Optimal perfomance fee and flow of funds in asset management contracts*

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Abstract

This paper investigates the importance of flow of funds as an implicit incentive in the asset management industry. We build a two-period binomial moral hazard model to explain the trade-offs between flow, performance and fees where effort depends on the combination of implicit (flow of funds) and explicit (performance fee) incentives. Two cases are considered. With full commitment, the investor's relevant trade-off is to give up expected return in the second period vis-à-vis to induce effort in the first period. The more concerned the investor is with today's payoff, the more willing he will be to give up expected return in the second period by penalizing negative excess return in the first period. Without full commitment, the investor learns some symmetric and imperfect information about the ability of the manager to obtain positive excess return. In this case, observed returns reveal ability as well as effort choices. We show that powerful implicit incentives may explain the flow-performance relationship with a numerical solution. Besides, risk aversion explains the complementarity between performance fee and flow of funds.

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1 Introduction

The observation of the capital movements in the asset management industry shows that the flow of funds responds to past observed performance. This article addresses this behavior as a response to incentives while we consider it an implicit form of action induction by the investor.

We build a two-period binomial moral hazard model to describe the interaction between two relevant types of incentives presented in asset management contracts. *Explicit* and *implicit* incentives are used in this relationship as to induce the agent to exert higher levels of effort. Explicit incentives are represented by the performance fee written in a contract and, hence, enforceable by a court of law. They usually depend on the actual excess return of the performance evaluation period, affecting players' utilities in this same period.

Explicit incentives' typical clauses found in these contracts are linear in excess return with the intercept (α) and slope (β) of the contract fixed during the whole life of the contract. Most contracts for the delegation of investment decisions are: I) low-powered[1] contracts with $\alpha > 0$, $\beta > 0$ and limited liability over a high-water mark; b) fixed management fee contracts with $\alpha > 0$ and $\beta = 0$ which resembles a salary contract. The limited liability and the high-water feature turns the linear contract into a convex one, resembling the payoff structure of a call option on the asset value of the fund.

On the other hand, implicit incentives are not written in any enforceable long-term contract. They depend on the history of excess returns once this information reveals ability and/or effort exerted by the portfolio manager. This incentive only affects utility in the periods following the investor's asset allocation decisions. According to Bolton & Dewatripont (2004), the term implicit refers to informal incentives like reputation building, career concerns, other informal rewards and *quid pro quos*. Its importance is related to the fact that it complements the design and specification of long-term contracting. In our case, flow of funds is a proxy for career concerns.

Loosely speaking, we should expect the flow of fund to vary positively when the observed past excess return is positive while past negative excess return should be followed by withdrawals from the fund. Meanwhile, the flow of funds is part of an intertemporal allocation of the investor's wealth, we show that it also plays the important role of a powerful implicit component in the optimal provision of incentives under a moral hazard setting.

As to perform the objectives set above, this work consists of two models to analyze the flow-performance relationship under an incentive framework. Initially, we assume that there is no heterogeneity among managers. Then, we relax this assumption and consider two types of managers with respect to the probability of deflowering positive excess return. In this case, the investor infers about the manager's ability in a Bayesian manner.

[1] Ghatak & Pandey (2000) build a multi-task moral hazard model to explain optimal low power contracts as to recover first-best risk choices. The lower power reduces the benefits associated with the tails of the excess return distribution. The outliers' probability increases when riskier strategies are chosen by the agent.

The paper is organized as follows. In this section, we describe the some of the literature related to our work, the typical contract found in the marketplace. Section 2 describe the basic model and its numerical solution. Section 3 shows the model with heterogeneous managers, Bayesian adjustment of the posterior probability distribution of returns and its respective numerical solution. Section 4 concludes. In the last section of the Appendix, we describe the typical contract found in the marketplace and explore the possible incentive problems that arise in this contract.

1.1 The literature

The literature documents the importance of flow of funds as a response to incentives in the asset management industry. While it is not conclusive whether past performance is a good predictor of future performance since the existence of performance persistence still needs to be verified; there is sufficient empirical evidence that indicates the flow of funds are relationship with lagged measures of excess returns.

Chevalier & Ellison (1997) analyze the importance of flow of funds as an implicit incentive by considering risk-taking choices of sequential periods controlling for recent performance measures. The possibility of losing assets under management due to poor performance creates powerful incentives for high effort exertion. Effort is related to the generation of greater returns with little volatility. Given the inexorable stochastic behavior of asset prices, this objective is not always attained during the course of an investment period. The article divides this period in two sub-parts and consider the variance of the fund's quota as a proxy for investment decisions towards risk. The results show that returns' variance is greater in the following sub-period after poor performance once managers choose riskier investment alternatives to recover from relative negative results in the first period, obtain a better performance ranking in the total period among its group peers and, therefore, do not lose ground in the competition for capital. So, one of the consequences of flow of funds response to past performance is to alter investment strategies. Another result describes the shape of the flow-performance relationship as a convex function once the lowest and highest quantiles of the past performers distribution are, respectively, the ones more penalized and more granted with capital than the ones who obtain intermediate returns, i.e., closer to the benchmark's return level. Finally, younger funds show more flow sensitivity to past returns than older funds.

Berk & Green (2004) build a parsimonious multi-period model to explain the empirical evidence in the flow-performance relationship as well as the inexistence of performance persistence and the compatibility of these two stylized facts with investors rationality. They assume competitive provision of capital by investors to heterogeneous managers with respect to the ability in generating positive excess return. Besides, ability has decreasing returns to scale[2] while

[2] This means that obtainning excess return gets increasingly harder the larger the volume managed by each portfolio manager.

both managers and investors learn about it from observing past performance. Learning is based on a Bayesian inference methodology as new performance information is used to determine the posterior probability distribution of returns. Hence, efficient portfolio allocation is represented by an infinitely elastic supply of capital to managers who are expected to perform above the benchmark until, in equilibrium, all investors earn zero expected excess return. A great sort of relevance of this recent paper comes from the fact that the results derived in their model are compatible with those verified in Chevalier & Ellison (1997). Namely, the importance of fund's age, size and fees as well as the perverse incentives towards "safer" investment strategies. Assets under management growth leads to "closet indexing" strategies as to satisfy the demand for intertemporal insurance by portfolio managers. On the contrary, their model is not able to provide a convex shape of the flow-performance function.

All these results arise without any consideration about information asymmetry neither moral hazard issues. Indeed, none explicit compensation scheme or explicit performance fee is considered in their model. Our model intends to be seen as an attempt to include these elements in a rational model of asset management contracting.

The problem of efficient incentive provision with short-term versus long-term contracts in a repeated moral hazard framework is explored in Rogerson (1985A). His analysis does not make use of the first-order approach which, of course, yields results that are as general as one would like them to be. In the model, each strategy is a set of contigent actions which affects the probability distribution of a certain outcome in a context where contracts describe a set of contigent wages. These wages depend on the history of outcomes and, in equilibrium, they represent a Pareto-optimal contract when the agent is not able to use credit markets to self-insure.

One important result of paper is the Borch-like wage relationship of the optimal contract which has the property that "the inverse of the manager's marginal utility of income evaluated at any wage must be equal to the conditional expected value of the inverse of next period's marginal utility of income". Hence, marginal rates of substitution between the principal and the agent are equal in expectation across time.

Yet, he demonstrates the relevance of the shape of the inverse of the agent's HARA[3] utility function in the term-structure of the expected wage function conditional to past outcomes. Convex shapes are related to decreasing wages over time. Consequently, the unconditional expected wage in one period is greater than or equal to the next period's wage[4]. Time preference skews the wage relationship towards the beginning or towards the end of the relationship. That is, impatient agents prefer early wages which can be described by a decreasing wage function while impatient principals prefer late wage payments related to an increasing wage function.

[3] HARA utility funtions exhibits linear risk tolerance in outcome.

[4] Opposite results hold when the inverse of the agent's utility function is concave while wages are equal for the logarithmic functional form. These two results do not hold if the principal and the agent's time preferences are not equal.

In this seminal paper, Rogerson shows that memory plays a fundamental role in a Pareto optimal contract such that current outcomes affect current compensation as well as future period's ones. Memory, in our model, would be associated with the implicit component of incentives since the flow of funds depends on the history of excess returns, complementing the design and specification of long-term contracting.

Repeated moral hazard creates the opportunity for intertemporal risk-sharing. Once incentives do not allow for complete insurance across different states of nature, we can expect current investment decisions to be affected in order to allow the manager to self-insure against costly capital losses. Therefore, good past performance could be followed by "closet indexing" or benchmarking of investment decisions while poor past performance could lead to riskier asset allocation as to avoid period-end low ranking among his peers which would lead to capital withdrawal with high probability. This behavior could be seen as a possible way to access credit markets where agents try to self-insure and smooth their consumption. In this manner, "closet indexing" resembles savings in the benchmark-return linked instruments. Contrarily, riskier asset allocation can be seen as leveraged decisions against a short position in the benchmark with the objective to extract surplus from future contractual relations.

According to Rey & Salanié (1990), a sequence of two-period contracts that are negotiated each period can mimic a long-term contract. The implicit incentive is a form of periodic renegotiation as well as a severe punishment to prevent shirking behavior from the portfolio manager. That is, flow of funds can be seen as the mechanism through which long-term contracting becomes unnecessary.

On the other hand, flow of funds may cause opposite and undesired consequences if the demand for time insurance raises concerns of another form of shirking behavior: "closet indexing". Whenever this behavior undermines risky but profitable investment opportunities, we could think of it as some inadequate aspect originated by the power of the implicit incentive. Since excessive risk-averse behavior might be undesired, one should provide appropriate incentives towards riskier decisions by writing lock-up clauses in asset management contracts. This set of clauses implies in an extra cost to withdrawal resources. The investor commits to internalizes some of the costs the manager would have if he had to sell actual positions to generate cash and payback investors. For example, selling during market liquidity constrained situations given negative shocks in the economy that create asset valuation uncertainties and high levels of risk aversion.

Implicit incentives usually appear in the literature as career concerns and/or periodical bonus payments. Holmström (1982) is the first to introduce incomplete and symmetric information to model career concerns. His original approach to consider a model with incomplete but symmetric information is used in our model as well as in Berk & Green (2004). Both the principal (investor) and agents (managers) learn about the managers' type in a Bayesian manner as past performance reveals the information regarding ability to generate excess returns.

Our approach can be seen as a possible specification of these general models.

We incorporate implicit incentives and limited liability into Rogerson's framework while use of some of the elements of the models developed by Holmström (1982).

A general formulation of the problem of combined implicit and explicit incentive provision is analyzed in Pearce and Stacchetti (1998) who show that implicit incentives contracts' efficiency is increased if short-term explicit contracts are written in the context of a repeated principal-agent model. They also show that risk-averse agents prefer implicit incentives that vary negatively with explicit incentives which represents a form a self-insurance against income fluctuation. We observe these predictions in our numerical example.

Levin (2003) studies relational incentive contracts and shows the conditions under which stationary explicit contracts are optimal and how incentives interact in the trade-off between efficiency, screening and dynamic enforcement in the case of hidden information. In the moral hazard case, enforcement compresses the information obtained from the noisy signal and leads to only two levels of performance. In this case, poor performance is followed by a termination of the relationship even if the performance measure is subjective.

A relevant application to executive compensation is found in Gibbons & Murphy (1992) who study the importance of career concerns and show that the optimal contract optimizes total incentives. They show that the greater is the importance of implicit incentives, the least powerful is the explicit component of the contract in an executive compensation contract. An asset management application is found in Heinkel and Stoughton (1994). They assume the existence of a linear contract and derive the optimal contract structure and retention policy. Using a different and simplified approach, we also find that the explicit incentive is less powerful in a two-period economic setting. Any contract only elicits partial information about the portfolio manager and, hence, *ex-post* performance measurement becomes crucial in defining the optimal retention policy.

Finally, Basak, Pavlova & Shapiro (2003) show that fund flows play a very significant role in altering risk exposure given its importance as an implicit incentive. In a dynamic asset allocation framework, they show that the risk shifting strategies (unobservable, hence, non-contractible) and its related change in returns' volatility depend on the year-to-date excess return as well as the threshold return values that induce greater positive and negative fund flows. Their model demonstrates that fear of capital withdrawal leads to the departing from benchmark volatility as year-end approaches if year-to-date return is below the benchmark and to "closet indexing" if positive year-to-date performance is observed. The perverse incentive towards greater risk exposure shifts volatility in both directions in comparison with the benchmark's volatility. Indeed, risk taking might be done exclusively by taking systematic risk, that is, risk enhancing strategies may be adopted without any idiosyncratic risk shifting.

Moreover, they show the importance of writing asset allocation restrictions in the investment policy contract as to avoid moral hazard in risk choices. The "benchmark restriction" helps to diminish this concern freely, without any monitoring cost. We, on the contrary, consider the benchmark restriction as given and calculate the optimal combination of implicit (flow of funds) and explicit

incentives (management and performance fee) in a linear contract with limited liability in the performance fee - a compensation measure related to excess return over the selected benchmark.

1.2 The typical compensation - linear schedule with limited liability and high-water mark

In this section, we describe the most common asset management contracts found in the market place. This section serves the purpose to describe possible problems arising from the use of simple explicit compensation schemes.

Typical explicit clauses of **contracts** are linear with fixed coefficients during the life of the relationship and the payoff of the manager depends on the excess return of the fund, r_t. Long-term contracts are unusual. When they do exist, their maturities are defined in terms of the number of days from the withdrawal request to the delivery of the resources back to the investor. This period is rollover everyday after the lockup period. Then, we will assume that such contract do not exist in our economic environment.

Actually, given some features in the explicit compensation structure, the contract is not linear. First, it presents limited liability in the excess return - calculated as the return of the fund, R_t, in excess of the return of a predetermined benchmark, R_t^0. Second, performance fee is calculated over the high-water mark of the benchmark; that is, performance fee is only paid if the return exceeds the greater of the two benchmarks - the benchmark itself or the highest historical quota value of the fund which is also always indexed by the benchmark as well. Then, the explicit incentive is convex in excess return and the payoff can be written as

$$w_t(r_t) = \Omega_{t-1}\left(r^{t-1}\right).\{\alpha + \beta.\max\left[r_t; 0\right]\} \tag{1}$$

where $\Omega_{t-1}\left(r^{t-1}\right)$ is the total amount of assets under management and $r^{t-1} = \left(r^1, ..., r^{t-1}\right)$ represents the history of cumulative excess return of the fund over the high-water mark.

We call the total amount of asset under management, in period t, of Net Asset Value (NAV) and write it as

$$\Omega_{t-1}\left(r^{t-1}\right) = q_{t-1}\left(r^{t-1}\right).p_{t-1} \tag{2}$$

where $q_{t-1}\left(r^{t-1}\right)$ is the outstanding number of quotas of the fund and $p_{t-1}\left(r^{t-1}\right)$ is the marked-to-market quota value of the fund net of taxes and transactions costs. In period t, it is given by

$$p_t = p_0.\prod_{s=1}^{t}\left(1 + R_s\right) \tag{3}$$

The excess return of the fund in period t is given by

$$r_t = \frac{p_t}{\hat{p}_t} - 1 \tag{4}$$

where the denominator is the high-water quota price. This extra feature of the contract is given by

$$\hat{p}_t = \left(1 + R_t^0\right) . \max\left(p_{t-1}; \hat{p}_{t-1}\right) \tag{5}$$

Therefore, the high-water mark is given by

$$\widehat{r}_{t-1} = \max\left[0; \left(1 - \frac{p_{t-1}}{\hat{p}_{t-1}}\right)\right] \tag{6}$$

The manager's static payoff consists of a fixed, α - the management fee - and an option on the value of the fund due to the existence of limited liability - the performance fee, β. From a finance theory perspective, this payoff is always greater than zero and it synthesizes an European call option on the fund's quota mark-to-market price that the portfolio manager holds against the investor[5].

The high-water mark, $\widehat{r}_{t-1}$, determines the strike price of the option. Because of the high-water mark feature and the growth rate of the benchmark, this option has a variable strike price[6]. The high-water mark guarantees that the option is almost certainly out-of-the money since $\widehat{p}_{t-1} \geq p_{t-1}$. The option is, at maximum, at-the money when $\widehat{p}_{t-1} = p_{t-1}$, i.e., $\widehat{r}_{t-1} = 0$. The distance between p_{t-1} and $\widehat{p}_{t-1}$ determines how much the option is out-of-the money. So, the greater is $\widehat{p}_{t-1}$, the higher is $\widehat{r}_{t-1}$ and this implies that $r_t = \widehat{r}_{t-1} > 0$ is the minimum rate of excess return that the manager need to achieve from his investments decisions in order to start deriving any positive marginal utility from the option.

From Braido and Ferreira (2003), we learn that options may robustly induce risk-taking, regardless of the specific functional form of the utility function. Higher strike prices transform a riskier portfolio selection that is a second-order stochastically dominated cumulative distribution of excess return into a lottery that first-order stochastically dominates all other portfolio choices, even if the excess return probability joint distribution is unknown to the manager/investor. It means that the likelihood of the portfolio manager to choose riskier strategies is greater when his compensation includes an option whose strike price is high enough.

From an incentives theory approach, this out-of-the-money option[7] represents a compensation structure in which the manager derives higher marginal benefits of exerting effort and taking risks from high levels of excess return. The manager has incentives to take more risks, if $\kappa_t\left(\Omega_{t-1}\left(r^{t-1}\right)\right)$ represents a mean preserving spread of the distribution of cumulative excess return. That is, the manager has incentives to make portfolio choices whose joint prior distribution of excess returns has heavier tails.

