente experiência brasileira se torna um caso de estudo interessante porque combina melhorias em estabilidade, equidade e crescimento da renda domiciliar, cuja influência sobre a tensão na sociedade precisava ser quantificada. Em primeiro lugar, nas últimas duas décadas houve uma crescente estabilização nas flutuações da renda individual e agregada do país. Além disso, houve uma retomada do crescimento na última década, verificada especialmente nas rendas domiciliares, com um crescimento na equidade das rendas individuais e entre grupos socioeconômicos. Como consequência desse processo, podemos compreender melhor a relação entre essas causas primitivas das mudanças no bem-estar social e outros subprodutos, como a emancipação da pobreza e a emergência de uma nova classe média.

1. Introduction

This paper has two objectives. First it provides a common methodology to measure different social tensions that can exist in a society due to many reasons. Secondly it analyzes sources of social tension and their trends in Brazil from 1992-2012. The following dimensions of social tensions have been considered in our theoretical and empirical analysis:

1. High inequality
2. Existence of poverty
3. Diminishing middle class and increased polarization
4. Growth volatility
5. Social immobility

The measurement of each social concern requires specific normative judgments, which are made explicit in a social welfare function. Although the paper provides a general framework of social welfare function, since the main objective of this paper is to measure social tensions in Brazil, the social welfare functions considered are those which can be estimated from Brazilian household surveys.

Many social welfare functions have been proposed in the literature. The main purpose of social welfare functions is to evaluate economic allocations to know what policies are working and what are not working. The different policies affect individuals differently; from any public policy perspective some individuals will lose and others will gain. So in any evaluation, the normative judgment cannot be avoided. The social welfare functions explicitly specify the normative judgment about the weights given to different individuals.

The main object of this paper is to apply social welfare functions to measure different social tensions which can arise due to many reasons. These tensions are social concerns, which can become a source of social upheaval. This paper attempts to embody these various social concerns in a social welfare function framework.

The paper is organized as following. Section 2 provides a general description of the social welfare function approach that will be applied to each dimension addressed here. From section 3 to section 7 we study each source of social tension one at the time: inequality, poverty, polarization, aggregate growth volatility and social mobility. Section 8 gauges each of these social tensions for Brazil covering the period from 1992 to 2012 using the nationwide survey called in short PNAD[2]. Per capita real household income is used as individuals' welfare measure. The last section provides the main conclusions of the paper.

[2] PNAD stands for Pesquisa Nacional por Amostras de Domicílios which is an annual household survey and has been conducted by the Instituto Brasileiro de Geografia e Estatística - IBGE. During the 20 year period covered here PNAD questionnaire did not change.

2. A general description of social welfare function

Social welfare depends on both mean income and its distribution. The mean income is generated in the society through the output that is produced in the economy. This output is generally measured by gross domestic product. It is the mal-distribution of income that provides the measures of social tension. There may be trade-off between mean income and social tension, which in economic literature is described as an equity/efficiency trade-off.

Social tension in any dimension is measured by the proportional loss of social welfare caused due to the existence of social tension. The level of social welfare can be evaluated by

$$W = \mu(1 - T) \qquad (1)$$

where μ is the mean income of the society and T is a measure of social tension. A growth process may be described as inclusive if the percentage increase in mean income exceeds the percentage fall in (1-T) or in other words if net social welfare increases. Inclusive growth is viewed as a multidimensional concept because it takes into account different dimensions of social tension.

No attempt has been made in the paper to combine different dimensions of social tension into an overall measure of social tension. Since different dimensions are based on different normative judgments it makes little sense to combine them into a single index. Each dimension is analyzed individually in order to identify which source of social tension has an increasing or decreasing trend over time. An increasing social tension can become a source of much social unrest so it is imperative to measure trends in individual social tensions.

The trends in social tension can be analyzed by the following decomposition derived from (1):

$$\Delta Ln(W) = \Delta Ln(\mu) + \Delta Ln(1 - T) \qquad (2)$$

which shows that growth rate of social welfare is sum of the growth rates of mean income and (1-T). The growth rate of (1-T) denoted by $\tau = \Delta Ln(1 - T)$ informs whether social tension is increasing or decreasing overt time. The negative value of τ implies increasing social tension while the positive value implies decreasing social tension. Or in other words the gain in growth rate of social welfare signifies decreasing social tension and loss in growth rate signifies increasing social tension. If, hypothetically social tension T were to decrease from 60 percent to 50 percent, a gain in growth rate of 22.3 percent will be realized.

A social welfare function is like a rule that provides a way to aggregate different utilities enjoyed by individuals in the society. Suppose there are n individuals in the society who have income distribution denoted by

$$\tilde{x} \approx [x_1, x_2, - - - - - - x_n]$$

then one can construct utility functions, $u_i(\tilde{x})$ that summarizes the individuals' value judgments: A person i prefers $\tilde{x}$ to $\tilde{y}$ if and only if $u_i(\tilde{x}) > u_i(\tilde{y})$. The social welfare function is then defined as a function of individual utilities:

$$W(\tilde{x}) = W[u_i(\tilde{x}), u_2(\tilde{x}), - - - - - -, u_n(\tilde{x})]$$

One of the reasonable restriction on this general social welfare function is that it is an increasing function in each individuals' utility, that way it is ensured that if everyone prefers $\tilde{x}$ to $\tilde{y}$, the society will also prefer $\tilde{x}$ to $\tilde{y}$.

One of the most used approach of aggregating the individual utilities is the utilitarian approach, which defines social welfare as the sum of individual utilities:

$W(\tilde{x}) = \sum_{i=1}^{n} u_i(\tilde{x})$

This is referred to as Benthamite welfare function. A slight generalization of this function is the weighted utility function:

$W(\tilde{x}) = \sum_{i=1}^{n} a_i u_i(\tilde{x})$

where the weights attached to the individual utility adds to 1, i.e. $\sum_{i=1}^{n} a_i = 1$. These weights inform how much importance is attached to different individuals' utility in the social welfare function.

In the social welfare function defined above, the individual preferences are defined over the entire distribution $\tilde{x}$ rather than over each individual's bundle of consumption or income. One possible judgment is that individuals care only about their own consumption and are not concerned about what others are consuming. In this case utility of individual i is denoted by $u_i(x_i)$, then the social welfare will have the form:

$W = W[u_1(x_1), u_2(x_2), - - - - - - - -, u_n(x_n)]$

This social welfare function is a function of the individuals' utility and is called individualistic social welfare function. The function is popularly known as Bergson-Samuelson's welfare function.

This section has discussed general forms of social welfare functions. The next section discusses a specific relationship between income inequality measured by the Gini index and social welfare function. Per capita real income is defined as per capita nominal income adjusted for prices.

3. Gini social welfare function

The Gini social welfare function proposed by Sen (1974) can be derived using the idea that individuals suffer loss of welfare when they find that their income is lower than others. An individual with income *x* compares his income with all other individuals in the society. He selects other individuals one by one and makes all possible comparisons. Suppose he selects an individual with income y and feels deprived upon discovering his income x is lower than income y and therefore suffers loss of welfare. He suffers no loss of welfare if higher the compared income. His welfare is then given by Kakwani (1986):

$u(x, y) = x \; if \; x \geq y$

$= x - (y - x) \; if \; x < y$

Suppose the income *x* of an individual is a continuous random variable with mean μ and probability density function *f(x)*, then in all pair wise comparisons his expected welfare is obtained denoted by u(x)

$$u(x) = x - \mu[1 - F_1(x)] + x[1 - F(x)] \qquad (3)$$

where *F(x)* is the distribution function which is interpreted as the proportion of population with income less than *x* and $F_1(x)$ defined as

$$F_1(x) = \frac{1}{\mu}\int_0^{\infty} Xf(X)dX$$

is the proportion of income enjoyed by individuals with income less than or equal to *x*.

Differentiating (3) twice gives

$$u'(x) = 2 - F(x)$$

$$u''(x) = -f(x)$$

which implies that the individual (expected) welfare is an increasing function of income and is concave. This is the basic requirement of any utility function.

The average welfare of the society is then obtained from (3) as

$$W_G = \int_0^{\infty} u(x)f(x)dx = 2\int_0^{\infty} x[1 - F(x)]f(x)dx = \mu(1 - G) \qquad (4)$$

where G is the Gini index and $\mu(1 - G)$ is the Gini social welfare function proposed by Sen in 1974.