Nevertheless, the high-water mark feature is designed to protect the investor from paying excessive performance fees. Suppose the manager performs

[5] See Goeztman, Ingersoll and Ross (2000)

[6] Even when the benchmark is zero, the high-water mark feature incorporates all the variability of the fund's history of return.

[7] And, we may say, increasingly outer-of-the-money if performance is poor or if the growth rate of the benchmark is high.

very well during a certain period of time and the value of the fund hits a record value. Now, imagine that the fund has negative performance in some subsequent periods. In this case, all positive performance that follows the poor performance period will only pay performance fee after the record high-water mark is broken again. Nevertheless, due the option-like nature of the compensation schedule, this contract feature ends up creating more incongruities in risk preferences between managers and investors. Benchmarks with high growth rates only enhance this one problem once the high-water mark will also grow at this rate.

Ghatak and Pandey (2000) build a multi-task model in which the choice of effort moves the average of the distribution of excess returns, in a first-order stochastic dominance sense, and that the risk choice is a mean preserving spread of this distribution, in a second-order stochastic dominance sense. Then, the incentive implications of risk-taking choice reduces the optimal power of the static contract, especially in the presence of limited liability. This reduction in the explicit incentive, β, objectives to diminish the marginal utility of the manager from high levels of return, inducing him to choose less risky investment alternatives. In their model, the optimal linear contract (α^*, β^*) recovers the first-best solution, that is, manager's actions are equal to the optimal combination of these weakly substitutes tasks in the case they are contractible.

We would expect that, depending on the strike price of the option, the optimal power of the contract, β, would change as the value of fund is closer or outer-of-the-money. Moreover, (α_t^*, β_t^*) also should be a function of the history of performance. Rogerson (1985), in a repeated moral hazard model, shows that memory plays a crucial role in determining future incentives if the distribution of today's return affect current incentives. However, in the asset management industry, we know that α and β are fixed at the start of the fund. This fact amplifies perverse incentives on risk choices, forcing the investor to use implicit features of the contract in order to recover an optimal compensation schedule and, hence, optimal effort and risk choices.

As a consequence, the investor has to monitor the performance of each manager and constantly revise the total amount of assets under management allocated at each portfolio manager. This is done by adjusting the flow of funds f_t. This flow is endogenous in the model and we build it as a function of the history of cumulative excess return, $f_t(r^t)$. As investors decide to let cash resources flow in, $f_t > 0$, or out, $f_t < 0$, of the fund, quotas are respectively created, $\Delta q_t > 0$, or redeemed, $\Delta q_t < 0$, at current quota marked-to-market prices, p_t.

Then, the total number of quotas in period $t-1$ is given by

$$q_{t-1}\left(r^{t-1}\right) = \sum_{s=1}^{t-1} \frac{f_s\left(r^s\right)}{p_s} \tag{7}$$

We obtain the flow of funds in each period t, $f_t(r^t)$, as a function of the cumulative return of the fund and the variation in the number of quotas

$$f_t\left(r^t\right) = p_t.\left(q_t\left(r^t\right) - q_{t-1}\left(r^{t-1}\right)\right) \tag{8}$$

Normalizing $p_0 = 1$ and after substituting (1) in (8), we obtain

$$f_t\left(r^t\right) = \prod_{s=1}^{t}\left(1+R_s\right).\left(q_t\left(r^t\right) - q_{t-1}\left(r^{t-1}\right)\right) \tag{9}$$

The fixed fee in the contract, the management fee α, is a factor expressed in annual percentage terms of the net asset value, $\Omega_{t-1}\left(r^{t-1}\right)$, being accrued in a *pro rata temporis* form. It is related to the fixed and some variable costs of managing the fund, including the marginal cost of using the manager's time[8] and/or ability. Once $f_t\left(r^t\right)$ is a function of the history of cumulative excess returns, even in the absence of any performance fee, $\beta = 0$, the manager would still have incentives to make effort and risky choices, trying to influence the perception of the market about his level of ability. Besides, $\Omega_{t-1}\left(r^{t-1}\right)$ also multiplies the option-like component of the manager's wage, affecting more intensively the effort and risk choices of the manager in each period. Then, the manager has great incentives to attract a high volume of assets under management.

Indeed, we argue that the flow of funds is the most important incentive feature of the compensation schedule. This dynamic implicit incentive depends on the history of cumulative excess returns, $f_t\left(r^t\right)$, and we call it **flow concern**.

This function determine the optimal choices of effort and risk as well as the optimal incentives, taking into consideration reputation effects that arise from the observed history of excess returns. Fama (1980) argues that this dynamic concern may recover first-best solutions removing moral hazard issues in risk-taking. Holmström (1982) demonstrated that risk-aversion and discounting play an important role in confirming Fama's previsions.

If we substitute (3) and (5) in (4), we can rewrite the excess return of the fund in period t as

$$r_t = \frac{\prod_{s=1}^{t}\left(1+R_s\right)}{\left(1+R_t^0\right).\max\left(\prod_{s=1}^{t-1}\left(1+R_s\right); \hat{p}_{t-1}\right)} - 1 \tag{10}$$

Observe that $\hat{p}_t$ is calculated recursively based on the history of cumulative excess return, r^t.

We may write the total payoff of the manager in each period t as

$$u_M\left(w_t(r_t)\right) = \Omega_{t-1}\left(r^{t-1}\right).\left(\alpha + \beta.\max\left[\frac{p_0.\prod_{s=1}^{t}\left(1+R_s\right)}{\left(1+R_t^0\right).\max\left(p_{t-1};\hat{p}_{t-1}\right)} - 1; 0\right]\right)$$

Since the main objective of this paper is to address the relative importance of implicit incentives compared to explicit incentives, we assume no high-water mark in the excess return in the model develop below.

[8] If we consider leisure in the model. However, we do not do so here

1.3 Data and stylized facts

Here, we present some of the empirical results found in Chevalier and Ellison (1997). Observe the convex shape of the flow-performance relationship.

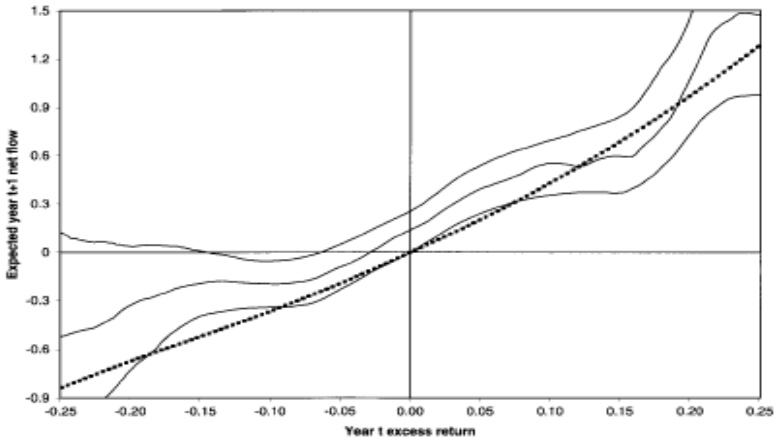


FIG. 3.—Flow of funds. The dashed line shows the flow of funds for two-year-old funds produced by the model (using the parameters reported in table 1) superimposed over the actual flow of funds plot (solid lines) for these funds as reported in Chevalier and Ellison (1997, fig. 1). Chevalier and Ellison report the estimated curve (middle line) as well as the 90 percent confidence intervals, the outer lines.

With the purpose to show the importance of age in the flow-performance relation, the graphical representation of the results found in Berk and Green (2004) is also presented here.

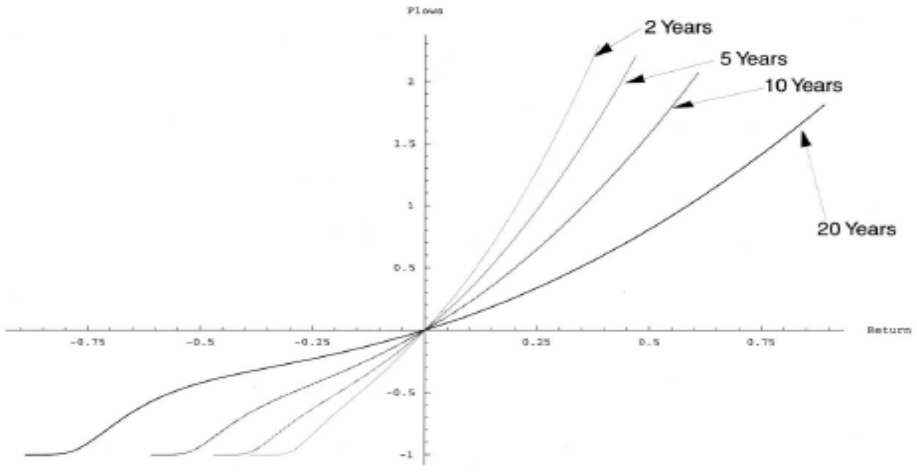


FIG. 1.—Flow of new funds as a function of return. The curves plot the response in the flow of new funds to the previous period's return (i.e., eq. [35]). The steepest curve shows the response for two-year-old funds (i.e., the return is from year 2 to year 3). The remaining curves show the response for five-, 10-, and 20-year-old funds, respectively. The parameter values used are $\phi = 0.03$, $\phi_0 = 0.065$, $f = 0.015$, $\gamma = 277$, and $\omega = 25$.

2 The model under full commitment

Consider a risk neutral investor[9] who hires a risk averse portfolio manager to invest a share of his assets[10] in an economy that lasts for two periods. We build a binomial model, i.e., there are two possible states of nature in each period. At the beginning of each period for each node of the decision tree, the investor decides the percentage of that share to be allocated with the portfolio manager, Ω_0, Ω_H and Ω_L. Besides, the contract that regulates this relationship describes the portion of the assets under management, m_f, that is paid to the manager in each node of the decision tree as management fee and the portion of the excess return (if positive), p_f, that is paid to the manager as performance fee. We still make two assumptions regarding the compensation schedule: a) the explicit incentive is stationary; i.e., they do not vary during the life of the contract and b) the explicit incentives are multiplied by the implicit incentives. These assumptions bring a lot of realism to the model since this schedule is the one frequently observed in the industry[11].

The portfolio manager has a time-separable utility function with impatient parameter, δ, and he is free to decide how to allocate the assets under his management in any possible investment alternative available in the economy. In order to make these decisions, the manager exerts costly and unobservable effort. Portfolio manager's effort decisions represent his set of feasible investment strategies and appear as more intense access to information, increased leverage, greater duration of fixed income instruments, open gap and credit risks, active day-trading, foreign exchange risks, etc. Thus, effort decisions affect the probability distribution of excess return *ex ante*, $\pi_{t,s}$ and they are considered to be **non-negative** and assume **continuous** values.

We still assume that the cost function of effort is monotonically increasing and twice continuously differentiable in effort; such that we have $\Psi(0) = 0$, $\lim_{e_s \to \infty} \Psi'(e_{t,s}) = \infty$, $\Psi'(\cdot) > 0$, $\Psi''(\cdot) > 0$ and $\Psi'''(\cdot) \geq 0$ which guarantees sufficiency conditions for interior solution and easy calculation of several static comparisons. In order to simplify the algebraic calculations, we define a exponential time-separable cost function

$$\Psi(e_{t,s}) = \frac{k}{2} (e_{t,s})^2 \tag{11}$$

The asymmetric information aspect of the model relies in the fact that $e_{t,s}$ is unobservable by the investor. In each period, the two states of nature are associated with two levels of excess return. The return of the investments made by the portfolio manager are compared to a pre-defined benchmark return, r_b. The investor, then, is not capable to know with certainty if excess return is due to effort or good fortune (luck). Indeed, the excess return, r_s, is a noisy signal

[9] The risk neutrality assumption is due to the standard justification that investors can diversify managers' specific risks away while each manager may not.

[10] We do not make any consideration about these assets or their associated markets.

[11] See Appendix 4 for a description of a typical contract found in the marketplace.

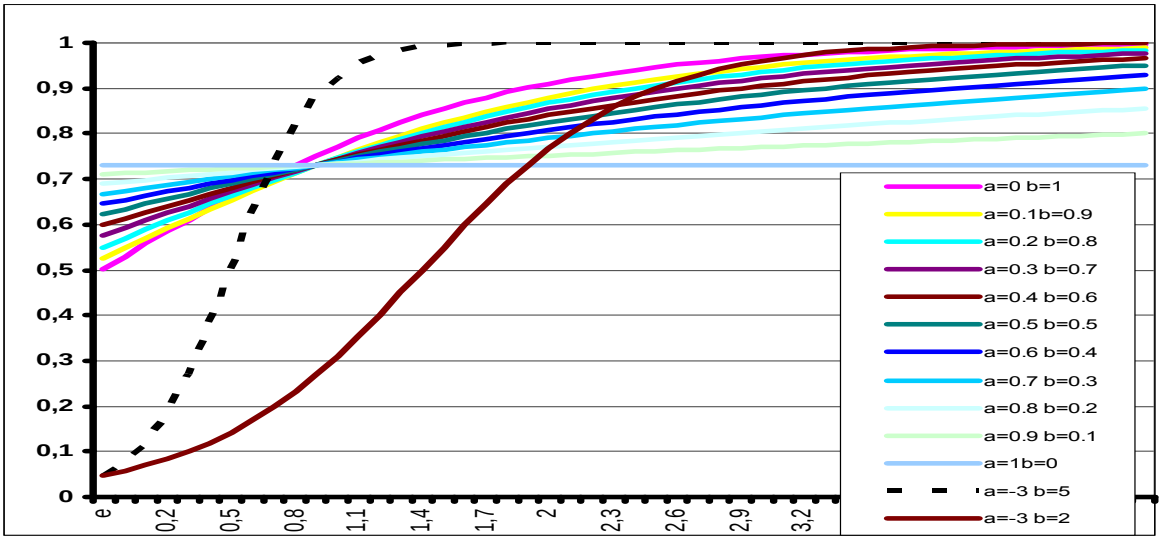
of $e_{t,s}$ and the portfolio manager is rewarded only on the basis of this noisy signal.

In the binomial model, high effort is associated with a higher excess return, r_H, and a particular compensation for the manager, (Ω_H, m_f, p_f). On the other hand, low effort is associated with a lower level of excess return, r_L, and a different compensation for the manager, $(\Omega_L, m_f, 0)$, assuming that $r_L < r_b$.

The function that describes the probability of obtaining a particular value of excess return is linear in effort and it is given by a logistic function

$$\pi_{t,s} = \frac{\exp(a + be_{t,s})}{1 + \exp(a + be_{t,s})}$$

The logistic function transforms a set of real number into the $[0, 1]$ interval. Effort is non-negative given the CPO´'s and the limited liability restrictions. If $a, b > 0$, then $\pi_{t,s} > 50\%$ which implies that it is not necessary too much effort induction to increase the expected return by affecting endogenous probabilities of return. Thus, the investor is better off if he provides less powerful contract either explicitly or implicitly. In this case, he will smooth the wage function such that effort choices and probabilities are equal in each node of the tree. To avoid this problem, it is necessary to impose restrictions on values each coefficient of the linear equation must assume such that it may assume negative values for low levels of effort choice. In this case, effort levels near zero would be associated with probabilities near zero. For a intercept $a = -3$, the probability of high return is equal to $\pi \approx 5\%$ when $e = 0$. For effort changes to affect probability levels, the effort coefficient b must be adjusted accordingly as well. In this paper, we will assume $b = [2, 3, 5]$. The shapes of the logistic function, given different parameters a and b, are shown in the graph below.



The coefficients of the function that transforms effort into the probability of occurrence of a particular state of nature are exogenously given in our model and

they determine the level of informativeness of the noisy signal, r_s. The intercept, a, can be seen as a parameter that only depends on specifics characteristics of each portfolio manager while the slope, b, represents the shift in the distribution of return derived from variations in effort. The greater is the value of b, the more dependent of effort is probability distribution of return.

If $b \approx 0$, then return is a sufficient statistic for managers' ability or, we may say, for specific features of the asset allocation strategies executed by the manager. Hence, return will only allow the investor to infer about manager's ability or about the implied risks of the portfolio; for example, given a particular investment regulation. In this case, moral hazard would not be an issue. On the other hand, when $b.e_{t,s}$ alters the probability of high return in a significant manner, return is a sufficient statistic for high levels of effort decisions executed by the manager and, thus, it should be used as a proxy of the manager compensation structure in our model. In this case, moral hazard in effort would play an important role in the maximization problem of the investor such that inducing optimal effort increases the value of the relationship.

In the model, expected return as well as variance depend explicitly on effort and are given by

$$E\left[r_{t,s}\right] = \frac{\exp(a + be_{t,s})}{1 + \exp(a + be_{t,s})} r_H - \left(1 - \frac{\exp(a + be_{t,s})}{1 + \exp(a + be_{t,s})}\right) r_L \qquad (12)$$

and

$$Var\left[r_{t,s}\right] = \left(\frac{\exp(a + be_{t,s})}{1 + \exp(a + be_{t,s})}\right) . \left(1 - \frac{\exp(a + be_{t,s})}{1 + \exp(a + be_{t,s})}\right) . \left(r_H - r_L\right)^2 \quad (13)$$

thus, **expected return and variance are endogenous** to effort decisions in our model.

The binomial distribution has an interesting relationship between the expected excess return and its variance. Low effort leads to low expected return and to low variance of returns as well. As $e_{t,s}$ increases and the probability of high return approaches half , both variance and expected return go up while variance attains its maximum at $\pi_{t,s} = 0.5$. So, medium effort is related to a greater average return but maximum variance. As $\pi_{t,s}$ goes to one, expected return reaches its maximum and variance is at its minimum again, that is, 0. In this model, the distribution of excess returns conditional on high effort, $\pi_{t,s}$, stochastically dominates in first-order the distribution of excess returns conditional on low effort, $(1 - \pi_{t,s})$. However, in a second-order stochastic dominance sense, the distribution of excess return conditional to low effort dominates the one conditional to high effort for $0 \leq \pi_{t,s} < \frac{1}{2}$. On the other hand, for $\frac{1}{2} \leq \pi_{t,s} \leq 1$, the distribution of excess return conditional to high effort dominates stochastically in a second-order sense the distribution of excess return conditional to low effort.

In economic terms, effort choice represents the reduced form of two tasks: effort choices increase expected return and risk choices shift variance of returns. In the interval $\pi_{t,s} \in \left[0; \frac{1}{2}\right]$, they are substitutes tasks. Only for higher than

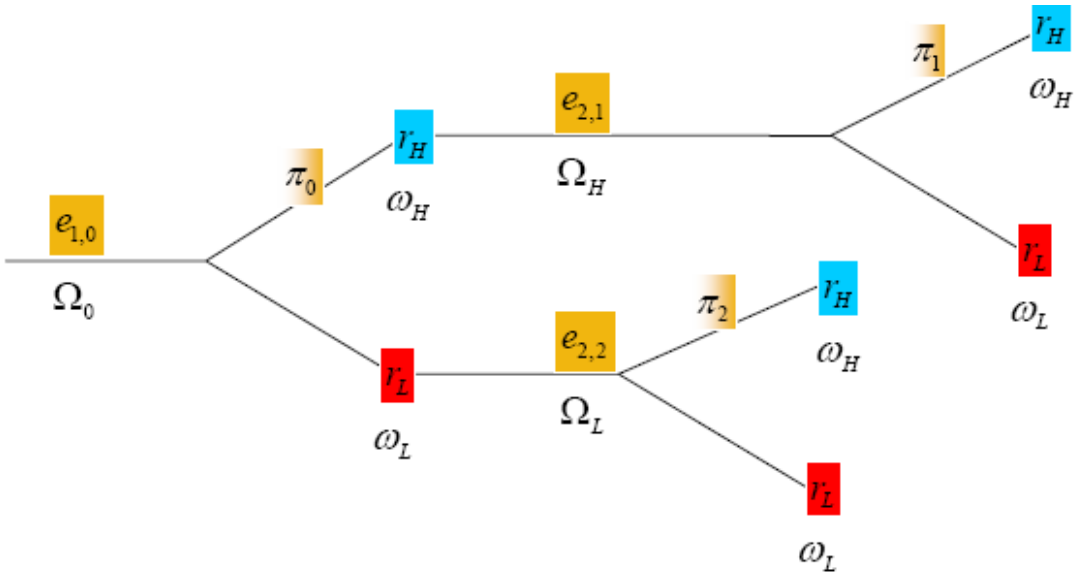
half effort choices becomes complementary tasks. Remember that it is less costly to induce complementary tasks than two substitute tasks in a second best environment since there are economies of scope when these tasks entails moral hazard. Then, these economies of scope only appear for levels of effort greater than half.

2.1 The timing of the model and the decision tree

The timing of the two period model is explained as follows. At the beginning of the first period, the investor simultaneously offers a contract $\{m_f, p_f, \Omega_{t,s}\}$ to the portfolio manager who receives fees $\omega = (m_f, p_f)$ for an initial investment Ω_0. The manager chooses his asset allocation strategy through a costly effort decision. Then, nature moves and a particular value of excess return, $r_{1,s}$, is realized.

At the end of the first period, the investor and the portfolio manager observe r_0 and, then, the investor changes Ω_0 to Ω_H or Ω_L, according to expected return in each node of the tree in the second period. Again, in the beginning of the second period, the manager chooses an state-dependent level of effort which will be followed by another nature move such that a particular value of excess return, $r_{2,s}$, is realized.

Decision tree graph goes here



2.2 The portfolio manager problem: optimal choice of effort

The portfolio manager maximizes expected utility by choosing the optimal levels of effort

$$\begin{aligned}\max_{e_0,e_1,e_2} U_M &= \sum_{t=0}^{2}\delta^t\left[\sum_{s_t=\underline{r}_t}^{s_t=\overline{r}_t} P\left(r_t|e_t\right)u\left(\Omega_{t,s}\omega_{t,s}\right)-\frac{k}{2}\left(e_{t,s}\right)^2\right] \qquad (14)\\ &= \left\{\begin{array}{c}\pi_0 u\left(\Omega_0\left(m_f+p_f\max\left[r_H-r_b,0\right]\right)\right)\\ +\left(1-\pi_0\right)u\left(\Omega_0\left(m_f+p_f\max\left[r_L-r_b,0\right]\right)\right)-\frac{k}{2}\left(e_0\right)^2\end{array}\right\}\\ &+\delta\left\{\begin{array}{c}\pi_0\left[\begin{array}{c}\pi_1 u\left(\Omega_H\left(m_f+p_f\max\left[r_H-r_b,0\right]\right)\right)\\ +\left(1-\pi_1\right)u\left(\Omega_H\left(m_f+p_f\max\left[r_L-r_b,0\right]\right)\right)-\frac{k}{2}\left(e_1\right)^2\end{array}\right]\\ +\left(1-\pi_0\right)\left[\begin{array}{c}\pi_2 u\left(\Omega_L\left(m_f+p_f\max\left[r_H-r_b,0\right]\right)\right)\\ +\left(1-\pi_2\right)u\left(\Omega_L\left(m_f+p_f\max\left[r_L-r_b,0\right]\right)\right)-\frac{k}{2}\left(e_2\right)^2\end{array}\right]\end{array}\right\}\end{aligned}$$

where the utility function of the portfolio manager presents the usual properties of concavity: $u_\Omega(\Omega=0,\omega=0)=u_\omega(\Omega=0,\omega=0)=\infty \lim_{(\Omega,\omega)\to\infty} u_\Omega(\cdot,\cdot)=u_\omega(\cdot,\cdot)=0$, $u_\Omega(\Omega,\omega)>0$, $u_\omega(\Omega,\omega)>0$, $u_{\Omega\Omega}(\Omega,\omega)<0$, $u_{\omega\omega}(\Omega,\omega)<0$. Risk aversion creates inefficiencies in the provision of effort due to the effects of moral hazard and, in this case, the risk neutral investor should pay a premium for a risk averse manager to participate.

The reservation utility of the portfolio manager is exogenously given and is equal to $\overline{U_M}$. The investor has all bargaining power and can make take-it-or-leave-it offers to the portfolio manager subject to providing him with an expected payoff which yields at least $\overline{U_M}$.

$$\frac{\partial U_M}{\partial e_0^*}=A\left(\omega,\Omega\right)=0 \qquad (15)$$

$$\frac{\partial U_M}{\partial e_1^*}=E\left(\omega,\Omega\right)=0 \qquad (16)$$

$$\frac{\partial U_M}{\partial e_2^*}=I\left(\omega,\Omega\right)=0 \qquad (17)$$

The optimal effort choice in the first period has a dynamic component represented by the present value of the difference in utility that the manager derive in each of the two possible states of nature in the second period. That is, optimal choice in the hidden action problem contains all elements of the compensation schedule, revealing the power of the implicit incentive in the dynamic moral hazard problem.

For a given a and b, when $p_f>0$ and $\Omega_H>\Omega_L$, the optimal choice of unobservable effort in the first period is higher in the dynamic problem than in the static version. With enough dynamic incentive $\Omega_H>\Omega_0$ and enough dynamic penalization $\Omega_L<\Omega_0$, it is possible to reduce the cost of implementing second-best solutions with a smaller distortion between p_f and 0, i.e., the power

of the explicit contract will be lower than in the static version. The importance of the implicit dynamic incentive is raised when the limited liability constraint binds.

In the second and last period of the relationship, the dynamic component vanishes and only the distortion in the explicit incentives matter for the manager, a solution that is similar to the static version of the hidden action problem. In fact, memory plays an important role by differentiating the compensation in each node of the second period. Memory appears while the implicit incentive depends on the return observed in the first period. Then, the optimal effort solution will obey

$$e_0^*(\omega,\Omega) \geq e_1^*(\omega,\Omega) \geq e_2^*(\omega,\Omega) \tag{18}$$

2.3 The investor problem: optimal provision of incentives

The risk-neutral investor maximizes expected profit by choosing the optimal levels of incentives

$$\max_{\substack{m_f,p_f,\Omega_H,\Omega_L \\ e_0,e_1,e_2}} V_I = \sum_{t=0}^{2}\left[\sum_{s_t=\underline{r}_t}^{s_t=\overline{r}_t} P(r_t|e_t)\,\Omega_{t,s}\,(r_{t,s}-m_f-p_f\max[r_s-r_b,0]) - (\Omega_{t,s})\,\overline{r_b}\right] \tag{19}$$

where $\overline{r_b}$ is the return of the outside investment alternative of the investor - the benchmark return can be obtained without any effort and incentive provision. When $(\Omega_{t,s}-1)>0$, the investor is borrowing at this benchmark rate and investing the resources in the fund. While $(\Omega_{t,s}-1)<0$, the investor is withdrawing resources from the fund and re-investing them in benchmark return-linked instruments. The investor observes excess return at the end of every period and decides to change the implicit incentive based on the history of excess returns. Excess return represents a noisy signal of effort with mean and variance respectively given by (12) and (13).

In equilibrium, the investor anticipates the optimal choice of actions taken by the portfolio manager and design an incentive compatible contract. When

$\overline{r_b} = 0$, the problem of the investor becomes

$$\begin{aligned}
\max_{\substack{m_f, p_f, \\ \Omega_H, \Omega_L}} V_I \quad = \quad & \left(\frac{\exp(a + be_0^*)}{1 + \exp(a + be_0^*)} \right) \Omega_0 \left(r_H - m_f - p_f \max \left[r_H - r_b, 0 \right] \right) \\
& + \left(1 - \frac{\exp(a + be_0^*)}{1 + \exp(a + be_0^*)} \right) \Omega_0 \left(r_L - m_f - p_f \max \left[r_L - r_b, 0 \right] \right) \\
& + \delta \frac{\exp(a + be_0^*)}{1 + \exp(a + be_0^*)} \left[\begin{array}{c} \frac{\exp(a+be_1^*)}{1+\exp(a+be_1^*)} \Omega_H \begin{pmatrix} r_H - m_f \\ -p_f \max \left[r_H - r_b, 0 \right] \end{pmatrix} \\ + \left(1 - \frac{\exp(a+be_1^*)}{1+\exp(a+be_1^*)} \right) \Omega_H \begin{pmatrix} r_L - m_f \\ -p_f \max \left[r_L - r_b, 0 \right] \end{pmatrix} \end{array} \right] \\
& + \delta \left(1 - \frac{\exp(a + be_0^*)}{1 + \exp(a + be_0^*)} \right) \left[\begin{array}{c} \frac{\exp(a+be_2^*)}{1+\exp(a+be_2^*)} \Omega_L \begin{pmatrix} r_H - m_f \\ -p_f \max \left[r_H - r_b, 0 \right] \end{pmatrix} \\ + \left(1 - \frac{\exp(a+be_2^*)}{1+\exp(a+be_2^*)} \right) \Omega_L \begin{pmatrix} r_L - m_f \\ -p_f \max \left[r_L - r_b, 0 \right] \end{pmatrix} \end{array} \right]
\end{aligned}$$

subject to the following participation constraints. We normalize the reservation utility to zero in each node and write

$$U_M \geq 0 \tag{20}$$

The incentive constraints are given by the CPOs' of the portfolio manager problem since we are imposing the first-order approach

$$e_{t,s}^* (\omega, \Omega) \in \arg\max U_M \tag{21}$$

It is necessary to write two limited responsibility constraints for the explicit incentives since the manager has limited liability in excess return and, thus, can only be penalized for exerting low levels of effort through the implicit incentive.

$$m_f > 0 \tag{22}$$

$$p_f \geq 0 \tag{23}$$

Since it is neither possible to borrow resources from the manager's fund nor to leverage positions in the fund by borrowing at the benchmark rate, there are also two short-selling and two borrowing constraints for the implicit incentives such that

$$0 \leq \Omega_H \leq 1 \tag{24}$$

$$0 \leq \Omega_L \leq 1 \tag{25}$$

All first-order conditions are shown in subsection 1 of the Appendix. The equilibrium solution $\left\{ m_f^*, p_f^*, \Omega_H^*, \Omega_L^* \right\}$ is algebraically intractable and can only have a numerical solution. The MatLab code and its results are shown, respectively, in subsection 2 and 3 of the Appendix.

2.4 Characterization of the optimal incentive contract

In equilibrium, the investor offers an incentive compatible contract $\left\{m_f^*, p_f^*, \Omega_H^*, \Omega_L^*\right\}$ that satisfies all the constraints of his problem. The investor provides total incentives that equalize the marginal excess expected return and the implied costs of effort induction. He does so by simultaneously combining and distorting both the implicit and the explicit incentive's compensation structure as to maximize the intertemporal excess expected return.

The explicit incentive reduces net excess expected return. In equilibrium, the investor sets m_f^* at the minimum possible level since $m_f^* > \min(m_f)$ only reduces net expected return while it does not affect the probability of high return. Indeed, if one sets $m_f^* > \min(m_f)$, it also presents the hazard to decrease the marginal utility that would be derived by a greater peformance fee and/or greater implicit incentive distortion. Regarding the performance fee, the investor sets $p_f^* \geq 0$ as to increase the probability of high return in each node of the decision tree. This result is natural since setting $p_f^* \geq 0$ induces positive effort, increases the probability of high return in all nodes of the tree and, thus, increases excess expected return. For a given solution $\{\Omega_H^*, \Omega_L^*\}$, the optimal level of performance fee, $\left(m_f^*, p_f^*\right)$, equalizes marginal excess expected return due to shifts in the probability distribution of return to the marginal cost of exerting effort in all nodes of the tree.

In the dynamic model. the investor desires to induce greater effort in the first period while its benefits are greater than the ones generated by effort executed in the second period. That is, investor faces a intertemporal trade-off between inducing effort in the first period - which increases expected return in the first period - *vis-à-vis* inducing effort in second period - increasing expected return in the second period. The investor distort the implicit incentive equilibrium allocations such that $\Omega_H^* > \Omega_L^*$ in most cases.

Then, the optimal contract is a combination of $\left\{\min\left(m_f^*\right), p_f^*, \Omega_H^*, \Omega_L^*\right\}$. Since we are imposing the first-order approach (FOA) - by substituting the portfolio managers' first-order conditions into the investor's objective function - it needs to be checked if the second-order conditions (SOC) satisfy the necessary and sufficient conditions for a local maxima. That is, we verify if at the solution found numerically, $\left\{m_f^*, p_f^*, \Omega_H^*, \Omega_L^*\right\}$, the Hessian matrix of the portfolio manager's maximization problem is negative semi-definite.

Another computational consideration that must be taken into account is that the choice of parameters must avoid situations in which the provision of implicit and explicit incentives have little effect in altering the distribution of returns. In these cases, the investor will minimize all incentives expenses and behave just like predicted by the case of complete information.

2.4.1 Comparison with the case where $\Omega_H^* = \Omega_L^* = 1$

The flow of funds serves two purposes. First, it determines the investor's asset allocation strategy. From a finance and portfolio allocation perspective, we know that the risk neutral investor should choose $\Omega^* = 1$ if net excess expected return is positive. On the other side, when net excess expected return is negative, the investor sets $\Omega^* = 0$.

Due to hidden action considerations, the flow of funds also plays the role of an implicit incentive as to avoid moral hazard in the execution of effort. In the dynamic model. the investor desires to induce greater effort in the first period while its benefits are greater than the ones generated by effort executed in the second period. That is, investor faces a intertemporal trade-off between inducing effort in the first period - which increases expected return in the first period - *vis-à-vis* inducing effort in second period - increasing expected return in the second period. Then, the investor distort the implicit incentive equilibrium allocations that may differ from the natural and trivial solution described above. Then, the flow of funds modify the allocation classical rule such that

$$\left(1 - p_f^*\right) E\left[r_{t,s}\right] - \left(\overline{r}_b + m_f^*\right) > 0 \Rightarrow 0 < \Omega_{t,s}^* \leq 1$$

and

$$\left(1 - p_f^*\right) E\left[r_{t,s}\right] - \left(\overline{r}_b + + m_f^*\right) \leq 0 \Rightarrow 0 \leq \Omega_{t,s}^* < 1$$

Moreover, since expected return is endogenous in this model and given (18), we have

$$E\left[r_{1,0}^*\right] > \delta E\left[r_{1,1}^*\right] \geq \delta E\left[r_{2,1}^*\right]$$

Indeed, there is economic value in providing distorted implicit incentives at the cost of destroying the relationship in the second period whenever one observes negative excess return in the first period. To maximize expected utility, the investor decides how much endogenous expected return to give up in the second period in order to obtain endogenous expected return derived from higher induced effort in the first period.