It is noted from (4) that the Gini social welfare function is the weighted average of incomes of all individuals with weight *v(x)* given by

$$v(x) = 2[1 - F(x)]$$

where the total weight adds to 1. It should be understood that the weight *v(x)* has been defined as a function of the whole distribution $\tilde{x}$ and not of x_i alone, which implies a more general social welfare function than the one that is additive separable. An additive social welfare function is obtained by adding up the individual welfare components which are independent of welfare (income) of others. This social welfare function captures the relative deprivation by making the weights *v(x)* depend on the ranking of all individuals. The lower a person is in rank, the greater the sense of deprivation with respect to others in the society. The deprivation suffered by an individual with income *x* is proportional to the number of persons who are richer than him.

The average deprivation suffered by the society is the Gini index which is the proportional loss of social welfare. Thus the Gini index is a measure of social tension caused due to the existence of inequality in the society. Atkinson's (1970) class of inequalities can also provide alternative measures of tensions due to inequality in the society. But in the analysis of social tension in Brazil, only the Gini social welfare function has been utilized.

4. Social tension due to existence of poverty

Poverty is viewed as income (or consumption) deprivation. A poverty situation emerges when some sections of a society have so little that they cannot meet their minimum basic needs as defined by the poverty line. The phenomenon of poverty has become a major source of social tension. Some economists have even suggested that poverty can trigger social unrest and ultimately the kind of sustained violence that reduces growth (Lustig, Arias and Rigolini, 2002).

As discussed in the previous section, inequality creates social tension which can be measured by the loss of social welfare. The relationship between inequality and social welfare has been widely discussed in the literature. Every inequality measure has an implicit social welfare function. But no such relationship has been established between various poverty measures and social welfare functions. This section derives social welfare functions corresponding to the class of Foster, Greer and Thorbecke (1984) measures which will be referred to as FGT measures.

It is assumed that the poor individuals suffer deprivation when they find that their income is less than the poverty line. The poverty line specifies the society's minimum standard of living. An individual suffers deprivation which results in loss of welfare when his income is less than the poverty line. The welfare of an individual with income *x* is given by

$$w(x) = x - g(z,x) \; if \; x < z \qquad (5)$$

$$= x \qquad if \; x \geq z$$

where *z* is the poverty line and

$$g(z,x) > 0 \;\; if \; x < z$$

$$= 0 \;\; if \; x \geq z$$

is the deprivation function suffered by the poor. The non-poor do not suffer any deprivation. Given (5), the average welfare of the society is given by

$$W = \mu - \int_0^z g(z,x) f(x) dx \qquad (6)$$

where μ is the mean income of the society. The second term in the right hand side (6) is the social deprivation due to the existence of poverty. The social deprivation needs to be measured in money metric so that the society knows how much average income is lost due to the existence of poverty or in other words social deprivation should be invariant to a positive transformation of the deprivation function. Clark, Hemming and Ulph (1981) introduced the idea of "equally distributed equivalent poverty gap" which denoting by $\bar{g}$ is specified as

$$\bar{g}^{\alpha} = \frac{1}{H} \int_0^z (z - x)^{\alpha} f(x) dx \qquad (7)$$

where *H* is the headcount measure of poverty. The social deprivation from (7) is then given

$$\int_0^z g(z,x)f(x)dx = H\bar{g} \qquad (8)$$

Substituting (7) and (8) into (6) gives the social welfare (average welfare of the society) as

$$W_\alpha = \mu - Hz(\frac{\theta_\alpha}{H})^{\frac{1}{\alpha}} \qquad (9)$$

where θ_α is the FGT class of poverty measures defined as

$$\theta_\alpha = \int_0^z (\frac{z-x}{z})^\alpha f(x)dx$$

where

θ_α = H, the headcount ratio when $\alpha = 0$,

= the poverty gap ratio when $\alpha = 1$,

= the severity of poverty ratio when $\alpha = 2$

The headcount ratio as Sen (1976) calls it is a crude measure of poverty because it is completely insensitive to depth and severity of poverty. The poverty gap ratio on the other hand takes into account the depth of poverty which is measured by the average income gap of the poor from the poverty line and the severity of poverty in addition takes account of inequality of income among the poor. Note that W_α is not defined for the headcount ratio when $\alpha = 0$.

W_α in (9) shows that like any inequality measure there is an implicit social welfare function for each member of the FGT class of poverty measures (except for the headcount ratio). The proportional loss of social welfare due to the existence of poverty is obtained from (8) as

$$P_\alpha = \frac{zH^{\frac{(\alpha-1)}{\alpha}}(\theta_\alpha)^{\frac{1}{\alpha}}}{\mu} \qquad (10)$$

which is the proposed measure of social tension caused due to the existence of poverty. The measure of social tension for poverty gap ratio is obtained by substituting $\alpha = 1$ in (10) as

$$P_1 = \frac{z\theta_1}{\mu} \qquad (11)$$

where θ_1 is the poverty gap ratio. Differentiating (11) with respect to μ while keeping inequality (measured by the Lorenz curve) constant gives

$$\frac{\partial P_1}{\partial \mu} = -Hz < 0$$

which demonstrates that social tension implied by poverty gap ratio always decreases when there is inequality neutral economic growth. Similarly social tension measure for severity of poverty is obtained by substituting $\alpha = 2$ in (10) as

$$P_2 = \frac{z\sqrt{H\theta_2}}{\mu} \qquad (12)$$

where θ_2 is the severity of poverty ratio. Differentiating (12) with respect to μ while keeping inequality constant gives:

$$\frac{\partial P_2}{\partial \mu} = -\frac{z[zf(z)\theta_2 + 2H(\theta_1+\theta_2)]}{2\mu^2\sqrt{H\theta_2}} < 0$$

which demonstrates that social tension implied by severity of poverty ratio also decreases when there is inequality neutral economic growth. Similarly it can be demonstrated that the entire class of social tension measures will always decreases when there is inequality neutral growth rate. Note that inequality neutral growth does not change the relative social tension measures caused due to inequality in the society such as the Gini index. Thus inequality neutral growth reduces the poverty tension but has no impact on inequality tension.

An inequality neutral growth increases the incomes of all individuals when the relative distribution defined by the Lorenz curve does not change. When growth is not inequality neutral, then the following four scenarios are possible:

1. Inequality tension increases but poverty tension decreases.
2. Both inequality and poverty tensions increase.
3. Both inequality and poverty tensions decrease.
4. Inequality tension decreases but poverty tension increases.

Ideally the society should aim at reducing both inequality and poverty tensions but such a scenario is not very common in developing countries. Recently, as discussed below, Brazil has achieved reduction in both inequality and poverty tensions (scenario 3). But scenario 1, whereby increase in inequality tension is accompanies by reduction in poverty tension is most widely observed in the developing world particularly in the Asia-Pacific region. This phenomenon is strongly prevalent in China where inequality has been increasing but at the same time poverty has been reducing sharply. These observations have led to a belief that developing countries face a tradeoff between inequality and poverty. Based on cross-country evidence Ravallion (2005) did not find any systematic tradeoff between measures of poverty and relative inequality.

The growth process endogenously determines both inequality and poverty tensions. The two kinds of social tensions may conflict in some cases and may not in other cases. They imply different social welfare functions and therefore have different social objectives both of which have intrinsic value to the society. The estimation of the two social tensions, however, is important to judge alternative policies.

5. Social tension and polarization

Recently, there has been a growing literature on the role of middle class in economic development. An emerging consensus among economists is that an increase in the size of the middle class leads to rise in per capita income and that increase in the middle income shares causes a rise in the growth rate (Easterly, 2001).

In addition, a greater income share of the middle class leads to better health and education outcomes. Birdsall (2007) even defined inclusive growth as growth which builds middle class. According to her, a small and weak middle class implies weak state institutions and hence unsustainable growth. Berkowitz and Jackson (2005) pointed out that a powerful middle class is conducive to lower inequality.

The phenomenon of "disappearing middle class" has become a concern among many economists (Wolfson 1994). The disappearing of middle class has occurred because the society is becoming more polarized. Foster and Wolfson (1992) introduced the idea of bi-polarization that is directly linked to the disappearing of the middle class.

A society is said to be polarized when it is divided into groups, with substantial intra group homogeneity and intergroup heterogeneity. Based on this definition, Esteban and Ray (1994) identified two distinct notions of polarization. The first is the alienation, which measures how far apart are different groups and the second is the identification, which measures how closely the members of a group are aligned with each other sharing common aspirations and values. The existence of such groups has potential for social conflicts.

Suppose a society is divided into three groups: the poor, middle class and the rich. Then the shrinking of the middle class and an increasing gap between the poor and the rich implies increasing polarization in the society. A polarized society has small middle class and sizable poor and rich classes with large income gap between them. This is basically the idea of bi-polarization as articulated by Foster and Wolfson (1992 and 2009).

The concept of polarization is directly linked to social tension. As has been argued in this paper, social tension can be measured by the loss of social welfare if one can find a social welfare function that embodies the essential elements of social tension caused due to polarization. This section derives such a social welfare function. A link between social welfare and polarization has not been discussed in the literature.

The notion of alienation is measured by the degree of spread from the middle position (median) to the tails of the income distribution. The larger spread from the median implies smaller middle class and larger polarization, which is the case where the rich become richer and the poor become poorer. This causes social tension. The social welfare function that incorporates the idea of alienation is derived as following.

A person is assumed to be alienated if his or her income spreads from the middle. Suppose m is the median income, then her alienation is given by the difference of the individual income from the median. The utility that takes account of the alienation from the median may be defined as:

$$u(x) = x - (m - x) \text{ if } x << m \qquad (13)$$

$$= x - (x - m) \text{ if } x >> m$$

So the average welfare of the society from (13) is obtained as

$$W_A = \mu - \frac{(m_2 - m_1)}{2} \qquad (14)$$

where m_1 and m_2 are the mean incomes of the population having income below and above the median income, respectively. W_A is the social welfare that takes into account alienation in the society. The proportional loss of social welfare due to alienation is given by

$$A = \frac{(m_2 - m_1)}{2\mu} \qquad (15)$$

The larger the A the greater is the alienation in the society. A is the proposed measure of social alienation.

The second aspect of bi-polarization refers to the case where incomes below the median or above the median become closer to each other. Nissanov, Poggi and Silber (2011) called this situation as a "bunching of the two groups in the sense that the gaps between the income below the median (or above the median) have been reduced". The polarization increases when the two groups become homogeneous.

The social welfare function in (14) gives equal weights to the income gaps from the median, which is the reason that it is completely insensitive to any transfer of income on either side of the median. To make it sensitive to such transfers, different weights need to be given to different income gaps. How should then weights be determined?

Suppose *v(x)* is the weight give to *x.* Since the society is most concerned with the welfare of middle income, the weight *v(x)* should be maximum at the median when *x=m,* which tappers off to 0 at the tails of the distribution. This means that *v(x)* should be an increasing function of *x* until it reaches the maximum value at *x=m* and then it should be decreasing with *x* until becomes 0 as income reaches infinity. A simple weighting scheme proposed is as follows:

$$v(x) = 4F(x) \quad if\ x < m \qquad (16)$$

$$= 4[1 - F(x)] \quad if\ x \geq m$$

So that the sum of all weights adds to 1:

$$\int_0^\infty v(x)f(x)dx = 1$$

Using (13) and (16) the average welfare of the society is then obtained as

$$W_B = \int_0^\infty u(x)v(x)f(x)dx = \mu - (m_2 - m_1) + 2\mu G \qquad (17)$$

which is the social welfare that takes account of polarization in the society. The proportional loss of social welfare due to polarization is given by

$$B = 2(A - G) \qquad (18)$$

where A is the measure of social alienation derived in (15) and G is the Gini index. B is a measure of social tension caused due to polarization in the society. This measure can also be written as

$$B = 2(G_B - G_W)$$

where G_B and G_W are the between and within group inequalities when the two groups are formed of the populations having income less and greater than the median income, respectively. The polarization measure B in (18) is similar to the measure proposed by Foster and Wolfson (1992). This is a measure of social tension due to the existence of polarization in the society.

6. Growth volatility and social tension

Growth rates inform whether and to what extent people are becoming better or worse off over time. Not surprisingly, therefore, magnitudes of growth rates are always the focus of attention when economists discuss alternative strategies of economic development. In practice there is large volatility in growth rates. They can fluctuate widely from negatives to positives. This volatility can create social tension particularly when people's standard of living is falling. The fluctuating growth rates create uncertainty among economic agents about their business decisions as to when they should make investments, or when they should hire and fire workers. The policy making becomes difficult for the government under the regime of volatile growth. The poor also suffer because they feel insecure when their incomes are widely fluctuating.

The volatility in growth rates is viewed here as causing social tension. This section develops a model to measure the loss of social welfare attributed entirely due to volatility in growth rates.

Suppose μ_t is the per capita income of the society in period t and there are n time periods, then a simple inter temporal social welfare function may be defined as

$$Ln(\mu^*) = \frac{1}{n}\sum_{t=1}^{n} Ln(\mu_t) \qquad (19)$$

where μ^* is the money metric social welfare for the entire n periods. Let r_t be the growth rate of per capita income in the year between $t-1$ and t, then definition $\mu_t = \mu_{t-1}(1+r_t)$ must hold. Substituting sequentially μ_t in terms of μ_1 gives

$$\mu_t = \mu_1(1+r_2)(1+r_3)----(1+r_t)$$

which on taking on logarithm of both sides gives

$$Ln(\mu_t) = Ln(\mu_1) + \sum_{j=2}^{t} ln(1+r_j)$$

which on substituting in (19) gives

$$Ln(\mu^*) = Ln(\mu_1) + \frac{n-1}{2}\sum_{t=2}^{n} w_t Ln(1+r_t) \qquad (20)$$

where

$$w_t = \frac{2(n-t+1)}{n(n-1)}$$

Such that $\sum_{t=2}^{n} w_t = 1$, which provides the relationship between the aggregate welfare level measured by μ^* and the growth rates.[3]

It is reasonable to assume that when all growth rates are equal, there is no volatility in growth in which case social welfare must be maximum so the loss of social welfare from the maximum provides a welfare measure of growth volatility. To measure the impact of volatility on social welfare, a counter factual is that all growth rates are equal to the average growth rate given by

$$\bar{r} = \sum_{t=2}^{n} w_t r_t \qquad (21)$$

which on substituting in (20) gives a new welfare function

$$Ln(\mu_M^*) = Ln(\mu_1) + \frac{n-1}{2} Ln(1+\bar{r}) \qquad (22)$$

Because of the concavity of logarithmic function, the following relationship will always hold:

$$\sum_{t=2}^{n} w_t Ln(1+r_t) \leq Ln(\sum_{t=2}^{n} w_t(1+r_t))$$

Which on using (21) immediately gives

$$\sum_{t=2}^{n} w_t Ln(1+r_t) \leq Ln(1+\bar{r}\,)$$

This equation holds for all values of growth rates r_t. Thus on comparing (20) and (22) leads to

$$Ln(\mu_M^*) \geq Ln(\mu^*)$$

Thus μ_M^* is the maximum value of money metric social welfare when there is no growth volatility, i.e. when all growth rates are equal. The loss of social welfare due to the volatility of growth rates is given by

$$V = (\mu_M^* - \mu^*) \qquad (23)$$

which is the proposed measure of social tension caused due to growth rate volatility. It can easily be verified that $V = 0$ when all growth rates are equal.

7. Social mobility and social tension

Social mobility is the movement of individuals or groups in economic and social position. The groups may refer to social classes such as the untouchable in India or ethnic and religious groups or even groups based on income, gender, race and age. A society can be said to lack social mobility if some social groups within it are unable to improve their social and economic status and are stuck in low paid jobs even when they work hard. Social barriers generally contribute to immobility in the society.

[3] This derivation follows Kakwani (1995).

There is a strong correlation between obtaining an education and increasing one's economic mobility. Despite the increasing availability of education for all, family background continues to play a huge role in determining one's economic success.

There is now a sizable literature on the measurement of income mobility. The measurement of income mobility in this literature is viewed as an extension of measurement of inequality over time. The pioneering work in this area is that of Shorrocks (1978) who developed a mobility index that informs to what extent the relative income of individuals have remained static or changed over time. The larger is the change in relative incomes over time the greater is the income mobility.

If the ranking of individuals by their incomes change in two periods, then this situation is also characterized as achieving income mobility. On the other side, if the ranking of individuals does not change in two periods, then there is no mobility in the society. King (1983) constructed an index which measures changes in the rank orders in the income distribution over time. Fields and Ok (1996, 1999a, 1999b) and Mitra and Ok (1998) viewed mobility in terms of distance between income distributions in two time periods. The measures of mobility proposed in the literature are largely based on the distance between income distributions between different periods. These measures can only be measured from the panel data of the same households, which are seldom available.