Let's consider two cases. First, assume that $r_H > r_b > r_L$. In the first case, there is no distortion in the implicit incentive such that $\Omega_H^* = \Omega_L^* = 1$. Once the incentive constraints depend on the difference of utilities in each node of the tree, effort choices will be equal

$$e_0^* = e_1^* = e_2^* = e$$

In this case, $\pi_0^* = \pi_1^* = \pi_2^* = \frac{\exp(a+be^*)}{1+\exp(a+be^*)} = \pi$ and the manager earns

$$U_M = (1+\delta)\left[\pi u\left(m_f + p_f\left(r_H - r_b\right)\right) + (1-\pi)\, u\left(m_f\right) - \frac{k}{2}e^2\right] \geq 0 \qquad (26)$$

In this case, only the explicit incentive, p_f, affects effort choices. The manager has to make only one choice of effort. From (22), the risk averse manager participation constraints is always greater than zero for all $m_f, p_f > 0$ and,

hence, the constraint is not binding ($\lambda = 0$). On his side, the risk neutral investor earns net excess expected return

$$V_I = (1+\delta)\left[\pi\left(r_H - m_f - p_f\left[r_H - r_b\right]\right) + (1-\pi)\left(r_L - m_f\right)\right] > 0 \tag{27}$$

From (27), the investor problem reduces to choosing m_f^* and p_f^*, subject to the optimal choice of effort made by the manager and its participation constraint (26).

Now, consider a second extreme case. Suppose that the investor offers full implicit incentive distortion. Then, we have $\Omega_H^* = 1$ and $\Omega_L^* = 0$. In this case, effort choices will be increasing

$$e_0^* > e_1^* > e_2^* = 0$$

In this case, In this case, $\pi_0^* > \pi_1^* > \pi_2^*$ and the participation constraint is given by

$$U_M = \left\{ \begin{array}{c} \pi_0 u\left(m_f + p_f\left(r_H - r_b\right)\right) + (1-\pi_0)\, u\left(m_f\right) - \frac{k}{2}\left(e_0\right)^2 \\ +\delta\left\{\pi_0\left[\pi_1 u\left(m_f + p_f\left[r_H - r_b\right]\right) + (1-\pi_1)\, u\left(m_f\right) - \frac{k}{2}\left(e_1\right)^2\right]\right\} \end{array} \right\} \geq 0 \tag{28}$$

From (28) and considering $m_f^* = \min\left(m_f\right)$, the investor has to calculate the p_f^* that maximizes effort. Due to the fact that it reduces the net expected return in all nodes of the tree, we have $p_f^* < r_H$. The investor earns an expected return equal to

$$\begin{array}{rcl} V_I & = & \pi_0\left(r_H - m_f - p_f\left[r_H - r_b\right]\right) + (1-\pi_0)\left(r_L - m_f\right) \\ & & +\delta\pi_0\left[\pi_1\Omega_H\left(r_H - m_f - p_f\left[r_H - r_b\right]\right) + (1-\pi_1)\, u\left(r_L - m_f\right)\right] \end{array} \tag{29}$$

If (29) > (27) for the same explicit contract, $\left(m_f^*, p_f^*\right)$, then it is optimal (in comparison with the first case described above) for the investor to fully distort the contract by offering a compensation scheme with powered implicit incentives. This results follows from the fact that the marginal benefit obtained in the first period is greater in module than the excess expected return given up in the second period.

From (29), the investor problem reduces to choosing p_f^*. Optimal choice of p_f^* will satisfy first-order conditions. The algebraic solution here is also intractable and we compare the numerical solution in this case with the numerical solution in case 1.

A third possible case is algebraically intractable and it has two possibilities. Either $\Omega_H^* = 1$ and $0 < \Omega_L^* < 1$ or $0 < \Omega_H^* < 1$ and $\Omega_L^* = 0$. In this two cases, $\Omega_H^* > \Omega_L^*$ and, then, $e_0^* \geq e_1^* > e_2^*$. It often occurs when the investor offers implicit incentive at a higher implied cost which is expressed in terms of giving up positive excess expected returns or seizing negative excess expected returns in the second period. In this case, the implicit incentive substitutes the explicit incentive in inducing effort whenever $p_f^* \approx 0$. On the other hand, variations in the implicit incentive that are very costly in terms of excess expected return

are less intense and they are compensated by more distortion in the explicit incentive. These results are shown in the next subsection.

This model may explain the little presence of complex long term explicit contracts in the asset management industry. That is, the investor writes a simple long term explicit contract and allows the powerful implicit incentive to depend on his beliefs at each node of the decision tree. In other words, possible agency problems derived from simple and/or incomplete explicit incentives may be partially solved by delegating power to the implicit incentive. Yet, flow of funds concerns may correct some wrong incentives and risk incongruities that may arise with the design of a simple explicit incentive mechanism.

Nevertheless, we may question if the implicit incentives' results present some dynamic inconsistency concerns from the perspective of the beginning of the second period. Since the investor may change his decision and decide not to give up positive excess expected return. This is due to the fact that effort induced in the first period is already executed and the first period probability distribution function of returns does not influence the second period's distribution. Therefore, implicit incentive distortion may represent a non-credible threat. We can adopt several strategies to solve this problem. For example, we may assume that the repeated game is played infinitely or that reputation concerns would force the investor to choose this costly allocation strategy. The alternative we use considers manager's heterogeneity with respect to ability and we expose it in the next section.

2.5 Numerical results

We assume that the risk aversion manager's preferences are represented by a HARA utility function and that the parameter of risk aversion of the function is given by ρ. The utility function takes the following functional form

$$u(\bullet) = \frac{1}{\rho - 1} \left(\Omega_{t,s}.m_f + \Omega_{t,s}.p_f.\max\left[r_{t,s} - r_b, 0\right]\right)^{\left(\rho - \frac{1}{\rho}\right)}$$

The coefficients of relative risk aversion to the variation of each compensation variable are given by

$$R^R_{m_f} = \left(\rho - 1 - \frac{1}{\rho}\right)\Omega_{t,s}$$

$$R^R_{p_f} = \left(\rho - 1 - \frac{1}{\rho}\right)\Omega_{t,s}.\max\left[r_{t,s} - r_b, 0\right]$$

$$R^R_{\Omega_{t,s}} = \left(\rho - 1 - \frac{1}{\rho}\right).\left(m_f + p_f \max\left[r_{t,s} - r_b, 0\right]\right)$$

The parameters values of the logistic function that defines the probability distribution function of return in each node of the tree are the coefficients a and b. The parameter of impatience is δ and the cost function coefficient is k.The high and low return depend on the level of the benchmark rate and the number

of days as well as on the benchmark percentage variation of the benchmark return obtained in each state of nature; respectively, r_B, $days$, $P_exc_ret_H$ and $P_exc_ret_L$. We calculate r_H and r_L in the following way

$$\begin{aligned} r_H &= \left((1+r_b)^{(days/252)} - 1\right) * P_exc_ret_H \\ r_L &= \left((1+r_b)^{(days/252)} - 1\right) * P_exc_ret_L \\ r_B &= \left((1+r_b)^{(days/252)} - 1\right) \end{aligned}$$

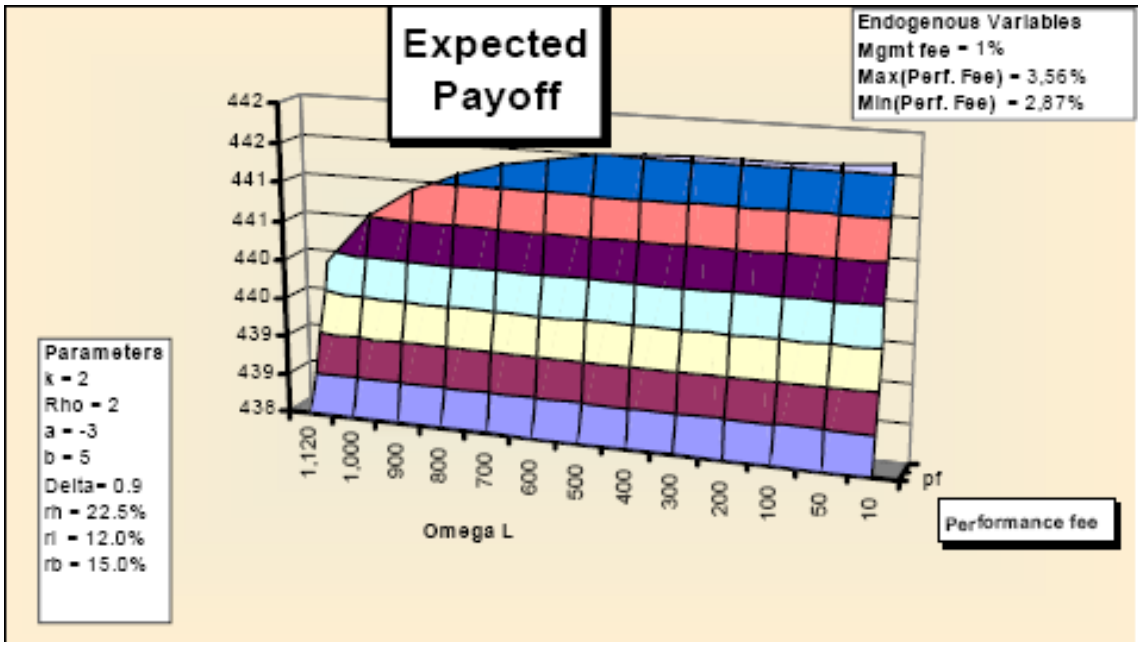
We use the fmincom function of the Optimization toolbox in the Matlab software. The MatLab code files are presented in subsection 2 of the Appendix 2. We make some adjustments in our problem to be able to solve it numerically. Since the fmincom function is a non-linear constrained minimization function, we multiply the investor objective function by minus one. We also have to multiply the participation constraint by minus one since the fmincom function read inequality constraints as functions smaller or equal than zero.

Given the nonlinearity existent in the problem, the global optimal solution depends on the initial guess values provided in the computational program. So, we need to run the code for distinct starting points and select the result with the greatest expected return.

We, however, adopt a different approach here. We fix the management fee, the funds in each state of the second period and calculate the optimal performance fee and the respective optimal effort choices for different levels of asset under management after poor performance. Then, we compare the respective expected return values.

Consider the case given the set of parameters shown in the graph below. The graph and the table below shows that the performance fee decreases and the expected payoff increases for higher levels of implicit incentives, i.e, higher distortion in the flow of funds after the observation of performance in the first period. The importance of the implicit incentive becomes more apparent when we observe that the marginal increase in the payoff occurs even the cost of withdrawing funds from funds that are expected to perform positively in the second period. Observe also that the investor gives up expected payoff in the second period by penalizing negative excess return in the first period with the

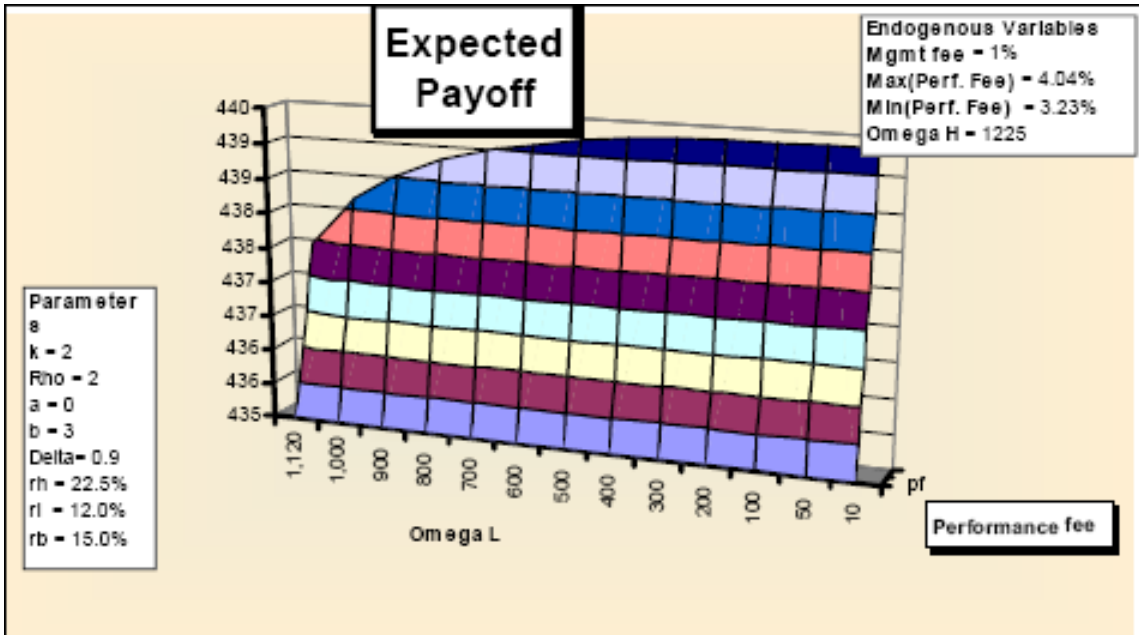
objective to gain from greater effort exertion in the first period.



pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
3.56%	1,120	1.32	1.30	1.28	0.9733	0.9707	0.9670	439.849	148.132%	147.947%	147.687%
3.18%	1,000	1.36	1.28	1.22	0.9784	0.9674	0.9573	440.534	148.488%	147.720%	147.013%
3.12%	900	1.40	1.27	1.19	0.9817	0.9668	0.9500	440.883	148.719%	147.676%	146.497%
3.04%	800	1.42	1.27	1.15	0.9839	0.9660	0.9400	441.118	148.873%	147.621%	145.801%
2.99%	700	1.44	1.27	1.11	0.9855	0.9655	0.9271	441.281	148.967%	147.584%	144.900%
2.96%	600	1.46	1.26	1.06	0.9868	0.9651	0.9094	441.397	149.075%	147.559%	143.655%
2.93%	500	1.48	1.26	1.00	0.9877	0.9649	0.8830	441.478	149.142%	147.542%	141.808%
2.92%	400	1.49	1.26	0.93	0.9885	0.9647	0.8397	441.531	149.195%	147.531%	138.782%
2.92%	300	1.50	1.26	0.83	0.9891	0.9647	0.7565	441.558	149.236%	147.528%	132.952%
2.94%	200	1.51	1.26	0.61	0.9896	0.9649	0.5183	441.551	149.269%	147.545%	116.283%
2.89%	100	1.51	1.26	0.05	0.9898	0.9643	0.0595	441.546	149.283%	147.503%	84.162%
2.88%	50	1.52	1.26	0.01	0.9898	0.9642	0.0508	441.578	149.289%	147.497%	83.557%
2.87%	10	1.52	1.26	0.00	0.9899	0.9642	0.0477	441.598	149.291%	147.494%	83.339%

We also want to compare the results above to those when the probability distribution function of return is less dependent on effort execution. Consider the case when $a = 0$ and $b = 3$. Observe that the minimum probability of high return is 50%, even when effort is closer to zero. In this case, it is more costly to induce optimal effort since it takes more performance fee to provide the appropriate incentives. Therefore, the expected payoff and excess expected

returns are lower as well as shown below



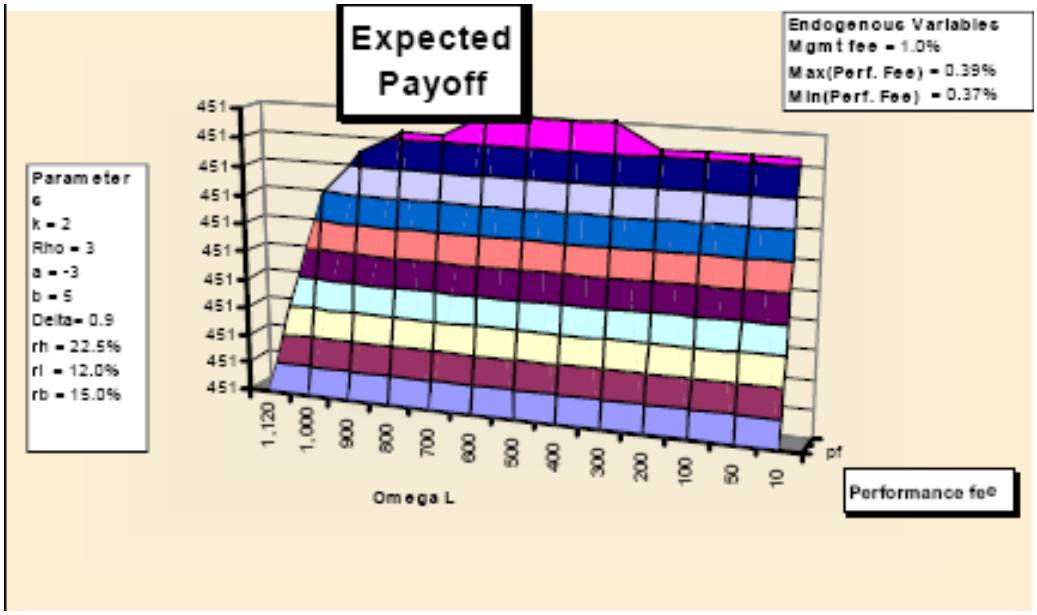
pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
4.04%	1,120	1.11	1.09	1.06	0.9659	0.9638	0.9598	437.431	147.611%	147.464%	147.184%
3.70%	1,000	1.18	1.07	0.99	0.9716	0.9611	0.9507	438.147	148.013%	147.275%	146.552%
3.55%	900	1.22	1.06	0.93	0.9751	0.9597	0.9425	438.527	148.254%	147.177%	145.973%
3.45%	800	1.26	1.05	0.88	0.9776	0.9587	0.9328	438.790	148.434%	147.112%	145.296%
3.38%	700	1.29	1.04	0.82	0.9796	0.9581	0.9209	438.977	148.572%	147.067%	144.460%
3.34%	600	1.32	1.04	0.75	0.9811	0.9576	0.9053	439.111	148.680%	147.035%	143.374%
3.31%	500	1.34	1.04	0.68	0.9823	0.9573	0.8842	439.206	148.764%	147.012%	141.891%
3.29%	400	1.36	1.03	0.59	0.9833	0.9571	0.8535	439.270	148.831%	146.995%	139.746%
3.27%	300	1.37	1.03	0.47	0.9841	0.9569	0.8060	439.309	148.884%	146.982%	136.418%
3.25%	200	1.39	1.03	0.33	0.9846	0.9567	0.7270	439.328	148.923%	146.971%	130.887%
3.24%	100	1.39	1.03	0.14	0.9850	0.9565	0.6024	439.336	148.950%	146.958%	122.168%
3.23%	50	1.40	1.03	0.05	0.9851	0.9565	0.5379	439.342	148.959%	146.953%	117.656%
3.23%	10	1.40	1.03	0.00	0.9852	0.9564	0.5034	439.345	148.963%	146.951%	115.239%

It also is possible to observe that the implicit incentive's power is lower once endogenous excess return is not as much affected as in the case above. Since it is more costly to induce effort through the implicit incentive, the investor is willing to use the performance to obtain greater expected payoff in all nodes of the decision tree. Now, the trade-off is different since it explores the benefits and costs of the usage of the performance fee in each period.