In all these studies, mobility is measured by the degree of volatility in individuals' incomes over time; the larger the volatility, the greater the mobility, which may itself contribute to an undesirable source of anxiety in the society. It has been argued in the previous section that volatility in growth causes social tension which results in a lower level of social welfare across time. This view makes the measurement of income mobility not very useful.

In this paper, social mobility is analyzed in terms of relative movement of social groups in their economic status. If the economic status of worse off social groups is improving at a faster rate than better off social groups, then the society may be defined as socially mobile. A society lacks social mobility if the worse off social groups are never able to improve their relative economic situation, which could be due to their family circumstances or other social barriers. This lack of social mobility is viewed here as causing social tension. The following methodology is proposed to measure such social tension.

Suppose a population is divided into k mutually exclusive social groups and a_i is the population share of the i^{th} group, then $\sum_{i=1}^{k} a_i = 1$ must hold. Further, if $f_i(x)$ is the density function of the i^{th} group, then the average social welfare enjoyed by the i^{th} group will be given by

$$W_i = \int_0^{\infty} u(x) f_i(x) dx \qquad (24)$$

where $u(x)$ has been defined in (3) and $W_G = \mu(1 - G)$ where G being the Gini index is the average welfare enjoyed by the whole society. The average welfare enjoyed by any group can then be compared with the average welfare enjoyed by the whole society. This will inform which group is enjoying more (or less) welfare than the whole society.

It can be easily shown that

$$f(x) = \sum_{i=1}^{k} a_i f_i(x) \quad (25)$$

where *f(x)* is the density function of the entire population. Now substituting (25) into (3) and using (24) gives

$$W_G = \sum_{i=1}^{k} a_i W_i \quad (26)$$

which demonstrates that the social welfare enjoyed by the whole society is the weighted average of the welfare enjoyed by each social group, where the weights is the population share of the social group. $100 \times a_i W_i$ is the percentage contribution of the i[th] social group to the total social welfare of the society.

The mobility of the society is determined to what extent socially worse off groups progress in relation to the whole society. The gap in economic status of different groups can be measured by the relative mean deviation:

$$RMD = \frac{1}{2W_G} \sum_{i=1}^{k} a_i |W_i - W_G| \quad (27)$$

The RMD is equal to 0 if all groups enjoy exactly the same welfare and equal to 1 if all groups except one have welfare equal to 0 and only one group enjoys all the welfare. It is possible that the relative welfare of some groups is negative because the average income earned by the group is less than the average deprivation suffered in which case the RMD can exceed 1. The negative welfare of a group implies that the group is extremely worse off in the society.

The society is defined to be mobile if the worse off groups improve their welfare more than the better off groups or in other words gap in economic status measured by social welfare decreases over time. The degree of mobility may be measured by the rate at which the RMD is declining over time. The increase in RMD over time implies that the worse of groups are relatively becoming even more worse off. An immobile society is the one when the some groups are never able to improve their economic status relative to whole society. Such situation causes social tension.

8. Empirical analysis of social tension in Brazil

The analysis presented in this section is based on Brazilian National Household Survey called PNAD for the 1992-2012 period. The survey has extensive information on personal and occupational characteristics of households and individuals. Per capita real household income is used as individuals' welfare measure[4]. The rich information provided by the survey allows for calculating various dimensions of social tensions and their trend in the 1992-2012 period. The trends are calculated for three periods: 1992-2001, 2001-2012 and 1992-2012.

[4] Per capita real income is defined as per capita nominal income adjusted for prices. The adjustment for prices is done by the consumer price index corresponding to the PNAD survey periods.

8.1 Social tension due to inequality

The debate on inequality in Brazil mainly revolves around the Gini index, which is the most widely used inequality measure. This is the main reason that social tension due to inequality in this paper is measured based on the Gini social welfare function. As discussed the Gini index is a relative measure of social tension.

Table 1 presents the estimates of social welfare, which is the per capita real income adjusted for the social tension caused by inequality as measured by the Gini index. The table does not give the values of social welfare for 2000 and 2010 because PNAD was not conducted during these census years. Although 1994 was not a census year, still no survey results were available for this year. In the estimation of trends, however, appropriate adjustments were done for the missing values for 1994, 2000 and 2010.

Let us first look at the entire period 1992-2012. The real per capita household income has been increasing at an annual rate of 2.13% while the social welfare has been increasing at higher rate of 3.04%, which implies that there is a gain in growth rate of 0.91% per annum in social welfare. This is happening because the Gini index has been decreasing at an annual rate 0.69%. Thus a decline in the Gini index of 1% leads to gain of 1.32% in growth rate of social welfare. Thus a reduction in inequality results in a substantial gain in social welfare.

Figure 1 depicting the trend in inequality shows that sustained decline in inequality occurred between 2001 and 2012. During this period the social welfare increased at an annual rate of 5.12% while the per capita real income increased at an annual rate of 3.65%, which implies a gain in annual growth rate of 1.47% in social welfare. This has occurred because the Gini index has been decreasing at an annual rate of 1.16%. Thus there has been a sharp decline in inequality in Brazil, which has led to a sharp increase in growth rate of social welfare. After decades of increasing or stagnated high inequality in Brazil, tides are turning as inequality has begun declining sharply since 2001.

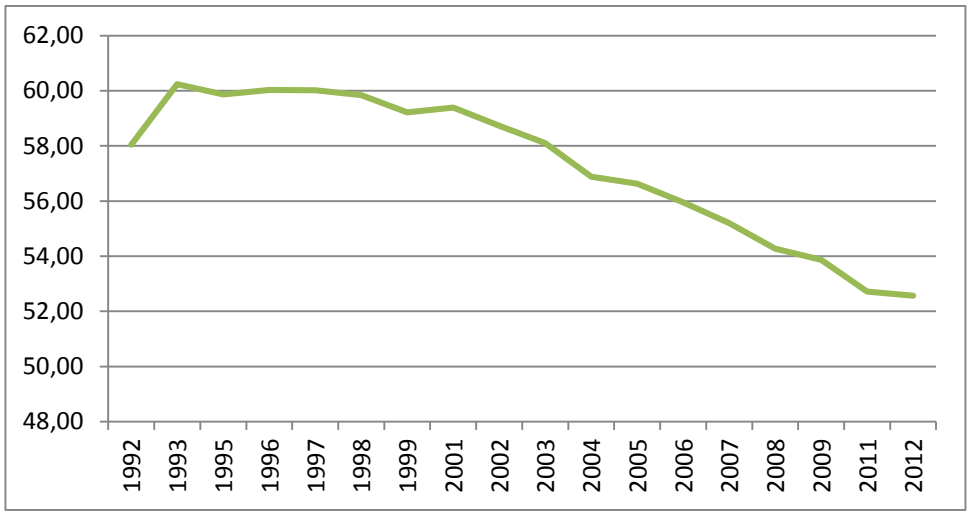
Table 1: Social tension in Brazil due to inequality

Year	Social welfare	Per capita income	Social tension
1992	199.1	474.7	58.05
1993	198.6	499.5	60.23
1995	248.5	619.2	59.86
1996	251.8	629.9	60.02
1997	251.5	628.9	60.02
1998	255.3	635.7	59.84
1999	244.6	599.6	59.21
2001	247.2	608.8	59.39
2002	251.5	609.4	58.73
2003	240.4	573.7	58.10
2004	255.6	592.8	56.89
2005	272.9	629.2	56.63
2006	303.2	688.2	55.95
2007	316.1	705.7	55.20

2008	338.1	739.4	54.27
2009	350.5	759.6	53.86
2011	381.8	807.4	52.72
2012	413.6	872.0	52.57
Growth rates: 1992-2001	2.67	2.77	0.08
Growth rates: 2001-2012	5.12	3.65	-1.16
Growth rates: 1992-2012	3.04	2.13	-0.69

Source: PNAD/IBGE. Prepared by the author.

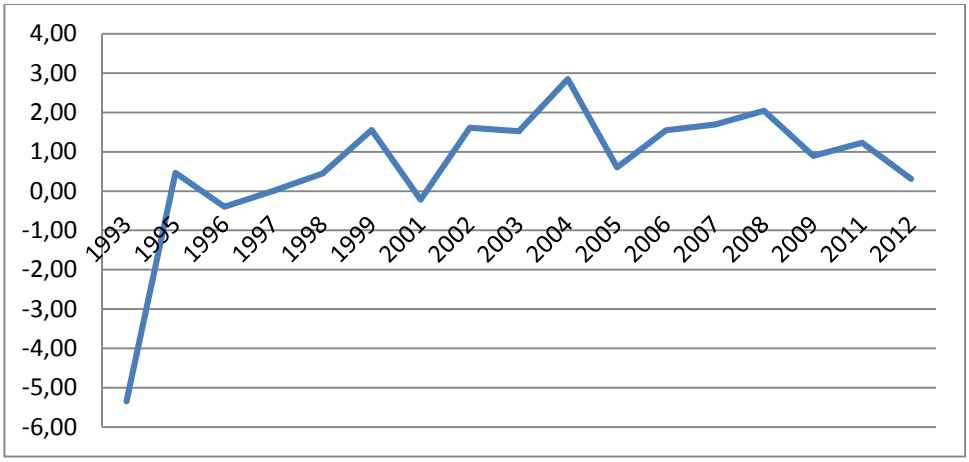
Figure 1: Social tension due to inequality: Brazil 1992-2012



Source: PNAD/IBGE. Prepared by the author.

The increase (decrease) in social tension leads to loss (gain) in growth of social welfare. Figure 2 shows that the gain in growth rate does not occur every year between 1992 and 2001 but however occurs every year between 2001 and 2012, although the gain has slowed down in the later years since 2008. The trend growth rate for the period 2001-2012 is equal to 1.47% per annum. The overall conclusion emerging is that in the new millennium growth process in Brazil has benefited the lower income groups more than the upper income groups resulting in consistently declining social tension due to inequality.

Figure 2: Gain/loss in growth rate in social welfare



Source: PNAD/IBGE. Prepared by the author.

8.2 Social tension due to existence of poverty

Poverty estimates for the headcount ratio, the poverty gap ratio and the severity of poverty are presented in Table 2. The estimates are based on two poverty lines: (1) extreme poverty with poverty line of 2005 PPP U$1.25 a day and (2) poverty with poverty line of 2005 PPP U$2.50 a day. All poverty measures show that poverty in Brazil has been decreasing sharply but the rate of decline in poverty has also increased sharply in the 2001-2012 period. Figure 3, depicting the percentage of poor and extremely poor shows that the rate of poverty decline accelerated in 2003 when Brazil introduced its flagship program call Bolsa Família.

In the period 2001-2012 the percentage of extreme poor declined at an annual rate of 8.9%, while the percentage of poor declined at a higher rate of 10.3% per annum, so the rate of poverty reduction among the extreme poor is lower than that among the poor. Since the extreme poor suffer much greater deprivation, the government policies should be more focused on reducing or even eliminating extreme poverty.

It is generally the case that poverty gap and severity of poverty decline at a faster rate than the percentage of poor. This is not happening in Brazil. What is the implication? In the period 2001-2012, the poverty gap for extreme poor has been declining at an annual rate of 6.6% which means that income gap from the poverty line for the extreme poor has been widening at an annual rate 2.3%. It can be concluded that the extremely poor who were unable to cross the poverty line have suffered falls in their income. The poverty alleviation programs in Brazil are not much helping those extremely poor who could not be lifted out of poverty.

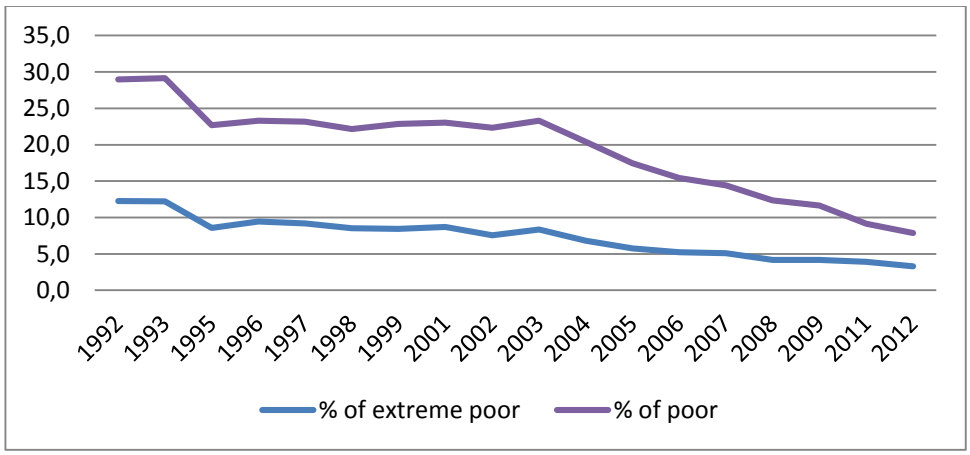
Table 2: Poverty in Brazil 1992-2012

Year	Poverty line PPP$1.25 per day			Poverty line PPP$2.50 per day		
	% of poor	Poverty gap	Severity	% of poor	Poverty gap	Severity
1992	12.3	5.6	3.6	29.0	13.2	8.2
1993	12.2	5.4	3.5	29.1	13.2	8.1
1995	8.6	3.8	2.5	22.7	9.8	5.9
1996	9.4	4.3	3.0	23.3	10.4	6.4
1997	9.2	4.1	2.8	23.2	10.2	6.2
1998	8.5	3.6	2.4	22.2	9.6	5.7
1999	8.4	3.7	2.4	22.9	9.8	5.8

2001	8.7	4.1	2.9	23.0	10.0	6.1
2002	7.6	3.3	2.2	22.3	9.0	5.3
2003	8.4	3.8	2.6	23.3	9.7	5.8
2004	6.8	3.0	2.1	20.4	8.2	4.8
2005	5.7	2.6	1.8	17.5	7.2	4.2
2006	5.2	2.2	1.5	15.4	5.9	3.5
2007	5.1	2.6	1.9	14.4	5.9	3.7
2008	4.2	2.1	1.5	12.4	5.0	3.1
2009	4.2	2.1	1.5	11.6	4.8	3.0
2011	3.9	2.2	1.7	9.1	4.3	2.9
2012	3.3	1.9	1.5	7.9	3.6	2.5
Growth rates: 1992-2001	-4.3	-4.1	-3.4	-2.9	-3.5	-3.8
Growth rates: 2001-2012	-8.9	-6.6	-5.4	-10.3	-9.5	-8.3
Growth rates: 1992-2012	-6.2	-5.1	-4.4	-5.7	-6.0	-5.7

Source: PNAD/IBGE. Prepared by the author.

Figure 3: percentage of extreme poor and poor



Source: PNAD/IBGE. Prepared by the author.

The social tension depends on what poverty measure is used. Table 3 presents the estimates of social tension for both poverty gap ratio and severity of poverty. Figure 4 shows that social tension due to extreme poverty decreased sharply between 1992 and 1995, then the rate of decline slowed down considerably until 2003 but accelerated sharply until 2012.

The social tension due to existence of poverty is much higher than that due to existence of extreme poverty. This is expected because the society is now concerned about the larger group of individuals who are suffering from poverty. Figure 5 shows that the social tension due to the existence of poverty has been declining at a faster rate.

The magnitude of social tension due to the existence of poverty is much smaller than that observed for social tension due to inequality but the rate of decline is much sharper. For the period 2001-2012 the trend rate of decline in social tension for severity of poverty is 10.79% while the decline for social tension due to

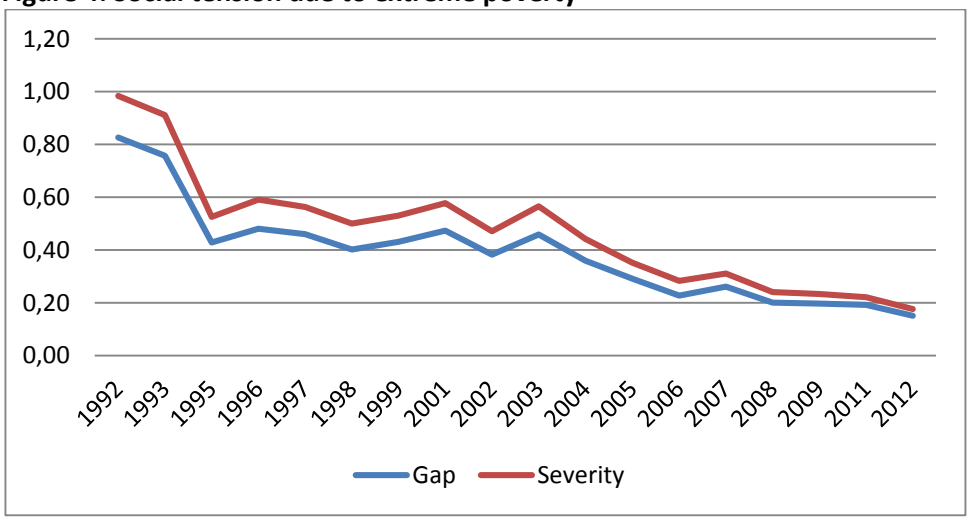
inequality is only 1.16%. A general conclusion is that the rate of decline of social tension due to poverty is much sharper than that due to inequality.