Greater risk aversion leads to less powerful incentive schemes. The more risk averse is the manager, the more efficient is the usage of the implicit and

the explicit incentive and, hence, less costly incentives induces more effort for greater levels of risk aversion. The more risk averse is the manager, the least flow distortion is necessary to induce effort in the first period, reducing the impact of the intertemporal trade-off as well as less performance fee is necessary to induce effort at each node of the decision tree. In this case, powerful implicit incentives are sub-optimal and are related to lower expected payoffs.

The graph and the table below show these results should be compared with the Graph 1 and Table 1, respectively.



pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
0.39%	1,120	1.86	1.65	1.61	0.9982	0.9948	0.9935	450.591	149.871%	149.633%	149.545%
0.38%	1,000	1.96	1.64	1.55	0.9989	0.9946	0.9913	450.678	149.923%	149.623%	149.389%
0.37%	900	2.01	1.64	1.49	0.9991	0.9946	0.9887	450.704	149.940%	149.621%	149.211%
0.37%	800	2.04	1.64	1.44	0.9993	0.9946	0.9851	450.718	149.949%	149.620%	148.955%
0.37%	700	2.07	1.64	1.37	0.9993	0.9946	0.9795	450.725	149.954%	149.620%	148.565%
0.37%	600	2.08	1.64	1.30	0.9994	0.9946	0.9705	450.729	149.958%	149.620%	147.933%
0.37%	500	2.09	1.64	1.21	0.9994	0.9946	0.9545	450.731	149.960%	149.620%	146.818%
0.37%	400	2.10	1.64	1.10	0.9995	0.9946	0.9227	450.730	149.962%	149.620%	144.590%
0.37%	300	2.11	1.64	0.94	0.9995	0.9946	0.8444	450.729	149.963%	149.620%	139.109%
0.37%	200	2.11	1.64	0.61	0.9995	0.9946	0.5105	450.724	149.963%	149.622%	115.734%
0.37%	100	2.11	1.64	0.02	0.9995	0.9946	0.0518	450.722	149.963%	149.620%	83.629%
0.37%	50	2.11	1.64	0.00	0.9995	0.9946	0.0480	450.723	149.963%	149.620%	83.363%
0.37%	10	2.11	1.64	0.00	0.9995	0.9946	0.0474	450.724	149.963%	149.619%	83.320%

Patient players, represented by greater levels of δ (delta), are less willing to trade-off expected return in the first period *vis-à-vis* expected return in the

second period. Then, more explicit incentive is used to induce higher effort. On the other hand, the portfolio manager is also more interested in smoothing consumption between the two periods.

In general terms, the optimal solution shows that the implicit incentive varies significantly in the second period depending on the expected return in each node. The investor provides powerful implicit incentives after the observation of high return in the first period and the upper bound (24) binds. On the other hand, after low return is observed in the first period, the investor penalizes the manager, withdrawing all or almost all resources from the manager and (25) binds. The dynamic implicit incentive component is so strong in this model that this result occurs even if the expected return is positive in the bad state of nature of the second period.

The intuition behind it is that since both the total payoff and the explicit incentives depend on the amount of implicit incentives, effort induction by using the flow of funds is more efficient under a moral hazard framework .

Risk aversion should explain the third result in which the two incentives are complementary; that is, the more powerful is the implicit incentive, the greater is the performance fee. A risk-averse manager has "flow of funds' insurance" when the implicit incentive is not used. The marginal utility derived from a marginal increase in the performance fee is lower in this case than when the implicit incentive is powerful; i.e., the flow of funds distort the total compensation of the portfolio manager. Hence, a smaller performance fee is necessary to induce optimal effort choices. On the other hand, when the portfolio manager is penalized for poor performance by the losing funds, the marginal utility derived by any marginal increase in the performance fee is higher since performance fee is affecting utilities in fewer states of nature. Once performance fee depends on the amount of resources invested in the fund, the total intertemporal compensation is more affected by a marginal increase in the performance fee. Therefore, the level of performance fee is greater when the investor uses the flow of funds to induce effort.

3 The model with two types of portfolio managers

Suppose that there two types of portfolio managers in the economy ($\overline{\eta}$ and $\underline{\eta}$) and that they are heterogeneous in the ability to generate positive excess return at each period and each node of the decision tree. In order to ease the algebraic calculations, the high ability portfolio manager, $\overline{\eta}$, is able to produce positive excess return with positive probability in the good state of nature of the binomial model while the low ability portfolio manager, $\underline{\eta}$, always produces negative excess return, never adding any value to the relationship. In this case, $P\left(r_H|\overline{\eta}\right) > 0$ and $P\left(r_H|\underline{\eta}\right) = 0$. We adopt a key simplification in the model and make the level of ability unknown to everyone in the economy, whether the investor or the manager. Therefore, the portfolio manager's type is an incomplete

and symmetric information. Only the prior distribution over η is commonly known and shared by all contracting parties *ex ante*.[12] Since the information is symmetric, there is no need for investors to offer menus of contracts in order to induce workers to self-select.

We further assume that the proportion of $\overline{\eta}$ in the economy is λ and the percentage of $\underline{\eta}$ is $(1-\lambda)$. In the first period, the investor has to infer the probability of return based on his belief of λ. In the second period, the investor uses his belief and the information derived from the the excess return observed in the first period to infer about the portfolio manager's type, η. Now, return is a noisy signal of effort and ability.

All the assumptions and notation remain the same unless for a new superscript in each effort function which indicates the type of the manager. Then, we describe effort as $e_{t,s}^{m} \in [0,1]$, $\eta = \overline{\eta}, \underline{\eta}$. In the first period, the probability of high return is given by the probability distribution of return conditional to the portfolio manager's expected level of ability

$$\pi_{1,0} = P(\overline{\eta}) . P(r_H|\overline{\eta}) + P(\underline{\eta}) . P(r_H|\underline{\eta})$$

The investor and the manager observe the realized return in the first period and learn about the manager's ability. Then, the investor adjusts the posterior distribution of return in a Bayesian way to obtain the probabilities of high return in each node of the second period.

$$\pi_{2,1} = P(r_H|\overline{\eta}) P(\overline{\eta}|r_H) + P(r_H|\underline{\eta}) P(\underline{\eta}|r_H)$$

$$\pi_{2,2} = P(r_H|\overline{\eta}) P(\overline{\eta}|r_L) + P(r_H|\underline{\eta}) P(\underline{\eta}|r_L)$$

while the probabilities that the manager is of a particular ability type given the return observed in the first period are respectively given by

$$P(\overline{\eta}|r_H) = \frac{P(\overline{\eta}) P(r_H|\overline{\eta})}{P(\overline{\eta}) P(r_H|\overline{\eta}) + P(\underline{\eta}) P(r_H|\underline{\eta})}$$

$$P(\underline{\eta}|r_H) = \frac{P(\underline{\eta}) P(r_H|\underline{\eta})}{P(\overline{\eta}) P(r_H|\overline{\eta}) + P(\underline{\eta}) P(r_H|\underline{\eta})}$$

$$P(\overline{\eta}|r_L) = \frac{P(\overline{\eta}) P(r_L|\overline{\eta})}{P(\overline{\eta}) P(r_L|\overline{\eta}) + P(\underline{\eta}) P(r_L|\underline{\eta})}$$

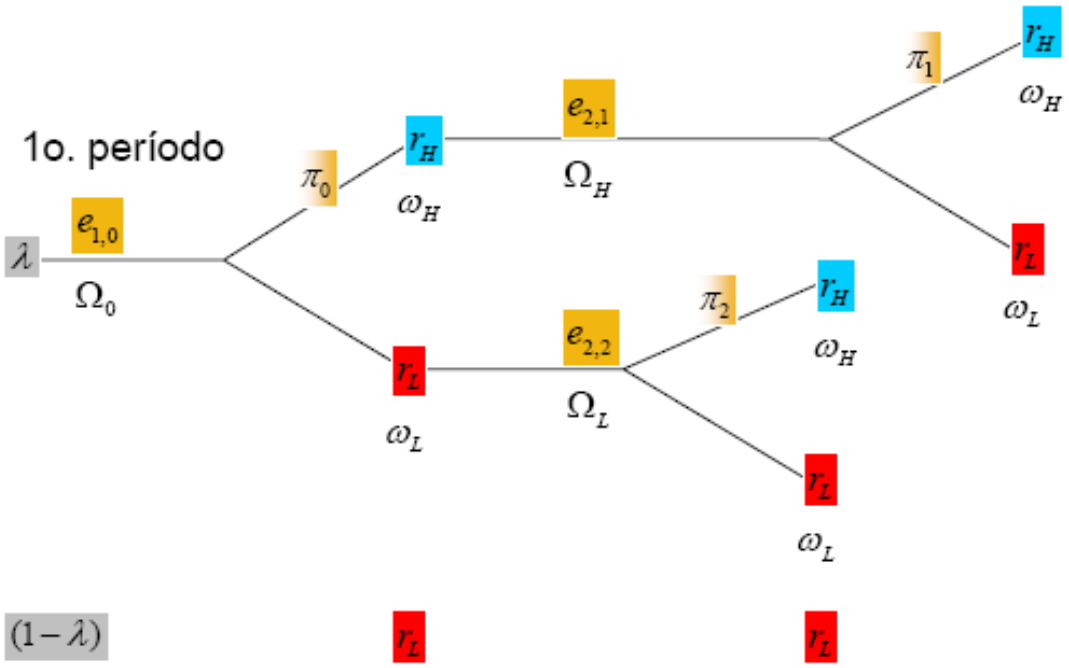
and

$$P(\underline{\eta}|r_L) = \frac{P(\underline{\eta}) P(r_L|\underline{\eta})}{P(\overline{\eta}) P(r_L|\overline{\eta}) + P(\underline{\eta}) P(r_L|\underline{\eta})}$$

The decision tree now comes with a line in the inferior part of the graph that represents the payoffs of the bad manager with the assumption that he never

[12] This idea was first introduced by Holmstrom (1982a).

performs positively.



3.1 The portfolio manager problem: optimal choice of effort

When the observation of return in the first period reveals hiss type, the portfolio manager solves

$$\begin{aligned}
\max_{e_0,e_1,e_2} U_M &= \sum_{t=0}^{2} \delta^t \left[\sum_{s_t=\underline{r}_t}^{s_t=\overline{r}_t} P\left(r_t|e_t\right) u\left(\Omega_{t,s}\omega_{t,s}\right) - \frac{k}{2}\left(e_{t,s}\right) \right] \qquad (30) \\
&= \pi_{1,0} u\left(\Omega_0\omega_H\right) + \left(1-\pi_{1,0}\right) u\left(\Omega_0\omega_L\right) - \frac{k}{2}\left(e_{1,0}\right)^2 \\
&\quad +\delta \left\{ \begin{array}{c} \pi_{1,0} \left[\begin{array}{c} \pi_{2,1} u\left(\Omega_H\omega_H\right) + \left(1-\pi_{2,1}\right) u\left(\Omega_H\omega_L\right) \\ -\frac{k}{2}\left(e^i_{2,1}\right)^2 \end{array} \right] \\ +\left(1-\pi_{1,0}\right) \left[\begin{array}{c} \pi_{2,2} u\left(\Omega_L\omega_H\right) + \left(1-\pi_{2,2}\right) u\left(\Omega_L\omega_L\right) \\ -\frac{k}{2}\left(e^i_{2,2}\right)^2 \end{array} \right] \end{array} \right\}
\end{aligned}$$

We consider the simplifying assumption that the bad manager never generate positive excess return, that is, $P\left(r_H|\underline{\eta}\right) = 0$ and $P\left(r_L|\underline{\eta}\right) = 1$. Then, the probabilities of high return in each node are given by

$$\pi_{1,0} = \lambda \left(\frac{\exp\left(a_{1,0} + b_{1,0} e_{1,0}\right)}{1 + \exp\left(a_{1,0} + b_{1,0} e_{1,0}\right)} \right)$$

$$\pi_{2,1} = \left(\frac{\exp\left(a_{2,1} + b_{2,1} e_{2,1}\right)}{1 + \exp\left(a_{2,1} + b_{2,1} e_{2,1}\right)} \right) P\left(\overline{\eta} | r_H\right) = \left(\frac{\exp\left(a_{2,1} + b_{2,1} e_{2,1}\right)}{1 + \exp\left(a_{2,1} + b_{2,1} e_{2,1}\right)} \right)$$

$$\pi_{2,2} = \left(\frac{\exp\left(a_{2,2} + b_{2,2} e_{2,2}\right)}{1 + \exp\left(a_{2,2} + b_{2,2} e_{2,2}\right)} \right) P\left(\overline{\eta} | r_L\right) = \left(\frac{\exp\left(a + b e_{2,2}^{i}\right)}{1 + \exp\left(a + b e_{2,2}^{i}\right)} \right) \left(\frac{\lambda\left(1 - \pi_{1,0}\right)}{1 - \lambda \pi_{1,0}} \right)$$

The reservation utility of the portfolio manager is exogenously given and is equal to $\overline{U_M}$. Again, the investor has all bargaining power and can make take-it-or-leave-it offers to the portfolio manager subject to providing him with an expected payoff which yields at least $\overline{U_M}$. Normalizing $\Omega_0 = 1$, the first-order conditions of the manager's problem are given by

$$\frac{\partial U_M}{\partial e_{1,0}^*} = A\left(m_f, p_f, \Omega_H, \Omega_L\right) \tag{31}$$

$$\frac{\partial U_M}{\partial e_{2,1}^*} = E\left(m_f, p_f, \Omega_H\right) \tag{32}$$

$$\frac{\partial U_M}{\partial e_{2,2}^*} = I\left(m_f, p_f, \Omega_L\right) \tag{33}$$

Observe that the optimal effort choice in the bad state of nature in the second period depend on the optimal effort choice in the first period. Calculating explicit expressions for $e_{1,0}^*$ and $e_{2,2}^*$ becomes algebraically intractable and the numerical solution also yields optimal effort choices. Given the Bayesian adjustment of posteriors, we know that $e_{1,0}^* > e_{2,2}^*$ and that $e_{2,1}^* > e_{2,2}^*$. However, we can not say anything about the relation ship between $e_{1,0}^*$ and $e_{2,1}^*$. As the numerical results show, depending on the parameter values the difference between them may have any sign.

3.2 The investor problem: optimal provision of incentives

Now, the risk-neutral investor solves

$$\begin{aligned}
\max_{\substack{m_f, p_f, \Omega_H, \Omega_L, \\ e_0, e_1, e_2}} U_I \quad = \quad & \pi_0 \Omega_0 \left(r_H - m_f - p_f \max\left[r_H - r_b, 0\right]\right) && (34) \\
& + \left(1 - \pi_0\right) \Omega_0 \left(r_L - m_f - p_f \max\left[r_L - r_b, 0\right]\right) && (35) \\
& + \delta \pi_0 \left[\begin{array}{c} \pi_1 \Omega_H \left(r_H - m_f - p_f \max\left[r_H - r_b, 0\right]\right) \\ + \left(1 - \pi_1\right) \Omega_H \left(r_L - m_f - p_f \max\left[r_L - r_b, 0\right]\right) \\ - \left(\Omega_H - \Omega_0 * \left(1 + r_H\right)\right) \overline{r_b} \end{array} \right] \\
& + \delta \left(1 - \pi_0\right) \left[\begin{array}{c} \pi_2 \Omega_L \left(r_H - m_f - p_f \max\left[r_H - r_b, 0\right]\right) \\ + \left(1 - \pi_2\right) \Omega_L \left(r_L - m_f - p_f \max\left[r_L - r_b, 0\right]\right) \\ - \left(\Omega_L - \Omega_0 * \left(1 + r_L\right)\right) \overline{r_b} \end{array} \right]
\end{aligned}$$

subject to the following constraints. We normalize the reservation utility to zero in each node and write the participation constraint as

$$U_M \geq 0 \tag{36}$$

An incentive compatible contract offered by the investor also satisfies the incentive compatibility constraints

$$e_0, e_1, e_2 \in \arg\max \quad \sum_{t=0}^{2} \delta^t \left[\sum_{s=1}^{2} P\left(r_t | e_t\right) u\left(\Omega_{t,s} m_f + p_f \max\left[r_{t,s} - r_b, 0\right]\right) - \frac{k}{2}\left(e_{t,s}\right)^2 \right] \tag{37}$$

The manager has limited liability in excess return and can only be penalized for exerting low levels of effort through the implicit incentive, reducing the total compensation in the second-period. Then, it is necessary to write two limited responsibility constraints for the explicit incentives such that

$$m_f > 0 \tag{38}$$

$$p_f \geq 0 \tag{39}$$

Since it is neither possible to borrow resources from the manager's fund nor to leverage positions in the fund by borrowing at the benchmark rate, there are also two short-selling constraints for the implicit incentives such that

$$0 \leq \Omega_H \leq 1 \tag{40}$$

$$0 \leq \Omega_L \leq 1 \tag{41}$$

The equilibrium solution $\left\{m_f^*, p_f^*, \Omega_H^*, \Omega_L^*, e_0^*, e_1^*, e_2^*\right\}$ is algebraically intractable and can only have a numerical solution.