Table 3: Social tension due to Poverty in Brazil 1992-2012

Year	Poverty line $1.25 a day		Poverty line $ 2.50 a day	
	Gap	Severity	Gap	Severity
1992	0.83	0.98	1.95	2.27
1993	0.76	0.91	1.84	2.15
1995	0.43	0.53	1.11	1.31
1996	0.48	0.59	1.15	1.36
1997	0.46	0.56	1.13	1.34
1998	0.40	0.50	1.05	1.24
1999	0.43	0.53	1.14	1.35
2001	0.47	0.58	1.15	1.37
2002	0.38	0.47	1.03	1.25
2003	0.46	0.57	1.18	1.42
2004	0.36	0.44	0.97	1.17
2005	0.29	0.35	0.80	0.95
2006	0.23	0.28	0.60	0.74
2007	0.26	0.31	0.59	0.72
2008	0.20	0.24	0.47	0.58
2009	0.20	0.23	0.44	0.55
2011	0.19	0.22	0.37	0.44
2012	0.15	0.18	0.29	0.35
Growth rates: 1992-2001	-6.92	-6.61	-6.30	-6.11
Growth rates: 2001-2012	-10.21	-10.79	-13.16	-12.97
Growth rates: 1992-2012	-7.26	-7.44	-8.10	-7.82

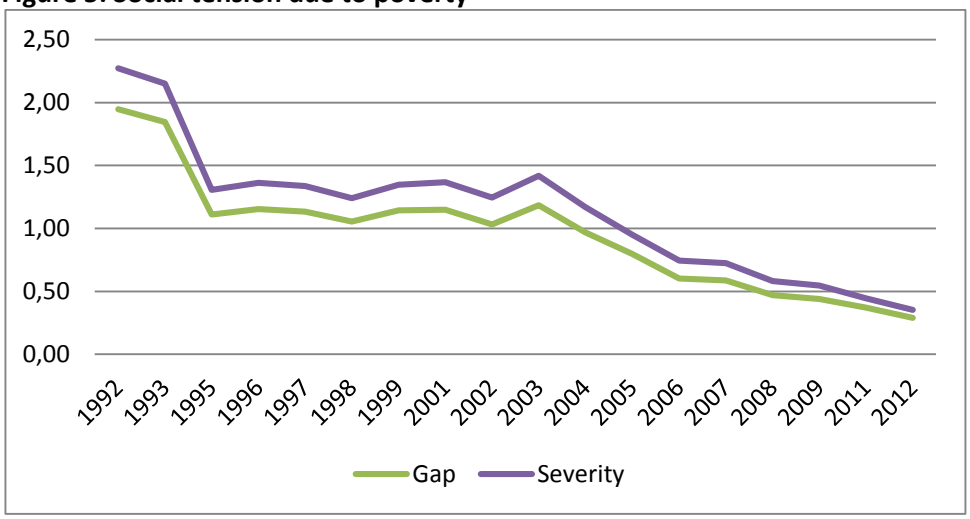
Source: PNAD/IBGE. Prepared by the author.

Figure 4: Social tension due to extreme poverty



Source: PNAD/IBGE. Prepared by the author.

Figure 5: Social tension due to poverty



Source: PNAD/IBGE. Prepared by the author.

8.3 Social tension due to alienation and polarization

As discussed social tension due to alienation and polarization are closely related to the diminishing middle class. It is assumed that society is concerned with the diminishing middle class, which occurs because the society is polarized. The alienation is concerned with the spread of distribution from the median; the larger is this spread, the smaller the middle class. The polarization in addition is concerned with the degree of homogeneity of the social groups.

Brazil has been suffering from high degrees of alienation and polarization but the empirical results presented in Table 4 show that both these sources of social tension have been increasing at annual rates of 0.06 and 0.03 percent respectively in the period 1992-2001. But in the subsequent period of 2001-2012 both social tensions due to alienation and polarization have been decreasing sharply at annual rates of 1.07 and 0.71 percent, respectively.

This study divides the Brazilian population into three mutually exclusive groups:

(i) the poor whose per capita income is less 50% of the median;
(ii) the middle class whose per capita income is above the 50% of the median and below the 150% of the median; and
(iii) the rich whose per capita income is above the 150% of the median.

In the determination of social classes, the median is used as the reference point. There is arbitrariness in the definition of middle class. In the literature there is no consensus on what range around the median should be used in defining middle class. The choice of range determines the size and share of the middle class so any number of alternatives can be justified. This arbitrariness surely weakens the entire analysis of middle class, which has become so much of focus in the economic literature.

The middle class definition used is given in (ii) and it has two components: (1) size of the middle class and (2) income share of the middle class. Figures 6 and 7 show that there is an inverse relationship between the size and income share of the middle class and alienation and polarization. In the period 1992-2001 when both alienation and polarization have been increasing, the size and income share of the middle class have been decreasing. But in the subsequent period of 2001-2012 when both alienation and polarization have been declining, the size and income share of the middle class have been increasing at annual rates of 1.40% and 3.53%, respectively. The share of middle class has been increasing at a much faster rate than the size of the middle class. This means that in relative terms, per capita income of the middle class is growing at a faster rate than the average income.

The alienation and polarization are closely related to middle class and can be measured without specifying any range. To see the closeness of this relationship, the following regressions were fitted using PNAD for the period 1992-2012:

Log (size middle class) = 9.1 - 1.25 Log (alienation) R^2=0.86
(29.6) (-10.1)

Log (share of middle class) = 16.9 - 3.2 Log (alienation) R^2=0.97
(28.5) (-23.3)

Log (size middle class) = 6.62 - 0.85 Log (polarization) R^2=0.43
(8.0) (-3.5)

Log (share of middle class) = 11.0 - 2.3 Log (polarization) R^2=0.55
(6.1) (-4.4)

These results show that the relationship between size and income share of the middle class with alienation is so strong that it may be possible to conclude that a decrease (increase) in alienation leads to an increase (decrease) in the size and income share of the middle class. Although this relationship is established in terms of the specific range of 50% to 150% of the median, simulations were performed with alternative ranges around the median. The conclusions emerging were found to be robust for alternative ranges. Thus one does not need to arbitrarily specify the range of the middle class in order to know whether middle class is increasing or decreasing. This analysis does not exactly inform how much is the size and income share of the middle class but it can tells whether the social concern due to diminishing middle class is increasing or decreasing for all possible ranges around median.

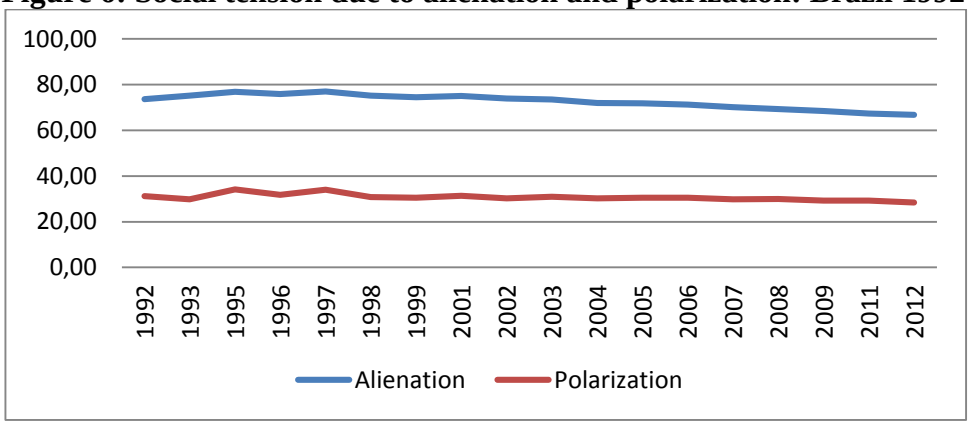
Table 4: Social tension due to alienation and polarization in Brazil 1992-2012

	Alienation	Polarization	Size of middle class	Share of middle class
1992	73.62	31.16	39.54	19.62
1993	75.16	29.85	40.25	18.60
1995	76.92	34.11	41.24	18.48
1996	75.88	31.71	37.78	17.21
1997	77.04	34.04	39.03	17.28
1998	75.20	30.73	39.54	18.01
1999	74.44	30.46	39.79	18.54
2001	75.07	31.36	39.88	18.49

2002	73.85	30.24	40.68	19.46
2003	73.56	30.92	41.13	20.08
2004	72.02	30.26	41.63	20.79
2005	71.84	30.44	42.65	21.40
2006	71.19	30.47	43.68	22.57
2007	70.10	29.80	43.14	23.08
2008	69.24	29.93	44.54	24.35
2009	68.51	29.30	44.76	24.86
2011	67.32	29.20	45.90	26.69
2012	66.78	28.43	46.67	27.16
Growth rates: 1992-2001	0.06	0.03	-0.08	-0.52
Growth rates: 2001-2012	-1.07	-0.71	1.40	3.53
Growth rates: 1992-2012	-0.65	-0.51	0.87	2.12

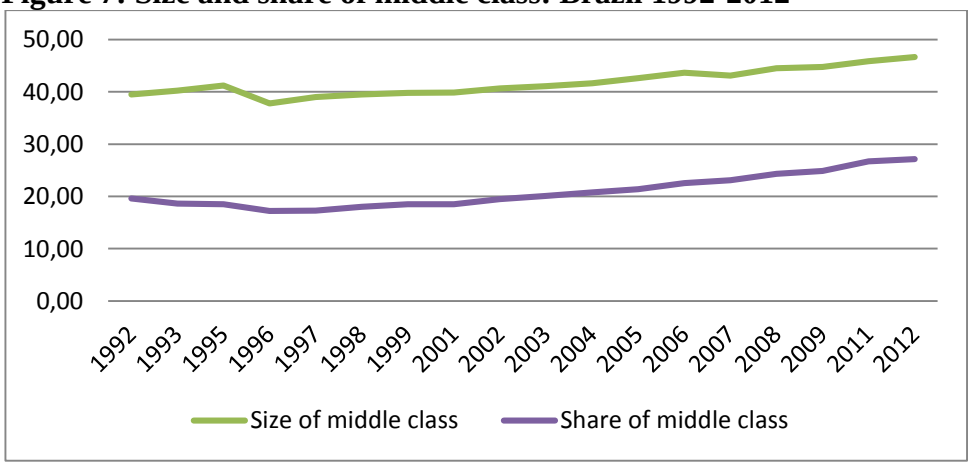
Source: PNAD/IBGE. Prepared by the author.

Figure 6: Social tension due to alienation and polarization: Brazil 1992-2012



Source: PNAD/IBGE. Prepared by the author.

Figure 7: Size and share of middle class: Brazil 1992-2012



Source: PNAD/IBGE. Prepared by the author.

8.4 Growth volatility and social tension

The social tension due to volatility of growth is measured by the loss of social welfare in a temporal social welfare function. There are year to year differences in growth rates between PNAD per capita income and per capita GDP. It is noted in Figure 8 that per capita incomes from PNAD has more volatile growth rates than per capita GDP. Table 5 presents the growth rates of per capita GDP, household income and income of the bottom 40% population. The volatility index presented in the last three rows of Table 5 is calculated for the three periods: 1992-2001, 2001-2012 and 1992-2012. The following conclusions are summarized.

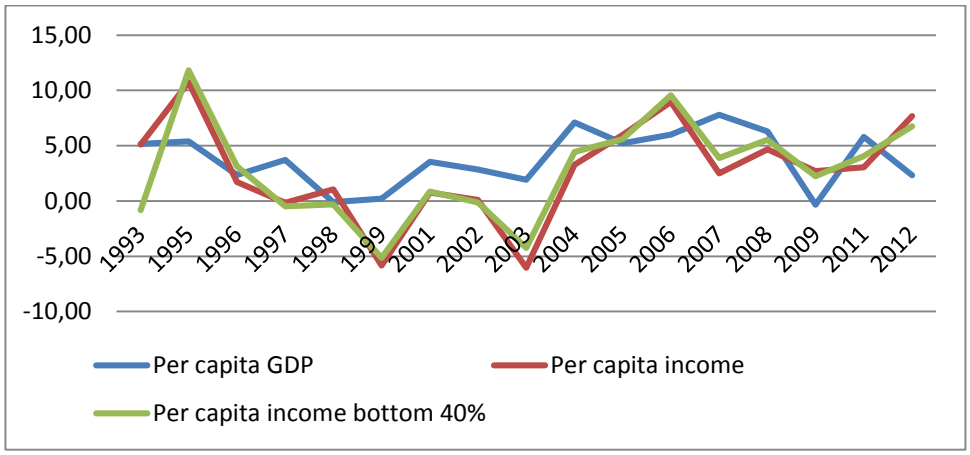
1. The per capita GDP has lower volatility than the per capita household income.
2. The bottom 40% of the population suffers higher volatility in per capita income than the whole population. This is an important observation revealing that the poor have not only lower incomes but also suffer from more volatile incomes.
3. There is greater volatility of growth rates in the period 1992-2001 than in the subsequent period 2001-2012.
4. The social welfare in the new millennium has not only improved but also has become less volatile.

Table 5: Growth rates of per capita GDP, income and income of bottom 40% population: Brazil 1993-2012

	Per capita GDP	Per capita income	Per capita income bottom 40%
1993	5.17	5.10	-0.82
1995	5.39	10.74	11.82
1996	2.35	1.71	3.14
1997	3.71	-0.16	-0.47
1998	-0.09	1.06	-0.30
1999	0.22	-5.84	-5.15
2001	3.53	0.76	0.85
2002	2.86	0.11	-0.15
2003	1.92	-6.04	-4.25
2004	7.10	3.27	4.45
2005	5.21	5.97	5.59
2006	5.99	8.97	9.56
2007	7.79	2.50	3.90
2008	6.30	4.67	5.53
2009	-0.35	2.70	2.24
2011	5.81	3.05	4.08
2012	2.34	7.69	6.75
Volatility: 1992-2001	1.82	2.96	3.32
Volatility: 2001-2012	1.02	0.56	0.73
Volatility: 1992-2012	1.21	1.12	1.28

Source: PNAD/IBGE. Prepared by the author.

Figure 8: Growth rates of per capita GDP, income and income of bottom 40% population



Source: PNAD/IBGE. Prepared by the author.

8.5 Social Immobility

In this section, social mobility is measured by the following three alternative divisions of the society:

1. Social groups by income: poor, middle class and rich
2. Social groups by age: children, adults and elderly.
3. Social groups by race: white, black/colored and others.

As discussed before a society is defined to be mobile if the worse off groups improve their welfare more than the better off groups or in other words gap in economic status measured by social welfare decreases over time. The degree of mobility is measured by the rate at which the relative mean deviation (RMD) is declining over time. The increase in RMD over time implies that the worse off groups are relatively becoming even more worse off. A perfectly immobile society is the one when the some groups are never able to improve their economic status relative to the whole society. Such a situation causes social tension.