Observe that the investor provides incentives in order to maximize expected utility as he learns about the manager's type. For all $\lambda < \overline{\lambda}$, it is optimal to offer full distortion in the implicit incentive structure. That is, for a particular belief about the percentage of bad managers in the economy and below this level, there is no cost in providing full distortion in the implicit incentive, i.e., when performance is poor in the first period, withdrawing all resources from the fund can be done without any cost.

3.3 Characterization of the optimal incentive contract

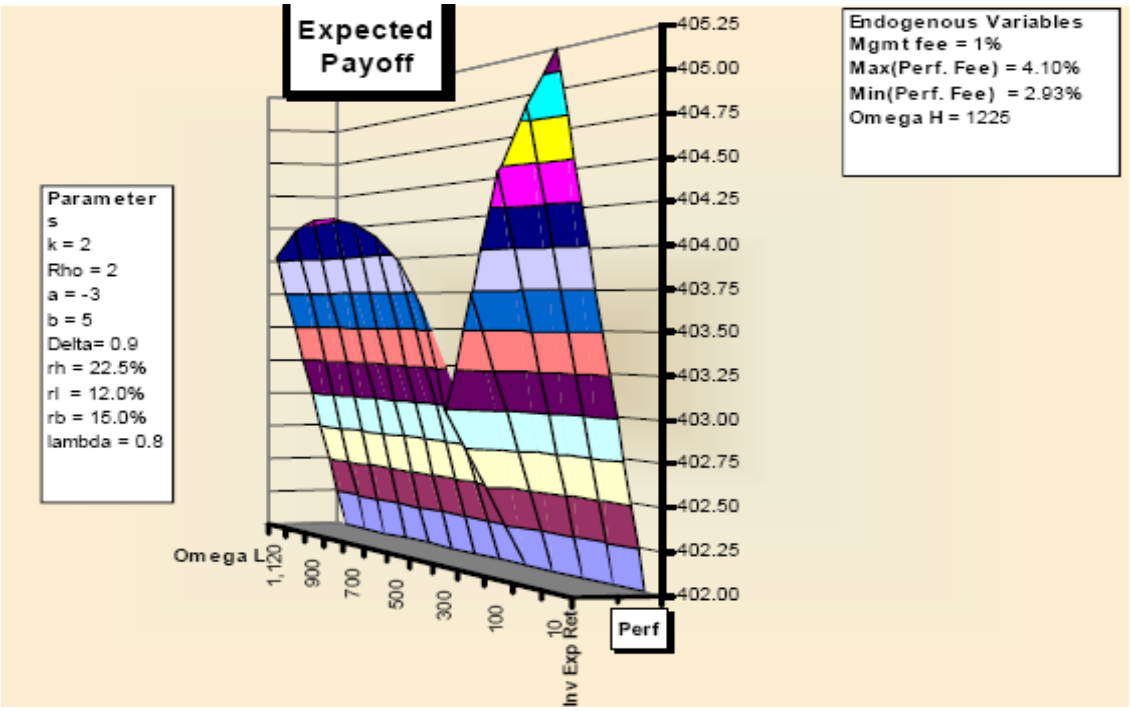
In equilibrium, the investor offers an incentive compatible contract $\left\{m_f^*, p_f^*, \Omega_H^*, \Omega_L^*\right\}$ that satisfies all the constraints of his problem. He also chooses $\{e_0^*; e_1^*; e_2^*\}$ that satisfy the incentive constraints. The investor provides total incentives that equalize the marginal excess return and the implied costs of effort induction. He does so by simultaneously combining and distorting both the implicit and the explicit incentive's compensation structure as to maximize the intertemporal excess expected return.

3.4 Numerical results

The computer codes are presented in subsection 4 of the Appendix. We execute the same procedure than the one described in the first model in Section 2.

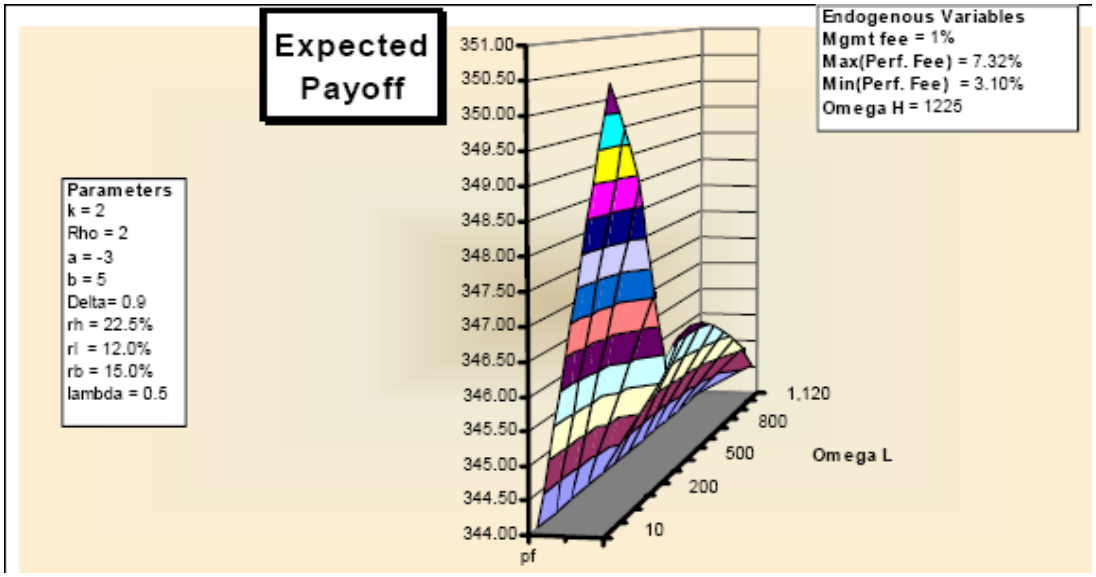
However, we build a performance fee schedule that uses the concept of the high water mark, that is, performance in the second period is only due to the excess return that compensates for the losses in the first period.

Consider each case below for different set of parameters as shown in the graphs and tables. They show that performance fee and expected payoff present a non-linear relationship according to different levels of implicit incentives, differently than the previous model. The complementary relationship between the explicit (performance fee) and the implicit incentive (Omega) remains in the model with learning. The performance fee always needs to go up, in order to compensate a more powerful implicit incentive does decrease the expected payoff. And again, the trade-off between expected return in the first period vis-à-vis expected return in the second period shows the power of the implicit incentive and its importance as a component of the contract since it recovers the relationship history.



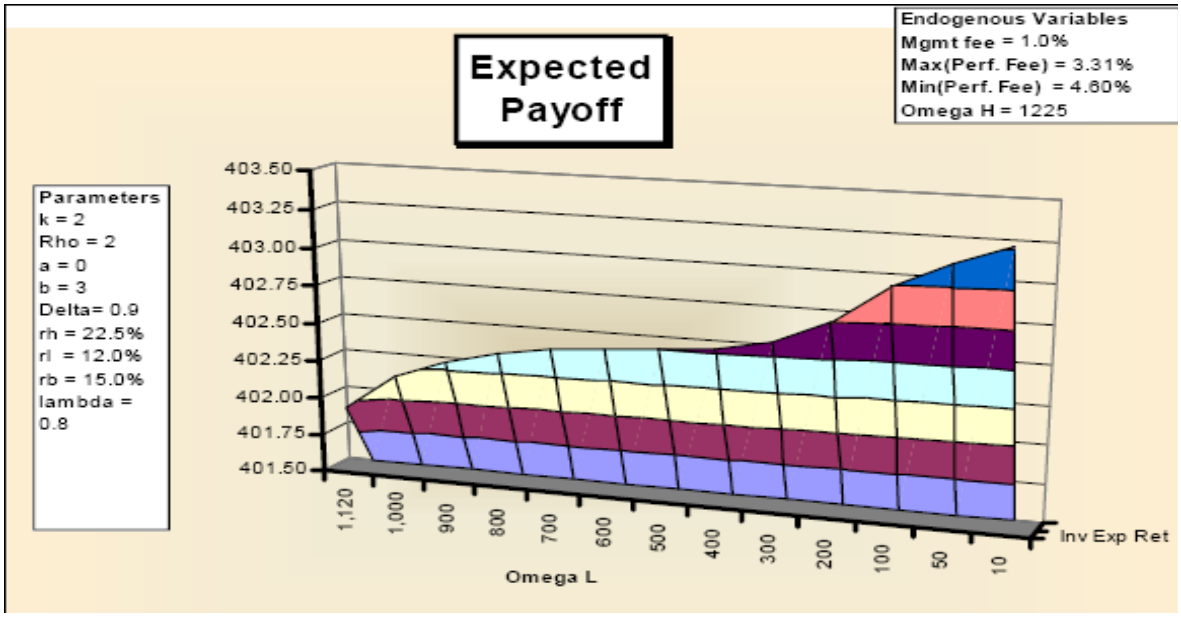
pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
4.03%	1,120	1.36	1.32	1.05	0.7825	0.9739	0.4216	404.024	134.774%	148.175%	109.513%
3.91%	1,000	1.39	1.32	1.01	0.7847	0.9731	0.4102	404.205	134.932%	148.120%	108.715%
3.80%	900	1.41	1.31	0.97	0.7861	0.9725	0.3989	404.271	135.027%	148.072%	107.922%
3.75%	800	1.42	1.31	0.93	0.7872	0.9721	0.3854	404.276	135.107%	148.044%	106.976%
3.73%	700	1.44	1.31	0.88	0.7882	0.9719	0.3682	404.229	135.174%	148.036%	105.776%
3.76%	600	1.45	1.31	0.82	0.7890	0.9721	0.3450	404.127	135.231%	148.048%	104.148%
3.84%	500	1.47	1.31	0.75	0.7898	0.9727	0.3107	403.955	135.283%	148.091%	101.752%
4.10%	400	1.49	1.33	0.65	0.7906	0.9743	0.2560	403.649	135.339%	148.204%	97.923%
3.05%	300	1.47	1.27	0.08	0.7896	0.9662	0.0319	402.981	135.275%	147.631%	82.233%
2.98%	200	1.47	1.27	0.04	0.7899	0.9653	0.0256	403.717	135.291%	147.574%	81.794%
2.94%	100	1.48	1.26	0.01	0.7901	0.9650	0.0228	404.464	135.306%	147.550%	81.598%
2.93%	50	1.48	1.26	0.00	0.7902	0.9649	0.0220	404.834	135.311%	147.542%	81.543%
2.93%	10	1.48	1.26	0.00	0.7902	0.9648	0.0217	405.127	135.313%	147.538%	81.517%

Now, a higher probability of bad managers in the economy decreases expected payoff while it increases the power of the contract. more performance fee needs to be paid and the powerful implicit incentive has a greater impact in generating endogenous return.



pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
5.37%	1,120	1.33	1.38	1.05	0.4874	0.9802	0.3066	344.598	114.121%	148.611%	101.462%
5.22%	1,000	1.35	1.37	1.01	0.4886	0.9796	0.2996	345.059	114.203%	148.571%	100.974%
5.16%	900	1.37	1.37	0.97	0.4895	0.9794	0.2927	345.388	114.263%	148.556%	100.490%
5.16%	800	1.38	1.37	0.93	0.4902	0.9794	0.2843	345.670	114.316%	148.555%	99.898%
5.21%	700	1.40	1.37	0.89	0.4909	0.9796	0.2735	345.905	114.363%	148.569%	99.142%
5.33%	600	1.41	1.38	0.84	0.4915	0.9800	0.2589	346.084	114.406%	148.600%	98.125%
5.57%	500	1.43	1.39	0.78	0.4921	0.9808	0.2382	346.183	114.448%	148.657%	96.674%
6.05%	400	1.44	1.40	0.69	0.4928	0.9823	0.2068	346.134	114.495%	148.760%	94.475%
7.32%	300	1.48	1.44	0.58	0.4938	0.9853	0.1595	345.691	114.565%	148.970%	91.164%
3.18%	200	1.39	1.28	0.03	0.4906	0.9674	0.0182	347.070	114.345%	147.720%	81.274%
3.12%	100	1.40	1.27	0.01	0.4908	0.9669	0.0167	348.851	114.356%	147.682%	81.167%
3.11%	50	1.40	1.27	0.00	0.4909	0.9667	0.0162	349.741	114.360%	147.670%	81.136%
3.10%	10	1.40	1.27	0.00	0.4909	0.9666	0.0160	350.451	114.362%	147.665%	81.121%

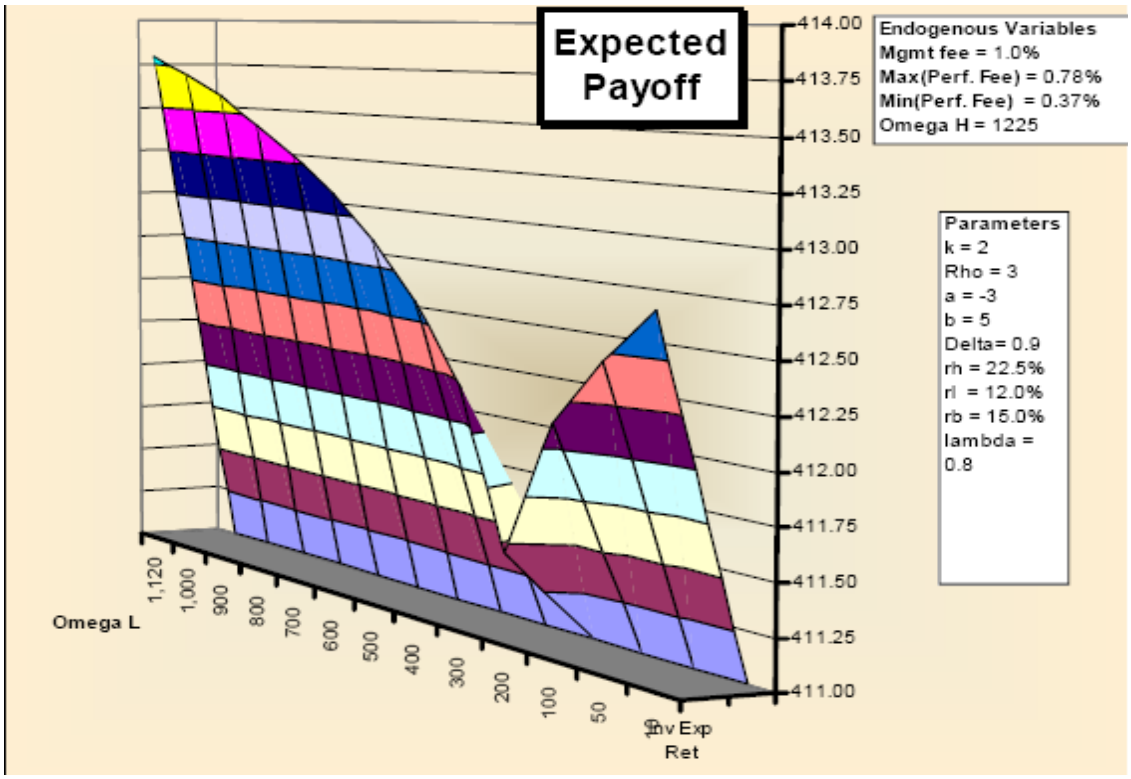
Now, we vary the parameters of the logistic function and observe that for given greater a increases probability and even in this case, greater implicit incentives increase expected payoff since less performance fee is needed to generate more effort.



pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
4.60%	1,120	1.18	1.13	0.75	0.7773	0.9675	0.4260	401.940	134.414%	147.728%	109.818%
4.34%	1,000	1.21	1.11	0.69	0.7793	0.9659	0.4157	402.178	134.554%	147.611%	109.096%
4.17%	900	1.23	1.10	0.63	0.7808	0.9648	0.4063	402.307	134.653%	147.533%	108.439%
4.05%	800	1.26	1.09	0.58	0.7820	0.9639	0.3957	402.390	134.738%	147.470%	107.702%
3.95%	700	1.28	1.09	0.52	0.7830	0.9631	0.3835	402.440	134.809%	147.416%	106.844%
3.86%	600	1.29	1.08	0.45	0.7838	0.9624	0.3687	402.468	134.867%	147.366%	105.809%
3.77%	500	1.31	1.07	0.38	0.7845	0.9616	0.3503	402.488	134.913%	147.313%	104.524%
3.66%	400	1.32	1.07	0.29	0.7850	0.9607	0.3273	402.518	134.949%	147.250%	102.910%
3.54%	300	1.33	1.06	0.20	0.7853	0.9596	0.2994	402.586	134.974%	147.171%	100.960%
3.41%	200	1.33	1.05	0.11	0.7856	0.9584	0.2698	402.733	134.991%	147.086%	98.888%
3.33%	100	1.34	1.04	0.04	0.7858	0.9575	0.2446	402.977	135.005%	147.027%	97.123%
3.31%	50	1.34	1.04	0.01	0.7859	0.9573	0.2356	403.130	135.011%	147.014%	96.490%
3.31%	10	1.34	1.04	0.00	0.7859	0.9573	0.2311	403.260	135.014%	147.010%	96.176%

Below, we see that risk aversion creates opportunities for less powerful contracts. The manager will exert higher effort if his utility is smoothed along time and states of nature. For a fixed fee, "he sells the management to the investor". It is a good explanation for low powered contracts that are very regulated. The discretionary power of the manager will be limited since it is very costly to the

manager to run risks to obtain performance fee.



pf	oml	e0	e1	e2	Prob 0	Prob 1	Prob 2	Inv Exp Ret	E [r0exc] (% cdi)	E [r1exc] (% cdi)	E [r2exc] (% cdi)
0.47%	1,120	1.85	1.69	1.40	0.7985	0.9956	0.4384	413.790	135.894%	149.693%	110.687%
0.46%	1,000	1.94	1.68	1.34	0.7990	0.9955	0.4351	413.702	135.931%	149.687%	110.458%
0.47%	900	1.98	1.68	1.30	0.7992	0.9956	0.4321	413.582	135.945%	149.692%	110.246%
0.48%	800	2.01	1.69	1.24	0.7993	0.9957	0.4281	413.440	135.953%	149.700%	109.968%
0.50%	700	2.04	1.70	1.18	0.7994	0.9959	0.4225	413.280	135.957%	149.711%	109.573%
0.53%	600	2.05	1.71	1.12	0.7994	0.9961	0.4139	413.098	135.961%	149.725%	108.972%
0.57%	500	2.07	1.72	1.04	0.7995	0.9963	0.3999	412.884	135.963%	149.743%	107.993%
0.64%	400	2.08	1.74	0.93	0.7995	0.9967	0.3748	412.612	135.965%	149.769%	106.234%
0.78%	300	2.09	1.78	0.80	0.7995	0.9973	0.3234	412.214	135.967%	149.808%	102.639%
0.38%	200	2.07	1.65	0.03	0.7995	0.9947	0.0249	411.451	135.964%	149.628%	81.741%
0.37%	100	2.07	1.64	0.00	0.7995	0.9946	0.0216	412.120	135.964%	149.621%	81.511%
0.37%	50	2.07	1.64	0.00	0.7995	0.9946	0.0212	412.460	135.964%	149.620%	81.483%
0.37%	10	2.07	1.64	0.00	0.7995	0.9946	0.0211	412.733	135.964%	149.620%	81.478%

4 Conclusions

The existence of optimal contracts with powerful implicit incentives is the most relevant result presented here. This power arises from the fact that, given the typical contract's characteristics, the flow of funds represents a less conflictive mechanism designed by the investor to induce the portfolio manager to exert higher levels of effort. While the implied cost of using explicit incentives reduce net expected return directly, the implicit incentive only affects the investor objective function when the benchmark return is greater than the endogenous expected return obtained by the portfolio manager. Besides, risk aversion explains complementary incentives. The performance fee is greater in the case the implicit incentive is less powerful.

The power of the flow of funds might also be an explanation for simple and incomplete explicit incentives. Indeed, implicit incentives' power might complement simple explicit incentives, given the general conditions encountered in the marketplace. Or we may say, powerful implicit incentives may correct some nuisances created by simple and incomplete linear explicit incentives that are detrimental to efficient risk choices executed by the portfolio manager.

However, it does not arise as an important incentive response without a relevant implied cost. First, expected returns are endogenous to effort provision. Second, the trade-off between incentives and performance may be so costly that it even represents a non-credible threat when the portfolio allocation decision is different than the usual solution without any intertemporal incentives consideration, i.e., full allocation if expected excess return is positive.

More importantly, powerful implicit incentives may negatively affect the portfolio manager's ability to take risks since the implied uncertainty of highly volatile flow of funds creates incentives to myopic investing. This greater income uncertainty reduces the utility of a risk averse manager and may lead to an increase in the likelihood of "closet indexing" of the fund when past excess return is positive and asset under management grows. On the other hand, it may also increase the likelihood of excessive risk-taking when past excess return is negative and the flow of fund's expected punishment may lead to all-or-nothing bets.

Rational investors should be "forward looking" decision makers. Since the investor can not observe effort executed by the manager, moral hazard issues arise and, hence, "backward looking strategies" maximize expected return. This result may explain an empirical regularity found in the asset management industry that seems to be unreasonable and inconsistent, once past return may not be indicative of future return.

If powerful implicit incentives raise flow concerns that are detrimental to optimal effort and risk-taking behavior, it would be desirable to spend time and resources in the designing of somewhat complex explicit incentives clauses that internalize the history of returns as well as pre-defined variables like investor and portfolio manager's investment profiles and objectives.

For instance, it might make sense to build a compensation structure that depend less on the total volume under management and design a mechanism in

which total incentives are more dependent on performance with a more powerful explicit incentive. The investor should compensate current and future performance rather than past performance to guarantee that he seizes all the possible benefits of the dynamic relationship in an asset management contract. Based on the comparison of the optimal performance fee given the power of implicit incentives, the investor could offer to risk averse managers contracts with lower implicit incentives; i.e., lock-up periods for the invested funds in exchange for a lower performance fee.

Possible extensions and generalizations to this work would allow for a continuum of states and more than two periods. Also, a further analysis of investment strategies alternatives given possible functional forms of the flow-performance relationship.

5 Appendix

5.1 Computer code for the binomial model with full commitment

5.1.1 Portfolio Manager Utility function

```
function resp = u(arg1,arg2,arg3,arg4,arg5)
    resp = ((arg1*arg2+arg1*arg3*arg4)^(arg5-1/arg5))/(1/(arg5-1));
```

5.1.2 Investor Utility function

```
function resp = vi(arg1,arg2,arg3,arg4,arg5)
    resp = arg1*(arg2-arg4-arg3*arg5);
```

5.1.3 Input values for model's parameter

```
% Parameters
    k = 2;
    a = -3;
    b = 5;
    delta = 0.8;
    rho = 2;
    om0 = 1000;
    cdi = 0.15;
    days = 252;
    cdiperch = 1.5;
    cdipercl = 0.8;
    rh = cdiperch*((1+cdi)^(days/252)-1);
    rl = cdipercl*((1+cdi)^(days/252)-1);
    rb = ((1+cdi)^(days/252)-1);
    if (rh-rb)>0;
    erh=rh-rb;
    else erh=0;
    end
    if (rl-rb)>0;
    erl=rl-rb;
    else erl=0;
    end
```

5.1.4 Investor's Maximization problem objective function and its gradient

```
function [f,GG] = probinv(x);
    tese_input;
    syms mf pf omh oml rho e0 e1 e2;
    % Investor utility function
```

```
F=vi(om0,rh,erh,mf,pf);
G=vi(om0,rl,erl,mf,pf);
H=vi(omh,rh,erh,mf,pf);
K=vi(omh,rl,erl,mf,pf);
M=vi(oml,rh,erh,mf,pf);
N=vi(oml,rl,erl,mf,pf);
% Probabilities of high return using optimal effort functions
A = exp(a+b*e0)/(1+exp(a+b*e0));
E = exp(a+b*e1)/(1+exp(a+b*e1));
I = exp(a+b*e2)/(1+exp(a+b*e2));
% Investor objective function
f = -(A*F+(1-A)*G-vi(om0-om0,rb,0,0,0)
+delta*A*(E*H+(1-E)*K-vi(omh-(om0*(1+rh)),rb,0,0,0))
+delta*(1-A)*(I*M+(1-I)*N-vi(oml-(om0*(1+rl)),rb,0,0,0)));
% Partial Derivatives
d_f_mf = diff(f,sym('mf'));
d_f_pf = diff(f,sym('pf'));
d_f_omh = diff(f,sym('omh'));
d_f_oml = diff(f,sym('oml'));
d_f_e0 = diff(f,sym('e0'));
d_f_e1 = diff(f,sym('e1'));
d_f_e2 = diff(f,sym('e2'));
% Gradient
GG = [d_f_mf d_f_pf d_f_omh d_f_oml d_f_e0 d_f_e1 d_f_e2];
% Solution values and parameters
mf = x(1);
pf = x(2);
omh = x(3);
oml = x(4);
e0 = x(5);
e1 = x(6);
e2 = x(7);
tese_input;
% Objective function and gradient evaluated at solution x
f = eval(f);
GG = eval(GG);
```

5.1.5 Investor's Maximization problem constraints and their derivatives

```
function [c,ceq,GC,Geq] = constr(x)
    tese_input;
    syms mf pf omh oml rho e0 e1 e2;
    % Portfolio Manager utility function
    P = u(om0,mf,pf,erh,rho);
    Q = u(om0,mf,pf,erl,rho);
```

```
S = u(omh,mf,pf,erh,rho);
T = u(omh,mf,pf,erl,rho);
Vu = u(oml,mf,pf,erh,rho);
W = u(oml,mf,pf,erl,rho);
% Probabilities of high return using optimal effort functions
A = exp(a+b*e0)/(1+exp(a+b*e0));
E = exp(a+b*e1)/(1+exp(a+b*e1));
I = exp(a+b*e2)/(1+exp(a+b*e2));
% Portfolio manager objective function
PM = A*P+(1-A)*Q-(e0^2)*(k/2)
+delta*A*(E*S+(1-E)*T-(e1^2)*(k/2))
+delta*(1-A)*(I*Vu+(1-I)*W-(e2^2)*(k/2));
% CPO´s of the portfolio manager with respect to optimal choice of incentives
d_PM_e0=diff(PM,sym('e0'));
d_PM_e1=diff(PM,sym('e1'));
d_PM_e2=diff(PM,sym('e2'));
% Participation constraint and efforts' upper and lower bounds constraints
c=-(A*P+(1-A)*Q-(e0^2)*(k/2)
+delta*A*(E*S+(1-E)*T-(e1^2)*(k/2))
+delta*(1-A)*(I*Vu+(1-I)*W-(e2^2)*(k/2)));
c0=d_PM_e0-0.000000001;
c1=d_PM_e1-0.000000001;
c2=d_PM_e2-0.000000001;
c3=-d_PM_e0-0.000000001;
c4=-d_PM_e1-0.000000001;
c5=-d_PM_e2-0.000000001;
% Partial derivatives of the constraints
d_c_mf=diff(c,sym('mf'));
d_c_pf=diff(c,sym('pf'));
d_c_omh=diff(c,sym('omh'));
d_c_oml=diff(c,sym('oml'));
d_c_e0=diff(c,sym('e0'));
d_c_e1=diff(c,sym('e1'));
d_c_e2=diff(c,sym('e2'));
d_c0_mf=diff(c0,sym('mf'));
d_c0_pf=diff(c0,sym('pf'));
d_c0_omh=diff(c0,sym('omh'));
d_c0_oml=diff(c0,sym('oml'));
d_c0_e0=diff(c0,sym('e0'));
d_c0_e1=diff(c0,sym('e1'));
d_c0_e2=diff(c0,sym('e2'));
d_c1_mf=diff(c1,sym('mf'));
d_c1_pf=diff(c1,sym('pf'));
d_c1_omh=diff(c1,sym('omh'));
d_c1_oml=diff(c1,sym('oml'));
```

```
d_c1_e0=diff(c1,sym('e0'));
d_c1_e1=diff(c1,sym('e1'));
d_c1_e2=diff(c1,sym('e2'));
d_c2_mf=diff(c2,sym('mf'));
d_c2_pf=diff(c2,sym('pf'));
d_c2_omh=diff(c2,sym('omh'));
d_c2_oml=diff(c2,sym('oml'));
d_c2_e0=diff(c2,sym('e0'));
d_c2_e1=diff(c2,sym('e1'));
d_c2_e2=diff(c2,sym('e2'));
d_c3_mf=diff(c3,sym('mf'));
d_c3_pf=diff(c3,sym('pf'));
d_c3_omh=diff(c3,sym('omh'));
d_c3_oml=diff(c3,sym('oml'));
d_c3_e0=diff(c3,sym('e0'));
d_c3_e1=diff(c3,sym('e1'));
d_c3_e2=diff(c3,sym('e2'));
d_c4_mf=diff(c4,sym('mf'));
d_c4_pf=diff(c4,sym('pf'));
d_c4_omh=diff(c4,sym('omh'));
d_c4_oml=diff(c4,sym('oml'));
d_c4_e0=diff(c4,sym('e0'));
d_c4_e1=diff(c4,sym('e1'));
d_c4_e2=diff(c4,sym('e2'));
d_c5_mf=diff(c5,sym('mf'));
d_c5_pf=diff(c5,sym('pf'));
d_c5_omh=diff(c5,sym('omh'));
d_c5_oml=diff(c5,sym('oml'));
d_c5_e0=diff(c5,sym('e0'));
d_c5_e1=diff(c5,sym('e1'));
d_c5_e2=diff(c5,sym('e2'));
% Gradient
GC = [d_c_mf d_c_pf d_c_omh d_c_oml d_c_e0 d_c_e1 d_c_e2;
d_c0_mf d_c0_pf d_c0_omh d_c0_oml d_c0_e0 d_c0_e1 d_c0_e2;
d_c1_mf d_c1_pf d_c1_omh d_c1_oml d_c1_e0 d_c1_e1 d_c1_e2;
d_c2_mf d_c2_pf d_c2_omh d_c2_oml d_c2_e0 d_c2_e1 d_c2_e2;
d_c3_mf d_c3_pf d_c3_omh d_c3_oml d_c3_e0 d_c3_e1 d_c3_e2;
d_c4_mf d_c4_pf d_c4_omh d_c4_oml d_c4_e0 d_c4_e1 d_c4_e2;
d_c5_mf d_c5_pf d_c5_omh d_c5_oml d_c5_e0 d_c5_e1 d_c5_e2];
% Equality constraints and their gradient
ceq=[];
Geq=[];
% Solution values
mf=x(1);
pf=x(2);
omh=x(3);
```

```
oml=x(4);
e0=x(5);
e1=x(6);
e2=x(7);
tese_input;
% Constraints and the gradient evaluated at solution x
c=[eval(c) eval(c0) eval(c1) eval(c2) eval(c3) eval(c4) eval(c5)];
GC = eval(GC');
```

5.1.6 Checking sufficiency conditions for local optima of the portfolio manager's problem

```
function h = hess_sign(x);
    syms mf pf omh oml rho e0 e1 e2;
    tese_input;
    % Optimal solution derived in the tese_opt m.file using the fmincom fucntion
    mf = x(1);
    pf = x(2);
    omh = x(3);
    oml = x(4);
    e0 = x(5);
    e1 = x(6);
    e2 = x(7);
    % Portfolio Manager utility function
    P = u(om0,mf,pf,erh,rho);
    Q = u(om0,mf,pf,erl,rho);
    S = u(omh,mf,pf,erh,rho);
    T = u(omh,mf,pf,erl,rho);
    Vu = u(oml,mf,pf,erh,rho);
    W = u(oml,mf,pf,erl,rho);
    % Probabilities of high return using optimal effort functions
    A = exp(a+b*e0)/(1+exp(a+b*e0));
    E = exp(a+b*e1)/(1+exp(a+b*e1));
    I = exp(a+b*e2)/(1+exp(a+b*e2));
    % Portfolio manager objective function
    PM = A*P+(1-A)*Q-(e0^2)*(k/2)
    +delta*A*(E*S+(1-E)*T-(e1^2)*(k/2))
    +delta*(1-A)*(I*Vu+(1-I)*W-(e2^2)*(k/2));
    % CPO´s of the portfolio manager with respect to optimal choice of incentives
    d_PM_e0=diff(PM,sym('e0'));
    d_PM_e1=diff(PM,sym('e1'));
    d_PM_e2=diff(PM,sym('e2'));
    d_PM_e0e0=diff(d_PM_e0,sym('e0'));
    d_PM_e0e1=diff(d_PM_e0,sym('e1'));
    d_PM_e0e2=diff(d_PM_e0,sym('e2'));
```

```
d_PM_e1e0=diff(d_PM_e1,sym('e0'));
d_PM_e1e1=diff(d_PM_e1,sym('e1'));
d_PM_e1e2=diff(d_PM_e1,sym('e2'));
d_PM_e2e0=diff(d_PM_e2,sym('e0'));
d_PM_e2e1=diff(d_PM_e2,sym('e1'));
d_PM_e2e2=diff(d_PM_e2,sym('e2'));
% Hessian matrix of the portfolio manager problem evaluated at solution x
HH = [d_PM_e0e0, d_PM_e0e1, d_PM_e0e2;
d_PM_e1e0, d_PM_e1e1, d_PM_e1e2; d_PM_e2e0,
d_PM_e2e1, d_PM_e2e2];
diag(eig(eval(HH)));
% Signals of eigenvalues of the hessian matrix
S = sign(diag(eig(eval(HH))));
% Counting the number of negatives and positive values
% positive eigenvalues
P=0;
% negative eigenvalues
N=0;
for i = 1: max(size(S))
if S(i,i)==1
P = P + 1;
else N = N + 1;
end
end
if P*N>0
h=0;
else if P==0
h=-1;
else h=1;
end
end
```