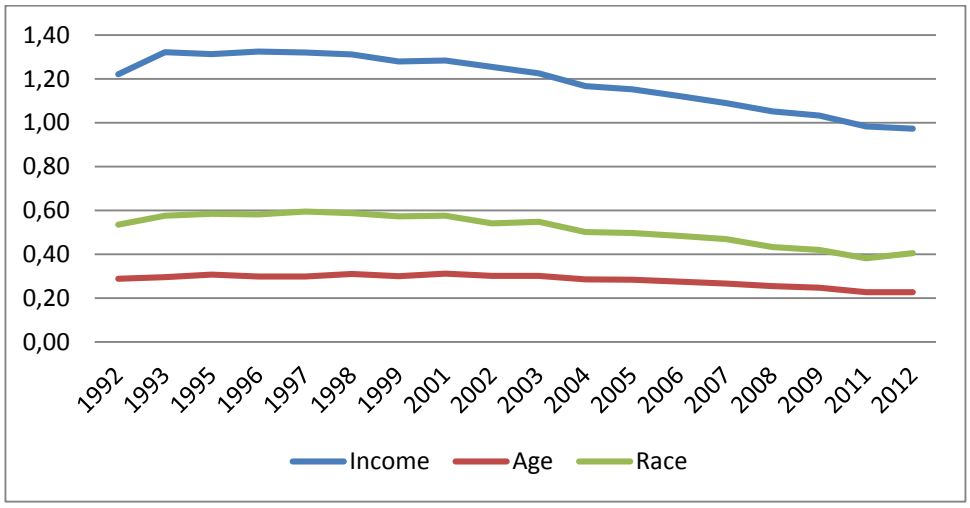
Table 6 presents the estimates of RMD for the three alternative social divisions. The RMD for the social groups divided by income (poor, middle class and rich) is greater than 1, which signifies that there is a very large gap in social welfare among the these groups. The RMD for social groups divided by age and race in turn is much less than 1 signifying much smaller gaps in their social welfare.

The growth rates of RMD presented in the last three rows in Table 6 inform that social mobility had a decreasing trend in the 1990s between 1992 and 2001 but then the tide reversed. In the new millennium period of 2001-2012, the social mobility began improving: the relatively worse off social groups improved their welfare more than the average welfare.

Table 6: Relative mean deviation by social classes: Brazil 1993-2012			
	Income	Age	Race
1992	1.22	0.29	0.53
1993	1.32	0.30	0.58
1995	1.31	0.31	0.58
1996	1.33	0.30	0.58
1997	1.32	0.30	0.59
1998	1.31	0.31	0.59
1999	1.28	0.30	0.57
2001	1.28	0.31	0.58
2002	1.25	0.30	0.54
2003	1.23	0.30	0.55
2004	1.17	0.28	0.50
2005	1.15	0.28	0.50
2006	1.12	0.27	0.48
2007	1.09	0.27	0.47
2008	1.05	0.25	0.43
2009	1.03	0.25	0.42
2011	0.98	0.23	0.38
2012	0.97	0.23	0.40
Growth rates: 1992-2001	0.19	0.68	0.52
Growth rates: 2001-2012	-2.63	-3.04	-3.61
Growth rates: 1992-2012	-1.56	-1.32	-2.05

Source: PNAD/IBGE. Prepared by the author.

Figure 9: RMD for the social groups by income, age and race in Brazil 1992-2012



Source: PNAD/IBGE. Prepared by the author.

9. Conclusions

This paper models and measures various dimensions associated with social tensions from a social welfare function framework. The advantage of this common approach pursued is to make explicit the assumptions and normative values associated with each dimension used. This allows a starting point to compare them in a more rigorous way which provides valuable insights.

Our empirical exercises applied to the Brazilian case in the 1992-2012 period allows to gauge how different social tensions evolved over time. The approach proves to be useful to organize possible relationships between these social tensions. In particular, the sharp drop of inequality observed in Brazil during 2001 to 2012 generated different implications. The main conclusions reached for this period are the following:

- **Social Welfare -** During this period the social welfare increased at an annual rate of 5.12% while the per capita real income increased at an annual rate of 3.65%, which implies a gain in annual growth rate of 1.47% in social welfare.
- **Poverty -** The magnitude of social tension due to the existence of poverty is much smaller than that observed for social tension due to inequality but the rate of decline is much sharper. For the period 2001-2012 the trend rate of decline in social tension for severity of poverty is 10.79% while that the decline for social tension due to inequality is only 1.16%. A general conclusion is that the rate of decline of social tension due to poverty is much sharper than that due to inequality, because it also incorporates positive growth trends.
- **Middle Class -** One particular contribution here is to derive from an explicit social welfare function alienation and polarization measures and to study their relationship with measures of the size of the middle class. Our results have shown that alienation movements which does not need to use specific income brackets, as do middle class definitions, are particular useful variables to predict both changes in the population size of the middle class as well as its share in total income.
  Brazil has been suffering from high degrees of alienation and polarization and both sources of social tension were roughly stable in the period 1992-2001. However, in the subsequent period of 2001-2012 both social tensions due to alienation and polarization have been decreasing sharply at annual rates of 1.07 and 0.71 percent, respectively. These are consistent with the occurrence of an emerging middle class.
- **Aggregate Risk -** The bottom 40% of the population suffers higher volatility in per capita income than the whole population. This is an important observation revealing that the poor have not only lower incomes but also suffer from more volatile income. The social welfare in the new millennium has not only improved but also has become less volatile.
- **Social Mobility -** An immobile society is one in which some groups are never able to improve their economic status relative to the whole society. Social mobility measured in this repeated cross-sectional environment is interpreted as how the relative welfare of disadvantaged groups such as children and afro-descendent move with respect to the movements in overall social welfare. After 2001, social mobility began improving: the relatively worse off social groups improved their welfare more than the society as a whole**.**

References

Atkinson, A.B. (1970) **On the Measurement of Inequality**, Journal of Economic Theory, Vol. 2, 244-263.

Berkowitz, D. and Jackson, J. E. (2005). **Entrepreneurship and the Evolution of Income Distributions in Poland and Russia**. Revised versions of a paper presented at the International Conference on Transitional Economies, Hanoi, Vietman.

Birdsall, N. (2007). **Income Distribution: Effects on Growth and Development**, CGD Working Paper 188. Washington DC: CGD.

Clark, S., Hemming, R. and Ulph, D.; (1981). **On indices for the measurement of poverty**. Economic Journal, 91, pp. 515–526.

Easterly, W. (2001). **The middle class consensus and economic development.** Journal of Economic Growth, Vol. 6, No. 4, December 2001, 317-336.

Esteban, J.; Ray, D. (1994). **On the measurement of polarization**. Econometrica, v. 62, n. 4, p. 819-851.

Fields, G.S., Ok, E.A., (1996). **The meaning and measurement of income mobility**. Journal of Economic Theory 71, 349–377.

_____. (1999a). **The measurement of income mobility: an introduction to the literature**. In: Silber, J. (Ed.), Handbook of Income Distribution Measurement. Kluwer Academic Publishing, Boston.

_____. (1999b). **Measurement of income movement**. Economica 66, 455–472.

Foster, J., Greer, J. and Thorbecke, E. (1984). **A Class of Decomposable Poverty Measures**, Econometrica, Vol. 52, No. 3, 761-66.

Foster, J. and Wolfson, M.C. (1992). **Polarization and the Decline of the Middle Class: Canada and the U.S.** Vanderbilt University and Statistics Canada, mimeo.

_____ (2009). **Polarization and the decline of the middle class: Canada and the U.S.** Journal of Economic Inequality 8(2): 133-273.

Kakwani, N. (1986), **Analyzing redistribution policies: A study using Australian data.** New York: Cambridge University Press.

_____. (1995).**Structural adjustment and performance in living standards in developing countries**. Development and Change, 26(3), July: 469__502.

King, M. 1983. **An index of inequality, with applications to horizontal equity and social mobility**. Econometrica 51, 99–115.

Lustig, N., Arias, O. and Rigolini, J. (2002). **Reducción de la pobreza y crecimiento económico. La doble causalidad**. Serie de informes técnicos del Departamento de Desarrollo Sostenible (POV-111) BID.

Mitra T. and E. A. Ok. (1998). **The Measurement of Income Mobility: A Partial Ordering Approach**. Economic Theory 12: 77-102.

Nissanov Z., Poggi A. and J. Silber (2011). **Measuring bi-polarization and polarization a survey**. in Deutsch J. and Silber J. (eds). The Measurement of Individual Well-Being and group Inequalities. Essays in memory of Z.M. Berrebi, chapter 3.

Ravallion, M. (2005). **Inequality is Bad for the Poor**, World Bank Policy Research, Working Paper 3677. Washington DC: World Bank.

Sen, A.K. (1974). **Informational Bases of Alternative Welfare Approaches: Aggregation and Income Distribution.** Journal of Public Economics, 4.

_____.(1976). **Poverty: An Ordinal Approach to Measurement**. Econometrica 45: 219-231.

Shorrocks, A.F., (1978). **The measurement of mobility**. Econometrica, Vol. 46, No. 5 (Sep., 1978), pp. 1013-1024.

Wolfson, M.C. (1994). **When Inequalities Diverge.** American Economic Review, May.