5.1.7 Non-linear Minimization problem function and print

```
function [best_result, ret_best_resul] = tese_opt;
    tese_input;
    syms mf pf omh oml rho e0 e1 e2;
    % Lower bounds x >= 0
    lb = [0.01,0,om0*(1+rh),10,0,0,0];
    % Upper bounds
    ub = [lb(1),0.4,lb(3),lb(4),5,5,5];
    % Specifing the optimization code features
    options = optimset('Display','iter','GradConstr','on','GradObj','on',
    'Hessian','on','Jacobian','on','MaxIter',20,'MaxFunEvals',100,'TolFun',1e-10,'TolCon',1e-
10);
    format long
```

```
% Initial guess values of the investor's control variables
mf = ub(1);
pf = ub(2);
omh = ub(3);
oml = ub(4);
e0 = 1.2;
e1 = 1.2;
e2 = 1.2;
x0 = [mf pf omh oml e0 e1 e2];
% Optimization function
[x,fval,EXITFLAG,OUTPUT,LAMBDA,GRAD,HESSIAN] =
= fmincon(@probinv,x0,[],[],[],[],lb,ub,@constr,options)
% Solution values
mf = x(1);
pf = x(2);
omh = x(3);
oml = x(4);
e0 = x(5);
e1 = x(6);
e2 = x(7);
tese_input;
% Portfolio Manager utility function
P = u(om0,mf,pf,erh,rho);
Q = u(om0,mf,pf,erl,rho);
S = u(omh,mf,pf,erh,rho);
T = u(omh,mf,pf,erl,rho);
Vu = u(oml,mf,pf,erh,rho);
W = u(oml,mf,pf,erl,rho);
% Probabilities of high return using optimal effort functions
A = exp(a+b*e0)/(1+exp(a+b*e0));
E = exp(a+b*e1)/(1+exp(a+b*e1));
I = exp(a+b*e2)/(1+exp(a+b*e2));
% Calculating output variables and functions
if (rh-rb)>0;
pfeeh=(pf)*(252/days);
else pfeeh=0;
end
if (rl-rb)>0;
pfeel=(pf)*(252/days);
else pfeel=0;
end
r0=rh*A+rl*(1-A);
r0exc=r0-rb;
Var0=(A)*(1-A)*(rh-rl)^2;
r1=rh*E+rl*(1-E);
r1exc=r1-rb;
```

```
Var1=E*(1-E)*(rh-rl)^2;
r2=rh*I+rl*(1-I);
r2exc=r2-rb;
Var2=I*(1-I)*(rh-rl)^2;
% Output print routine
parameters = [k a b delta rho cdi days cdiperch cdipercl om0]'
x0'
ret_best_result = [pfeeh pfeel r0exc r1exc r2exc r0 Var0 r1 Var1 r2 Var2]'
[c,ceq,GC,Geq] = constr(x);
result_iter(1:7) = x(1:7);
result_iter(8) = A;
result_iter(9) = E;
result_iter(10) = I;
result_iter(11) = -fval;
result_iter(12:18) = -c(1:7);
best_result = result_iter'
hess_sign(x)
% Saving minimization problem's results
save 'resultados_one' parameters best_result ret_best_result
```

5.2 Computer code for the binomial model with learning

5.2.1 Portfolio Manager Utility function

```
function resp = u(arg1,arg2,arg3,arg4,arg5)
    resp = ((arg1*arg2+arg1*arg3*arg4)^(arg5-1/arg5))/(1/(arg5-1));
```

5.2.2 Investor Utility function

```
function resp = vi(arg1,arg2,arg3,arg4,arg5)
    resp = arg1*(arg2-arg4-arg3*arg5);
```

5.2.3 Input values for model's parameter

```
% Parameters
    lambda = 0.8;
    k = 2;
    a = 0;
    b = 3;
    delta = 0.9;
    rho = 2;
    om0 = 1000;
    cdi = 0.15;
    days = 252;
    cdiperch = 1.5;
    cdipercl = 0.8;
    rh = cdiperch*((1+cdi)^(days/252)-1);
```

```
rl = cdipercl*((1+cdi)^(days/252)-1);
rb = ((1+cdi)^(days/252)-1);
if (rh-rb)>0;
erh=rh-rb;
else erh=0;
end
if (rl-rb)>0;
erl=rl-rb;
else erl=0;
end
if 2*(rh-rb)>0;
erhh=rh-rb;
else erhh=0;
end
if (rh+rl-2*rb)-erhh>0;
erhl=rh+rl-2*rb;
else erhl=0;
end
if 2*(rl-rb)>0;
erll=rl-rb;
else erll=0;
end
if (rh+rl-2*rb)-erll>0;
erlh=rl+rh-2*rb;
else erlh=0;
end
```

5.2.4 Investor's Maximization problem objective function and its gradient

```
function [f,GG] = probinv(x);
    tese_input;
    syms mf pf omh oml rho e0 e1 e2;
    % Investor utility function
    F=vi(om0,rh,erh,mf,pf);
    G=vi(om0,rl,erl,mf,pf);
    H=vi(omh,rh,erhh,mf,pf);
    K=vi(omh,rl,erhl,mf,pf);
    M=vi(oml,rh,erlh,mf,pf);
    N=vi(oml,rl,erll,mf,pf);
    % Probabilities of high return using optimal effort functions
    A = lambda*(exp(a+b*e0)/(1+exp(a+b*e0)));
    E = exp(a+b*e1)/(1+exp(a+b*e1));
    I = ((lambda-lambda*A)/(1-lambda*A))*(exp(a+b*e2)/(1+exp(a+b*e2)));
    % Investor objective function
    f = -(A*F+(1-A)*G-vi(om0-om0,rb,0,0,0)
```

```
+delta*A*(E*H+(1-E)*K-vi(omh-om0*(1+rh),rb,0,0,0))
+delta*(1-A)*(I*M+(1-I)*N-vi(oml-om0*(1+rl),rb,0,0,0)));
% Partial derivatives
d_f_mf=diff(f,sym('mf'));
d_f_pf=diff(f,sym('pf'));
d_f_omh=diff(f,sym('omh'));
d_f_oml=diff(f,sym('oml'));
d_f_e0=diff(f,sym('e0'));
d_f_e1=diff(f,sym('e1'));
d_f_e2=diff(f,sym('e2'));
% Compute the gradient evaluated at x
GG = [d_f_mf d_f_pf d_f_omh d_f_oml d_f_e0 d_f_e1 d_f_e2];
mf=x(1);
pf=x(2);
omh=x(3);
oml=x(4);
e0=x(5);
e1=x(6);
e2=x(7);
tese_input;
f = eval(f);
GG=eval(GG);
```

5.2.5 Investor's Maximization problem constraints and their derivatives

```
function [c,ceq,GC,Geq] = constr(x)
    tese_input;
    % Endogenous variables
    syms mf pf omh oml rho e0 e1 e2 k;
    % Portfolio Manager utility function
    P = u(om0,mf,pf,erh,rho);
    Q = u(om0,mf,pf,erl,rho);
    S = u(omh,mf,pf,erhh,rho);
    T = u(omh,mf,pf,erhl,rho);
    Vu = u(oml,mf,pf,erlh,rho);
    W = u(oml,mf,pf,erll,rho);
    % Probabilities of high return using optimal effort functions
    A = lambda*(exp(a+b*e0)/(1+exp(a+b*e0)));
    E = exp(a+b*e1)/(1+exp(a+b*e1));
    I = ((lambda-lambda*A)/(1-lambda*A))*(exp(a+b*e2)/(1+exp(a+b*e2)));
    % Portfolio Manager's Problem and its CPOs
    PM=((A*P+(1-A)*Q-k*(e0^2)/2)
    +delta*A*(E*S+(1-E)*T-k*(e1^2)/2)
    +delta*(1-A)*(I*Vu+(1-I)*W-k*(e2^2)/2));
    d_PM_e0=diff(PM,sym('e0'));
```

```
d_PM_e1=diff(PM,sym('e1'));
d_PM_e2=diff(PM,sym('e2'));
% Participation Constraint
c=-((A*P+(1-A)*Q-k*(e0^2)/2)
+delta*A*(E*S+(1-E)*T-k*(e1^2)/2)
+delta*(1-A)*(I*Vu+(1-I)*W-k*(e2^2)/2));
% Incentive Constraints
c0=d_PM_e0-0.00000001;
c1=d_PM_e1-0.00000001;
c2=d_PM_e2-0.00000001;
c3=-d_PM_e0-0.00000001;
c4=-d_PM_e1-0.00000001;
c5=-d_PM_e2-0.00000001;
% Partial derivatives of the constraints
d_c_mf=diff(c,sym('mf'));
d_c_pf=diff(c,sym('pf'));
d_c_omh=diff(c,sym('omh'));
d_c_oml=diff(c,sym('oml'));
d_c_e0=diff(c,sym('e0'));
d_c_e1=diff(c,sym('e1'));
d_c_e2=diff(c,sym('e2'));
d_c0_mf=diff(c0,sym('mf'));
d_c0_pf=diff(c0,sym('pf'));
d_c0_omh=diff(c0,sym('omh'));
d_c0_oml=diff(c0,sym('oml'));
d_c0_e0=diff(c0,sym('e0'));
d_c0_e1=diff(c0,sym('e1'));
d_c0_e2=diff(c0,sym('e2'));
d_c1_mf=diff(c1,sym('mf'));
d_c1_pf=diff(c1,sym('pf'));
d_c1_omh=diff(c1,sym('omh'));
d_c1_oml=diff(c1,sym('oml'));
d_c1_e0=diff(c1,sym('e0'));
d_c1_e1=diff(c1,sym('e1'));
d_c1_e2=diff(c1,sym('e2'));
d_c2_mf=diff(c2,sym('mf'));
d_c2_pf=diff(c2,sym('pf'));
d_c2_omh=diff(c2,sym('omh'));
d_c2_oml=diff(c2,sym('oml'));
d_c2_e0=diff(c2,sym('e0'));
d_c2_e1=diff(c2,sym('e1'));
d_c2_e2=diff(c2,sym('e2'));
d_c3_mf=diff(c3,sym('mf'));
d_c3_pf=diff(c3,sym('pf'));
d_c3_omh=diff(c3,sym('omh'));
d_c3_oml=diff(c3,sym('oml'));
```

```
d_c3_e0=diff(c3,sym('e0'));
d_c3_e1=diff(c3,sym('e1'));
d_c3_e2=diff(c3,sym('e2'));
d_c4_mf=diff(c4,sym('mf'));
d_c4_pf=diff(c4,sym('pf'));
d_c4_omh=diff(c4,sym('omh'));
d_c4_oml=diff(c4,sym('oml'));
d_c4_e0=diff(c4,sym('e0'));
d_c4_e1=diff(c4,sym('e1'));
d_c4_e2=diff(c4,sym('e2'));
d_c5_mf=diff(c5,sym('mf'));
d_c5_pf=diff(c5,sym('pf'));
d_c5_omh=diff(c5,sym('omh'));
d_c5_oml=diff(c5,sym('oml'));
d_c5_e0=diff(c5,sym('e0'));
d_c5_e1=diff(c5,sym('e1'));
d_c5_e2=diff(c5,sym('e2'));
% Gradient of the inequality constraints
GC = [d_c_mf d_c_pf d_c_omh d_c_oml d_c_e0 d_c_e1 d_c_e2;
d_c0_mf d_c0_pf d_c0_omh d_c0_oml d_c0_e0 d_c0_e1 d_c0_e2;
d_c1_mf d_c1_pf d_c1_omh d_c1_oml d_c1_e0 d_c1_e1 d_c1_e2;
d_c2_mf d_c2_pf d_c2_omh d_c2_oml d_c2_e0 d_c2_e1 d_c2_e2;
d_c3_mf d_c3_pf d_c3_omh d_c3_oml d_c3_e0 d_c3_e1 d_c3_e2;
d_c4_mf d_c4_pf d_c4_omh d_c4_oml d_c4_e0 d_c4_e1 d_c4_e2;
d_c5_mf d_c5_pf d_c5_omh d_c5_oml d_c5_e0 d_c5_e1 d_c5_e2];
% Gradient of the equalitiy constraints
Geq = [];
% Solution x
mf=x(1);
pf=x(2);
omh=x(3);
oml=x(4);
e0=x(5);
e1=x(6);
e2=x(7);
tese_input;
% Evaluating constraints at solution x
c = [eval(c) eval(c0) eval(c1) eval(c2) eval(c3) eval(c4) eval(c5)];
GC = eval(GC');
ceq = [];
Geq = [];
```

5.2.6 Checking sufficiency conditions for local optima of the portfolio manager´s problem

```
function h = hess_sign(x);
```

```
tese_input;
syms mf pf omh oml rho e0 e1 e2;
% Portfolio Manager utility function
P = u(om0,mf,pf,erh,rho);
Q = u(om0,mf,pf,erl,rho);
S = u(omh,mf,pf,erhh,rho);
T = u(omh,mf,pf,erhl,rho);
Vu = u(oml,mf,pf,erlh,rho);
W = u(oml,mf,pf,erll,rho);
% Probabilities of high return using optimal effort functions
A = lambda*(exp(a+b*e0)/(1+exp(a+b*e0)));
E = exp(a+b*e1)/(1+exp(a+b*e1));
I = ((lambda-lambda*A)/(1-lambda*A))*(exp(a+b*e2)/(1+exp(a+b*e2)));
% Portfolio Manager obejctive function and its partial derivatives
PM=((A*P+(1-A)*Q-k*(e0^2)/2)
+delta*A*(E*S+(1-E)*T-k*(e1^2)/2)
+delta*(1-A)*(I*Vu+(1-I)*W-k*(e2^2)/2));
d_PM_e0=diff(PM,sym('e0'));
d_PM_e1=diff(PM,sym('e1'));
d_PM_e2=diff(PM,sym('e2'));
d_PM_e0e0=diff(d_PM_e0,sym('e0'));
d_PM_e0e1=diff(d_PM_e0,sym('e1'));
d_PM_e0e2=diff(d_PM_e0,sym('e2'));
d_PM_e1e0=diff(d_PM_e1,sym('e0'));
d_PM_e1e1=diff(d_PM_e1,sym('e1'));
d_PM_e1e2=diff(d_PM_e1,sym('e2'));
d_PM_e2e0=diff(d_PM_e2,sym('e0'));
d_PM_e2e1=diff(d_PM_e2,sym('e1'));
d_PM_e2e2=diff(d_PM_e2,sym('e2'));
% Solution x
tese_input;
mf=x(1);
pf=x(2);
omh=x(3);
oml=x(4);
e0=x(5);
e1=x(6);
e2=x(7);
% Hessian matrix of the portfolio manager problem evaluated at solution x
HH = [d_PM_e0e0, d_PM_e0e1, d_PM_e0e2; d_PM_e1e0,
d_PM_e1e1, d_PM_e1e2; d_PM_e2e0, d_PM_e2e1, d_PM_e2e2];
% Signals of eigenvalues of the hessian matrix
S = sign(diag(eig(eval(HH))));
P=0; % positive eigenvalues
N=0; % negative eigenvalues
% Counting th enumber of negatives and positive values
```

```
for i = 1: max(size(S))
if S(i,i)==1
P = P + 1;
else N = N + 1;
end
end
if P*N>0
h=0;
else if P==0
h=-1;
else h=1;
end
end
```

5.2.7 Non-linear Minimization problem function and print

```
function [best_result, ret_best_resul] = tese_opt;
    tese_input;
    syms mf pf omh oml rho e0 e1 e2;
    % Lower bounds x >= 0
    lb = [0.01,0,om0*(1+rh),10,0,0,0];
    % upper bounds
    ub = [lb(1),0.4,lb(3),lb(4),5,5,5];
    % Specifing the optimization code features
    options = optimset('Display','iter','GradConstr','on','GradObj','on',
    'Hessian','on','Jacobian','on','MaxIter',100,'MaxFunEvals',10000,'TolFun',1e-
14,'TolCon',1e-14);
    format long
    % Initial guess values of the investor's control variables
    mf = ub(1);
    pf = 0.03;%(ub(2)-lb(2))/4;
    omh = ub(3);
    oml = ub(4);
    e0 = 1.3;
    e1 = 1;
    e2 = 0.1;
    x0 = [mf pf omh oml e0 e1 e2];
    % Minimization procedure
    [x,fval,EXITFLAG,OUTPUT,LAMBDA,GRAD,HESSIAN] =
    = fmincon(@probinv,x0,[],[],[],[],lb,ub,@constr,options)
    % Verification of the optimal solution
    mf = x(1);
    pf = x(2);
    omh= x(3);
    oml= x(4);
    e0 = x(5);
```

```
e1 = x(6);
e2 = x(7);
% Portfolio Manager utility function
P = u(om0,mf,pf,erh,rho);
Q = u(om0,mf,pf,erl,rho);
S = u(omh,mf,pf,erhh,rho);
T = u(omh,mf,pf,erhl,rho);
Vu = u(oml,mf,pf,erlh,rho);
W = u(oml,mf,pf,erll,rho);
% Probabilities of high return using optimal effort functions
A = lambda*(exp(a+b*e0)/(1+exp(a+b*e0)));
E = exp(a+b*e1)/(1+exp(a+b*e1));
I = ((lambda-lambda*A)/(1-lambda*A))*(exp(a+b*e2)/(1+exp(a+b*e2)));
pfee=(pf)*(252/days);
r0=rh*A+rl*(1-A);
r0exc=1+((r0-rb)/rb)*(252/days);
Var0=A*(1-A)*(rh-rl)^2;
r1=rh*E+rl*(1-E);
r1exc=1+((r1-rb)/rb)*(252/days);
Var1=E*(1-E)*(rh-rl)^2;
r2=rh*I+rl*(1-I);
r2exc=1+((r2-rb)/rb)*(252/days);
Var2=I*(1-I)*(rh-rl)^2;
[c,ceq,GC,Geq] = constr(x);
[f,GG] = probinv(x);
result_iter(1:7) = x(1:7);
parameters = [k a delta om0 cdi days cdiperch cdipercl b lambda rh rl rb]'
'x0'
x0'
best_result = result_iter'
ret_best_result=[x(1) x(2) x(3) x(4) x(5) x(6) x(7) A E I -f -c(1) c(2) c(3)
c(4) c(5) c(6) c(7)
hess_sign(x) pfee r0exc r1exc r2exc r0 Var0 r1 Var1 r2 Var2 ]'
% saving results of minimization problem
save 'resultados_one' parameters best_result ret_best_result
```

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